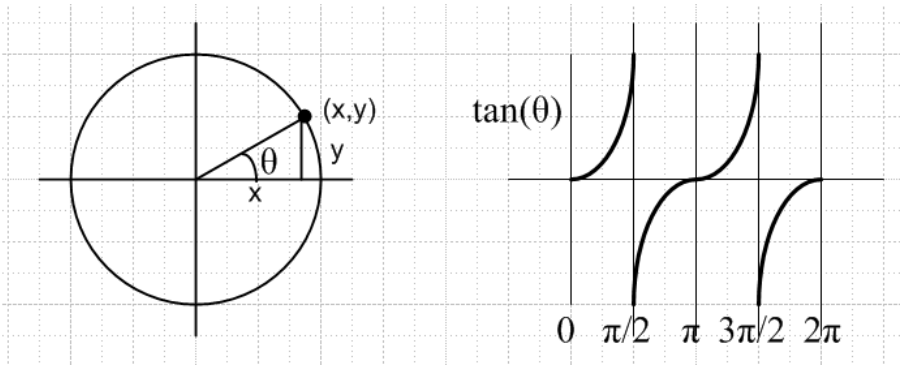
Dealing with arctangent PhL 2.5.11

Motivation: Every time I encounter this confusion, I waste a whole day getting it straightened out again. So just maybe I can somehow "write it down" here where I can find it next time confusion strikes.

tan(θ) π/2 π 3π/2 2π 0

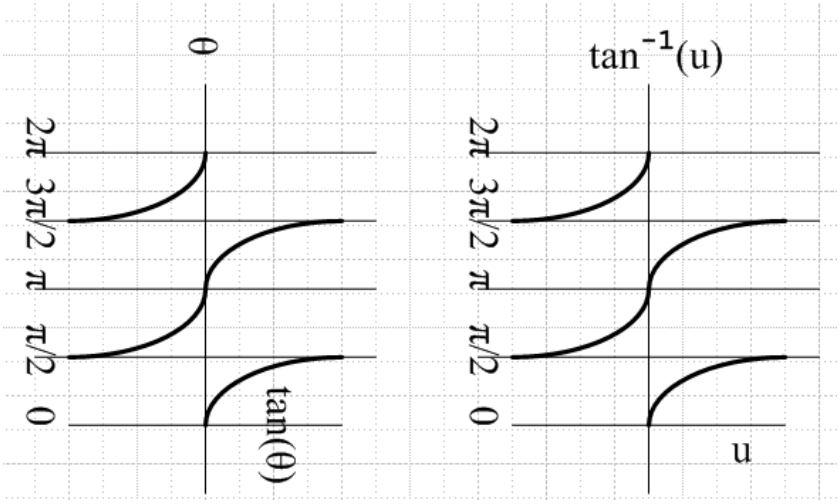
An arctan function which returns θ in (0,2π)

Consider this graphic: $\tan^{-1}(x)$



As θ goes around the circle from 0 to 2π, we trace out the function tan(θ) as shown. No problem.

Now the confusion comes with the arctangent. We can rotate and reflect the picture on the right to get the $\tan^{-1}$ picture and then we relabel it appropriately:



You can see that the resulting graph involves three different "standard branches" of the $\tan^{-1}(u)$ function. At the bottom we have one half of the "principle branch" as defined in Schaum. Given nothing but u = y/x, say, we cannot tell which of these three branches we are on!

If we ignore the *official* principle branches, we could regard the above as just two branches. But things like Schaum and Maple want to use the official principle branch. So we have to do this:

$\tan^{-1}(y/x) = \tan^{-1}_{P}(y/x) + q$

where P means principle branch. and the adder q comes from this table

Quadrant	q	signx	sign y
I	0	1	1
II	π	-1	1
III	π	-1	-1
IV	2π	1	-1

A Maple implementation of this arctan function

The Maple call arctan(y,x) does not do what I am describing here, it is intended for (-π,π) range of θ.

So you have to roll your own function to do this. Here is my Maple code to do this:

```
arctan2Pi := proc(x,y)
    local q;
    if x = 0 and y = 0 then print("arctan2Pi(0,0) error." ); RETURN(0) fi;
    if x = 0 and y > 0 then RETURN(Pi/2) fi;
    if x = 0 and y < 0 then RETURN(3*Pi/2) fi;
    if x > 0 and y = 0 then RETURN(0) fi;
    if x < 0 and y = 0 then RETURN(Pi) fi;
    if x > 0 and y > 0 then q := 0 fi;
    if x < 0 and y > 0 then q := Pi fi;
    if x < 0 and y < 0 then q := Pi fi;
    if x > 0 and y < 0 then q := 2*Pi fi;
    RETURN(arctan(y/x)+q)
  end;
```

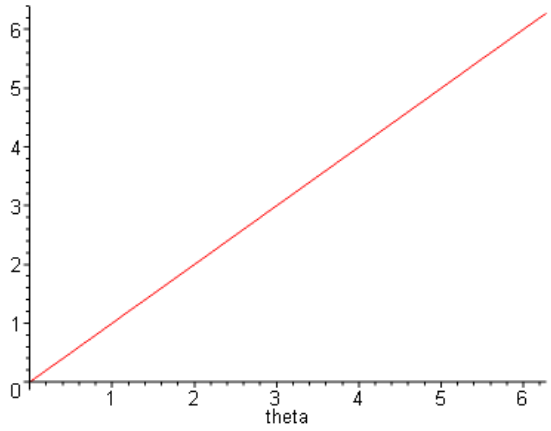
It seems to work. Now, suppose you want to use u which ranges from u_0 to $u_0+2\pi$ as your angle. Then

$u = \theta+u_0$

$u = \text{arctan2Pi}(x,y) + u_0$

Here is a convincing test of the above routine:

```
x := cos(theta):
y := sin(theta):
plot(arctan2Pi(x,y), theta=0..2*Pi);
```



If you know sinθ and cosθ, how to you find θ in range (0,2π) ?

Imagine a little unit radius circle on which we have X = cosθ and Y = sinθ. Then the answer to the question is this:

θ = arctan2Pi(cosθ,sinθ)

This requires the routine above to compute arctan(sinθ/cosθ) which is a division then $\tan^{-1}$, I guess that is not so bad.