The Jigger Problem PhL 2.28.10

The problem here is to find the Green's Function for an oblate hyperboloid upper half where the neck is filled with metal, so the upper half looks like the upper half of a metal "jigger". There is no "lower bloid" to worry about in the jigger problem.

1. The Smythian form and picture **1**

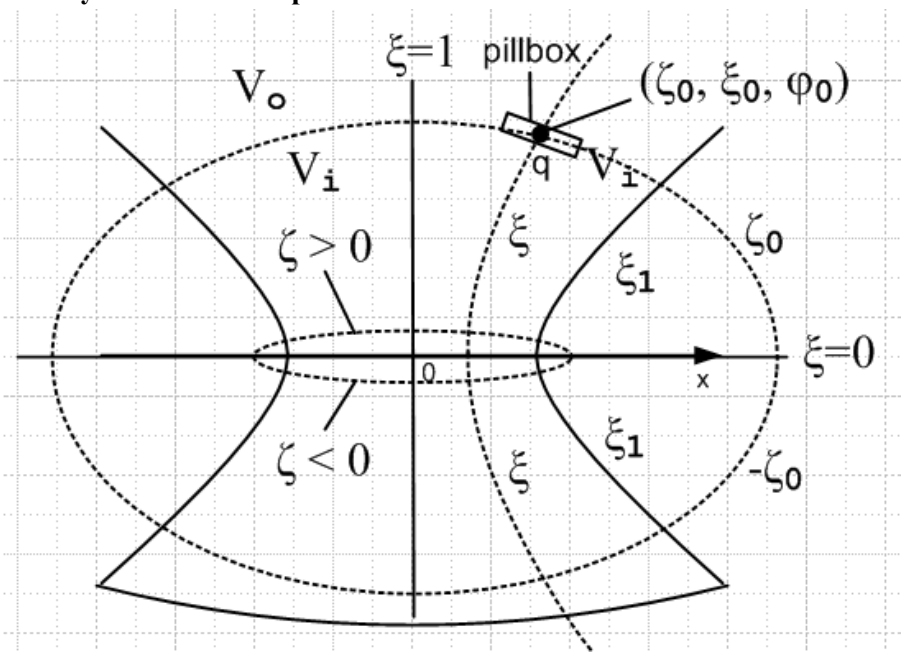
2. Condition V = 0 at the neck which is ζ = 0 ..... **1**

Status after doing all the neck matches: 2

3. Apply Gauss's Law to pillbox around point charge q **2**

4. Final result and comparison with the full bloid solution: **3**

1. The Smythian form and picture



If we follow the text of bloid attempt 4, we find that the Smythian form must be this:

$$V_o(\zeta,\xi) = \Sigma_{nm} [B_{nm}\ p_n^m(\zeta_0) + C_{nm}\ q_n^m(\zeta_0)]\ q_n^m(\zeta)\ P_n^m(\xi) \cos(m\varphi)$$
$$V_i(\zeta,\xi) = \Sigma_{nm} [B_{nm}\ p_n^m(\zeta) + C_{nm}\ q_n^m(\zeta)\ ]\ q_n^m(\zeta_0)\ P_n^m(\xi) \cos(m\varphi) \qquad \text{//upper } \zeta \geq 0$$

This is what it has to *look like* to match at the boundary between V_o and V_i . We can write this as:

$$V(\zeta,\xi) = \Sigma_{nm} [B_{nm}\ p_n^m(\zeta_<) + C_{nm}\ q_n^m(\zeta_<)\ ]\ q_n^m(\zeta_>)\ P_n^m(\xi) \cos(m\varphi) \qquad \text{//upper } \zeta \geq 0$$

2. Condition V = 0 at the neck which is ζ = 0. We have at the neck:

$$0 = V_i(0+,\xi) = \Sigma_{nm} [B_{nm}\ p_n^m(0+) + C_{nm}\ q_n^m(0+)]\ q_n^m(\zeta_0)\ P_n^m(\xi) \cos(m\varphi) \qquad \text{//upper}$$

and from leg prop doc we have

$$p_n^m(0\pm) = \sqrt{\pi}\ 2^{m+1} / [\ \Gamma(1/2 - n/2 - m/2)\ \Gamma(1 + n/2 - m/2)\ ]$$
$$q_n^m(0\pm) = (-1)^m \sqrt{\pi}\ 2^{m-1}\ [\ \Gamma(1/2 + n/2 + m/2) / \Gamma(1 + n/2 - m/2)]$$

so our condition at the neck is this:

$$B_{nm}\ p_n^m(0+) + C_{nm}\ q_n^m(0+) \quad \Rightarrow \quad C_{nm} = -[\ p_n^m(0+)/\ q_n^m(0+)]\ B_{nm}$$

$$[\ p_n^m(0)/\ q_n^m(0)] =$$

$$= 2^{m+1}\sqrt{\pi}\ / [\ \Gamma(1/2 - n/2 - m/2)\ \Gamma(1 + n/2 - m/2)\ ]\ /\ (-1)^m \sqrt{\pi}\ 2^{m-1}\ [\ \Gamma(1/2 + n/2 + m/2) / \Gamma(1 + n/2 - m/2)]$$

$$= (-1)^m\ 2^2 / [\ \Gamma(1/2 - n/2 - m/2)\ \Gamma(1 + n/2 - m/2)\ ]\ *\ [\Gamma(1 + n/2 - m/2)/\ \Gamma(1/2 + n/2 + m/2)]$$

$$= (-1)^m\ 2^2 / [\ \Gamma(1/2 - n/2 - m/2)\ ]\ *\ [1/\ \Gamma(1/2 + n/2 + m/2)]$$

$$= 4\ (-1)^m / [\Gamma(1/2 + n/2 + m/2)\ \Gamma(1/2 - n/2 - m/2)] \qquad \text{// which agrees with Attempt 4}$$

Let's define

$$g_3(n,m) = 1/ [\ \Gamma(1/2 + n/2 + m/2)\ \Gamma(1/2 - n/2 - m/2)\ ]$$

so

$$[\ p_n^m(0)/\ q_n^m(0)] = 4\ (-1)^m\ g_3(n,m)$$

Our relation is then:

$$C_{nm} = -[\ p_n^m(0+)/\ q_n^m(0+)]\ B_{nm} = -4\ (-1)^m\ g_3(n,m)\ B_{nm}$$

Status after doing all the neck matches:

$$V(\zeta,\xi) = \Sigma_{nm} [B_{nm}\ p_n^m(\zeta_<) + C_{nm}\ q_n^m(\zeta_<)]\ q_n^m(\zeta_>)\ P_n^m(\xi) \cos(m\varphi) \qquad \text{//upper} \quad \zeta \geq 0$$

with condition:

$$C_{nm} = -4\ (-1)^m\ g_3(n,m)\ B_{nm}$$
$$g_3(n,m) = 1/ [\ \Gamma(1/2 + n/2 + m/2)\ \Gamma(1/2 - n/2 - m/2)\ ]$$

3. Apply Gauss's Law to pillbox around point charge q.

This section is exactly word for word as in the bloid solution, where we apply only to the upper bloid. The conclusion is that no condition is put on C_{nm} and that

$B_{nm} = (2-\delta_{m,0})\,(q/[4\pi c_1\varepsilon])\;P_n^m(\xi_0)\,\sec[(n+m)\pi/2]\,f(n,-m)/\,K_n^m(\xi_1)$

4. Final result and comparison with the full bloid solution:

$V(\zeta,\xi) = \Sigma_{nm}\,[B_{nm}\;p_n^m(\zeta_<) + C_{nm}\;q_n^m(\zeta_<)\,]\;q_n^m(\zeta_>)\,P_n^m(\xi)\cos(m\varphi)$ //upper $\zeta \geq 0$

with condition:

$B_{nm} = (2-\delta_{m,0})\,(q/[4\pi c_1\varepsilon])\;P_n^m(\xi_0)\,\sec[(n+m)\pi/2]\,f(n,-m)/\,K_n^m(\xi_1)$
$C_{nm} = -4\,(-1)^m\,g_3(n,m)\,B_{nm}$
$g_3(n,m) = 1/\,[\,\Gamma(1/2 + n/2 + m/2)\,\Gamma(1/2 - n/2 - m/2)\,]$

so we write our jigger solution as

$V(\zeta,\xi) = \Sigma_{nm}\,B_{nm}\,[p_n^m(\zeta_<) - 4\,(-1)^m\,g_3(n,m)\,q_n^m(\zeta_<)]\;q_n^m(\zeta_>)\,P_n^m(\xi)\cos(m\varphi)$ //upper $\zeta \geq 0$

We can now compare this to our full bloid result:

$V(\zeta,\xi) = \Sigma_{nm}\,B_{nm}\,[p_n^m(\zeta_<) - 2(-1)^m\;g_3(n,m)\,q_n^m(\zeta_<)]\;q_n^m(\zeta_>)\;P_n^m(\xi)\cos(m\varphi)$ //upper $\zeta \geq 0$
$V(\zeta,\xi) = \Sigma_{nm}\,B_{nm}\,[\quad 2(-1)^m\;g_3(n,m)\;q_n^m(\zeta_<)]\;q_n^m(\zeta_>)\;P_n^m(\xi)\cos(m\varphi)$ //lower $\zeta \leq 0$

So our jigger solution is the same as the upper bloid solution with a factor $2 \rightarrow 4$!!