The Green's for Bloid Maze PhL 2.1.10 last update: 2.28.10

[Note added much later, 12.9.10: I just read all of this Maze doc, it describes a fascinating month-long journey (Feb 2010) by a novice maze-walker who set out with inadequate knowledge. I did not understand the key SL fact that you have to have the right two coordinates be oscillatory for your geometry of interest. For the bloid, that means you need the $Q_{ip-1/2}(i\xi)$ Mehler type functions, but I knew nothing about that at the time. I kept getting hints in this direction, but I did not understand them. Probably there is in fact a QQ orthogonality theorem for these functions, and the large ξ behavior (that was the bane of this entire voyage) is tamed by the form n = ip-1/2. I should redo this entire problem at some point using the Mehler stuff. I think if you solve the problem that way, then close the ip contour, you might get the "cone type" solution I came up with. It is amusing (now) to see how the wrong Smythian form will *always* have a problem somewhere. It *has* to show a problem because there can only be one correct solution.]

My original Green's for hyperboloid problem I now call the open bloid problem, and if there is a plate in the hole region, I call it the jigger problem. The idea of a jigger problem only arose as I was writing these review notes.

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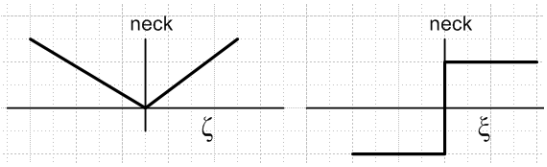
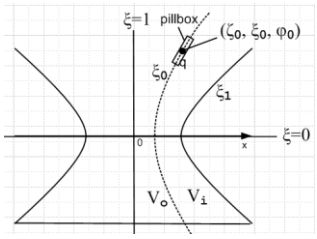
The hedge maze of problem solving.

This problem provides such a good and painful example of "what happens" in problem solving. I like the maze analogy. The maze is built of cement floor and walls with a cement ceiling to close it off, adding to the isolated painful atmosphere. Lighting is very dim, so you cannot see the tiny terminal signs which say "Wrong Way, Dead End" until you get right up to them, and only then can you start to backtrack your last path segment. You don't know how many decision points you should back through, at least one. The good news is that there is a written record of the turns you made, so you can always backtrack to any point you were previously at, and in fact you can back up all the way to the starting point and just start over. For problems that have solutions, there is always a successful threading path through the maze (perhaps several), but finding such a path can take at first several hours, then several days, then maybe several weeks or more. Perhaps this is what "research" is like, I don't have much experience doing such, but I certainly am gaining it now. So we can review the maze path I have taken since starting this problem.

Wrong Turn #1. At the opening door of the maze I took an *immediate wrong turn* which I will now explain. I selected the path which said "use the same oblate coordinate parameterization that you used in the oblate spheroid problem that you did earlier". So I had spheroids with constant ζ, and then each hyperboloid had a discontinuous ξ at the surface of the "neck" of the bloid. Here are two graphs showing how the coordinates ζ,ξ vary as you slide down on a hyperbola like the one shown below, moving from the upper region to the lower region





In the spheroid problem, this horrible behavior occurred inside the spheroid which was a "don't care" part of that problem. Here, the behavior occurs right at the neck of the bloid where everything is supposed to be super smooth. So allowing "bad behavior" of the coordinates in a smooth active region of the problem was my first whopping mistake. At least ζ is continuous, but its derivative is not. One can no doubt solve the problem with these coordinates, but a lot of extra work I think would be needed -- and there is already plenty of work available without having 2 of your 3 coordinates being singular at the neck.

Valid Work Done Despite. I continued with these bad coordinates, but was able to do some "productive work" with them, only because the Smythian Forms I built don't *a priori* assume any particular parameterization of the oblate coordinates. That is in the details later when you try to use the forms.

(re doc: " Green's Function for an Oblate Bloid ") In **Section 1** I constructed a form of the solution which satisfied the boundary conditions. Then in **Section 2** I compute $\partial_{\boldsymbol{\xi}}$ on my two forms, took the difference, and then in **Section 3** I did the usual Gauss's Law pillbox application and arrived at the characteristic double-delta function equation all these Smythe Form Green's problems lead to. Since all my work was in the upper bloid section, I stayed away from the "problem area" of the neck, so I think all these results were OK and independent of parameterization. After all, in both the spheroid and bloid parameterizations, the upper bloid region is treated the same way. The double delta function arises from treatment of the Green's point charge as a surface charge density. So here is where I stood at this point:

Smythian form:

$$V_o(\zeta,\xi,\varphi) = \Sigma_{nm} E_{nm} [P_n^m(\xi_1) Q_n^m(\xi_0) - Q_n^m(\xi_1) P_n^m(\xi_0)] P_n^m(\xi) Q_n^m(j\zeta) e^{im\varphi}$$
$$V_i(\zeta,\xi,\varphi) = \Sigma_{nm} E_{nm} [P_n^m(\xi_1) Q_n^m(\xi) - Q_n^m(\xi_1) P_n^m(\xi)] P_n^m(\xi_0) Q_n^m(j\zeta) e^{im\varphi}$$

Double-delta equation which exists at the boundary $\xi = \xi_0$

$$(q/\varepsilon c_1)\, \delta(\varphi-\varphi_0)\, \delta(\zeta-\zeta_0) = \Sigma_{nm} (-1)^m [(n+m)! / (n-m)!] E_{nm} Q_n^m(j\zeta) P_n^m(\xi_1) e^{im\varphi}$$

In **Section 4** I then used $\int d\varphi\, e^{-im\varphi'} e^{im\varphi} = 2\pi\, \delta_{m',m}$ to remove the φ delta function and this left me with the following single-delta equation:

$$(q/\varepsilon c_1)\, \delta(\zeta-\zeta_0) = 2\pi\, \Sigma_n (-1)^m [(n+m)! / (n-m)!] E_{nm} Q_n^m(j\zeta) P_n^m(\xi_1) e^{im(\varphi-\varphi 0)}$$

The whole goal of the problem is to find viable coefficients E_{nm} and then the problem is solved, you plug those into the Smythian form and hope that things converge.

The obvious next step requires some sort of Q function orthogonality on some ζ interval. Without such an orthogonality rule, I see even now no way of obtaining a simple expression for the E_{nm} coefficients. We need something like this to "break the n sum", exposing an isolated E_{nm}. But no book shows any such orthogonality integral rule. For P functions yes, for Q functions of imaginary argument, no. In other words, we need a "diagonalization" here of this nm "integral equation".

<u>Note added 12.9.10</u>. Here are the atomic forms for oblates expressed in current doc symbols

$$\text{(1)} \quad \overset{\text{osc}}{[P_n^m(\xi), Q_n^m(\xi)]}\ \overset{\text{expo}}{[P_n^m(i\zeta), Q_n^m(i\zeta)]}\ \overset{\text{osc}}{[\sin(m\varphi),\cos(m\varphi)]}$$

$$\text{(2)} \quad \overset{\text{expo}}{[P_{i\tau-1/2}^m(\xi), Q_{i\tau-1/2}^m(\xi)]}\ \overset{\text{osc}}{[P_{i\tau-1/2}^m(i\zeta), Q_{i\tau-1/2}^m(i\zeta)]}\ \overset{\text{osc}}{[\sin(m\varphi),\cos(m\varphi)]}$$

I was using the first form where ζ is expo. THAT was the problem. The second form is what we need, but I did not know about the second form at this time.

Wrong Turn #2. Made anxious by this fact, I was at least encouraged by the following: There is presumably some kind of Sturm-Liouville problem that could "go on" in the ζ dimension, and this SL problem somehow is producing the $Q_n^m(j\zeta)$ functions as its eigenfunctions, and these should somehow be orthogonal, and that is the path to finishing the problem. I knew that $Q_n^m(j\zeta) \sim \zeta^{-n}$ asymptotically so the Q functions were the correct limit-point finite-s-norm solutions. Also, the parameterization I was using at this time was $\zeta \geq 0$ everywhere, so I could ignore the unpleasant fact that $Q_n^m(j\zeta)$, at least as $Q_1(j\zeta)$, blows up as ζ^1 in the negative ζ direction. This unpleasant fact returns below, but not right away.

So at this point I looked into this SL problem. I decided that $(j0,+j\infty)$ was the right "interval" in z for this problem. At the singular end, I knew from SL theory that you just want a solution that is integrable, and I presumed that was the Q function $Q_n^m(j\zeta)$. At the 0 end, I needed some "homo BC". With no justification at all, I assumed the homo BC that $Q_n^m(j0+) = 0$. To my great pleasure, this led to a "spectrum" for n (discrete and finite) and it led to my desired orthogonality condition. I did some Maple work and some analytic work and convinced myself that the orthogonality problem was viable:

$$(u^{\mu}_{\nu}, u^{\mu}_{\nu'}) = \int_0^{\infty} d\zeta \; Q_{\nu}^{\mu}(i\zeta)^* \, Q_{\nu',}^{\mu}(i\zeta) = K_{\nu}^{\mu} \, \delta_{\nu,\nu'}$$

$\nu = \mu - N \qquad N = 2,4,6... \qquad$ but stops at last N such that $\nu > -1/2$

Just what the doctor ordered! I wrote all this up in **Sections 5 and 6**. And then I wrote my "final answer" to the problem in **Section 7.** [So what I was thinking was this: instead of having the φ and ξ problems be the SL dimensions, let's try having φ and ζ be the SL dimensions. Then n is determined not by ξ to be integers, but by ζ. Of course this leads to a conflict with the "bad parameterization" because then you need n = integers for $P_n^m(\xi)$ to be finite at ξ = -1, *and* you need the ζ spectrum whatever it is.]

But then in **Section 8** I discovered my wrong turn #2. By imposing the BC that $Q_n^m(j0+) = 0$, a quick glance at the Smythe form shows that this assumption causes the potential to be uniformly 0 on the entire disk that fills the neck of the bloid! I call this region "in the hole" of the bloid. I felt (and feel now) this was unacceptable on physical grounds. For example, if the point charge were put very close to this hole plane, perhaps a little off axis, the potential close to the point charge won't be V=0 in the hole plane, but will be V of a point charge close to the charge.

So my SL homo BC, though giving a reasonable "SL problem", was giving me a non-physical solution to the Green's Bloid problem. Better to say, was giving me a *no* solution to the bloid problem. It might be the solution of a *different* problem where the neck is filled by a metal plate so V=0 there [jigger]. I guess I have inadvertently solved that problem along the way, maybe I will comment more on that problem below. This different problem of course only involves half the bloid, so the issue of parameterization of the coordinates is a non issue.

So there I was, at the end of another maze path staring at a sign "Wrong Way, Dead End".

Pause to consider that different problem which I might call the "jigger" problem. [This jigger problem is not discussed in the raw notes doc.] This is the problem with a metal plate going across the hole. I thought this is the problem that my "final answer" is for, and here is that final answer:

$$V(\zeta,\xi,\varphi) = (q/[2\pi c_1 \varepsilon]) \, \Sigma_{nm} \, (1/K_n^m) \, (-1)^m \; [(n-m)!/(n+m)!] \, e^{im(\varphi-\varphi_0)}$$

$$* \; [Q_n^m(\xi_<) - \{ Q_n^m(\xi_1)/P_n^m(\xi_1)\} P_n^m(\xi_<)] \, P_n^m(\xi_>) \, Q_n^m(j\zeta) \, [Q_n^m(j\zeta_0)]^*$$

$= g(\zeta,\xi,\varphi \mid \zeta_0,\xi_0,\varphi_0)$ for half-hyperboloid with metal plate in the hole

$$\xi_> = \max(\xi,\xi_0) \qquad \xi_< = \min(\xi,\xi_0) \qquad \Sigma_{nm} = \Sigma_{m=2}^{\infty} \; \Sigma_{n=(0/1)}^{m-2}$$

where the K_n^m are those Q function orthogonality factors which at least can be calculated though I had no closed form for them.

I want to examine this solution now from a "quantum numbers" point of view. We of course know that the φ problem restricts μ = m to be integers, 0,±1, ±2 etc. These are the only "allowed values" but of course maybe not all of the values will be viable when we study the "separation constant cross-linkage" to the functions of the other coordinates. That is the only study of the φ dimension that we need do. At least something is simple in oblate coordinates.

Next, we consider the $P_n^m(\xi)$ type function with integer m. In our jigger problem, since we are dealing only with the upper half bloid, we deal with ξ in the range (0,1). I looked closely at this situation in my

"Legendre with z=1 cut..." doc, and the conclusion is that there are no restrictions at all on the value of n. That is to say, no general complex value of n causes $P_n^m(\xi)$ to diverge, regardless of m. However, there are certain integer values of n for which $P_n^m(\xi) \equiv 0$. Those values are $-m < n < m$ (only when $m > 0$, this is Theorem 2 of leg prop). For such values, n is "too small" for the corresponding m in the classical sense we all know about.

Now, the SL spectrum we got above was $n = m-N$ $N = 2,4,6...$ with $n \geq -1/2$. So n cannot be a negative integer because such a negative integer would result in $Q_n \sim \zeta^{-n-1}$ which then blows up in the SL normalization integral. Thus, we only have spectrum when m = positive integer.

Wrong Turn #3 in the Jigger Problem. So suddenly we see that something is wrong even with this jigger problem solution. We consider our Smythian form:

$$V_o(\zeta,\xi,\varphi) = \Sigma_{nm} E_{nm} [P_n^m(\xi_1) Q_n^m(\xi_0) - Q_n^m(\xi_1) P_n^m(\xi_0)] P_n^m(\xi) Q_n^m(j\zeta) e^{im\varphi}$$
$$V_i(\zeta,\xi,\varphi) = \Sigma_{nm} E_{nm} [P_n^m(\xi_1) Q_n^m(\xi) - Q_n^m(\xi_1) P_n^m(\xi)] P_n^m(\xi_0) Q_n^m(j\zeta) e^{im\varphi}$$

In the "outer" region V_0 we have the product $P_n^m(\xi) Q_n^m(j\zeta)$ where m = integer. The only allowed terms have m being positive integers starting with m = 2. Then for each such m, we have n = m-2,m-4.....(0 or 1). But these integers are exactly a subset of those for which $P_n^m(\xi) = 0$, because for these values of n, we have n being "too small" for its m (and $m > 0$). Therefore, for every single term in our V_o sum, $P_n^m(\xi) Q_n^m(j\zeta) = 0$ and therefore our entire $V_o = 0$!!! So here we are once again: "Wrong Way, Dead End". And this is for a problem which I think is *simpler* than the open bloid problem.

Current Status [2.1.10 or so]. I now think I should focus on this simpler jigger problem before continuing on the open bloid problem. The reason is that in this simpler problem: (0) The jigger problem avoids the issue of which oblate parameterization we select; (1) we can ignore issues of cuts in the Legendre function $Q_n^m(j\zeta)$ at $\zeta = 0$, so we use the official $Q_{0,0L}$ Bateman P and Q functions on the 0,0L sheet, and we bypass that whole world of confusion; (2) we are facing the Q orthogonality issue squarely with a well-defined SL problem in the ζ dimension, $Q_n^m(j0+) = 0$; (3) If I cannot solve the jigger problem, then something very basic must be wrong, such as the Smythe form is not appropriate. That would be good to find out.

What happened in Section 8. As noted above, I discovered that my "final solution" was causing V = 0 everywhere on the hole disk. I then moved to the better coordinate parameterization for ζ and ξ . I argued that $Q_n^m(j\zeta)$ is still an "atom" factor if we let ζ go negative as required in the new mode. At this point, I did not realize it, but I was talking about taking ζ "down through the cut", and I did not realize that this was going to damage large ζ behavior. I went on to talk about the value $Q_n^m(0)$ which was correct since things are now continuous in the hole. What this is of course is

$$Q_n^m(0) = Q_n^m(0)_{0,0R} = Q_n^m(0+)_{0,0L} \qquad // \; Q_n^m(0-)_{0,0L} \text{ no longer relevant}$$

Then in this new viewpoint, I considered a Smythian form with E_{nm} for $\zeta>0$, and F_{nm} for $\zeta<0$. I then examined the implications of matching the potential and all three of its derivatives in the hole. Here is a what I found:

potential match: $E_{nm} Q_n^m(0) = F_{nm} Q_n^m(0)$

ξ gradient match:	$E_{nm}\, Q_n^m(0) = F_{nm}\, Q_n^m(0)$	$\partial_n = (1/h_1)\,\partial_\xi$	$h_1 = c_1\xi / \sqrt{1-\xi^2}$
φ gradient match:	$E_{nm}\, m\, Q_n^m(0) = F_{nm}\, mQ_n^m(0)$	$\partial_n = (1/h_3)\,\partial_\varphi$	$h_3 = c_1 \sqrt{1-\xi^2}$
ζ gradient match:	$E_{nm}\, Q_n^m{}'(0) = F_{nm}\, Q_n^m{}'(0)$	$\partial_n = (1/h_2)\,\partial_\zeta$	$h_2 = c_1\, \xi$

I knew we could not have all $Q_n^m(j0) = 0$ for all n,m in our sum since this causes $V_o = 0$ in the hole. So in "where do we go from here" I decided to just set $F_{nm} = E_{nm}$ as a simple solution to all the matches.

Section 7A is then a continuation of the bloid solution with this new ξ,ζ parameter mode.

In **Idea #1** I thought "let's just forget about $Q_n^m(0) = 0$, and instead think of $(-i\infty, i\infty)$ as our SL interval. I conjectured that this might give a continuous n spectrum (without justification) and noted that if that were the case, we could get $\delta(n-n')$ in our orthogonality and that would be hard to deal with. I called it a "measure problem". *In retrospect* (see Idea #2) for this SL interval, you have to use the $Q_n^m(j\zeta)_{0,0R}$ functions, but they blow up on southern rays so I think there are no finite s norm solutions at all for this SL interval.

So in **Idea #2** I realized that $Q_n^m(j\zeta)$ was really "going through the cut". This led to a long digression in "Legendre with z=1 cut taken to the right.doc" and I was able to understand what going through the cut meant. I then realized that on rays below the cut, $Q_n^m(j\zeta)$ was not going as ξ^{-n-1} but like ξ^n, and this was ruining my entire SL problem. Function $Q_1(z)$ served as a good prototype. Maze path dead end. [Side comment added later: notice that n = -1/2 gives $\xi^{-1/2}$ going up or down, as in n = ip - 1/2.]

Idea #3 (pasted in later, originally written here, so out of temporal order) was go back and use the well behaved $Q_\nu^\mu(z)_{0,0L}$ functions, and choose ν = odd only, which makes $Q_\nu^\mu(z=0\pm)$ both equal, which lets us get away from the issue of requiring $Q_\nu^\mu(0) = 0$, which gave V = 0 on the hole plane (jigger problem). But then when I required continuity of the *derivative*, that in turn forced $Q_\nu^{m+1}(0) = 0$ and we came right back to the jigger problem. So this idea flopped, another maze path blockage.

My last gasp was then **Idea #4** where I tried doing everything as I was doing, except use $Q_n^m(j|\zeta|)$ to get a fix on the lower ray blowup problem. While grinding through this approach, I was forced to sharpen certain facts about P and Q functions which then got added to the Leg prop doc as Theorems. I then ground through the usual steps. Having V be continuous at the hole was automatic now. But I also needed V' to be continuous, to avoid "charge" appearing in the hole, and this meant I needed $Q_\nu^m{}'(0+) = 0$. This requirement then forced a certain n,m spectrum. But then, just as with the jigger problem above where we had $Q_\nu^m(0+) = 0$, we end up with the $Q_n^m(j\zeta)$ spectrum being orthogonal to the $P_n^m(\xi)$ spectrum, and we end up with $V_0 \equiv 0$ everywhere. Another dead end in the maze! Probably this failure is closely related to the Idea #3 failure, but I am not going to dwell on the connection.

This brings us to the end of the raw notes "Green's Function for an Oblate Bloid.doc" [non cone method]

Summary what I learned about the 0,0R sheet Legendre functions.

Part I:

I referred to the usual Bateman sheet as 0,0L where we have winding number 0 about both z = ±1 branch points, and the cuts are pulled off to the Left. I learned that if you want to "travel through the cut" -- which is something Smythe did in his "plate with hole in E field" problem -- you need to use the 0,0L function in the upper half plane, and the 0,1L function in the lower half plane, and this combination I referred to as the 0,0R plane, R being meaning just the z=+1 cut is pulled to the right. This frees up the entire (-1,1) z axis region, and in particular z = 0 which was the point I had to make my Q function pass through. If you just think about it, if you start on 0,0L and pass through the cut say near z = 0, you are in fact moving to the 0,1L sheet. Then as you rotate the z=1 branch cut CCW you simply *reveal* this sheet, and make it become the lower half plane of the 0,0R sheet.

Once this was understood, I wanted to figure out how to compute the P and Q functions on this new sheet 0,1L. This led to a lot of pain, but it is all doable. The work is in located here

\Math\ODEs and Special Functions\Legendre Functions\ Legendre with z=1 cut taken to the right.doc

I kept using the function $Q_1(z) = (z/2)\ln[(z+1)/(z-1)] - 1$ as my prototype for study. You can see the cut right there. As you move down through the cut, the phase of (z-1) gains $e^{+i2\pi}$ because you are doing your single CCW wind, and this creates a "new term" which is $-i\pi z$ in the function. That is to say,

$$Q_1(z)_{0,1L} = Q_1(z)_{0,0L} - i\pi z$$

which in turn tells us that

$$Q_1(z)_{0,0R} = Q_1(z)_{0,0L} \qquad \mathrm{Im}(z) > 0$$
$$Q_1(z)_{0,0L} = Q_1(z)_{0,0L} - i\pi z \qquad \mathrm{Im}(z) < 0$$

The $-i\pi z$ adder exactly compensates for the discontinuity in $Q_1(z)_{0,0L}$ as you pass through the cut. However, this same extra term "wrecks" the large z behavior of the Q function!! Instead of being $Q \sim z^{-1-1}$ along an outgoing ray, you now have $Q \sim z^{+1}$ along an outgoing ray in the lower half plane (while in the upper half plane you still have $Q \sim z^{-1-1}$). This of course is a disaster for the SL program I had been working on, at least for this particular Q function, and gave a hint of problems to come with the general Q function.

Nevertheless, this exercise did explain very well the behavior of the following function:

$$Q_1(i\zeta)_{0,0R} = -1 + \zeta \cot^{-1}(\zeta)$$

which appeared in the Smythe hole problem work. If you plot this function, it does in fact blow up as ζ^1 if you go down, but decays as ζ^{-2} if you go up. Going down, $\cot^{-1} \to \pi$ so you get $\sim \pi\zeta$ which is our $-i\pi z$ adder thing shown above. Going up $\cot^{-1} \to 1/\zeta - 1/3\zeta^3$ giving $-1 + \xi(1/\zeta - 1/3\zeta^3) = -1 + 1 - 1/3\zeta^2 = -1/3\zeta^2$ so we do get the predicted ζ^{-2} decay.

I was able to get the $Q_1(i\zeta)_{0,0R} = -1 + \zeta \cot^{-1}(\zeta)$ expression by parameterizing the various angles in the log form as apropos for the 0,0R sheet. When you can just "write out" a function like this in ζ, then "going down through the cut" just means you take the exact same functional form as the continuation through the cut. There are no tricky (z-1) factors whose angles you have to worry about any more.

So this was a good payoff, to understand the meaning of this $-1 + \zeta \cot^{-1}(\zeta)$ function, and to understand why it was violating the Q asymptotic behavior on a down ray.

Part II:

I was hoping that there would be some "spectrum" for which these new Q functions $Q_\nu^\mu(z)_{0,0R}$ would decay in both the up and down directions. [n = ip - 1/2 , 12.9.10] I thought of this as "killing off" the blow up in the down direction. Before I could try to kill this thing off, I had to find out how to even *write* the general Q function on the 0,0R sheet. Basically, to do this, I had to use the famous formula

$$e^{-i\pi\mu}Q_\nu^\mu(z)_{0,0L} = \csc(\pi\mu)\,(\pi/2)\,\{\,P_\nu^\mu(z)_{0,0L} - \Gamma(\nu+\mu+1)/\Gamma(\nu-\mu+1)P_\nu^{-\mu}(z)_{0,0L}\,\}$$

to get

$$e^{-i\pi\mu}Q_\nu^\mu(z)_{0,1L} = \csc(\pi\mu)\,(\pi/2)\,\{\,e^{-i\pi\mu}\,P_\nu^\mu(z)_{0,0L} - \Gamma(\nu+\mu+1)/\Gamma(\nu-\mu+1)\,e^{+i\pi\mu}\,P_\nu^{-\mu}(z)_{0,0L}\,\}$$

where you see the new phases which arise from "winding" the P functions using an F form for these functions that is valid in a radius 2 disk about the z = 1 branch point. The new phases cause the RHS to "no longer be a Q function" in the 0,0L sense, so all bets were then off on the large z behavior of this thing. [But it is a solution of the Legendre ODE.]

So, I then expended a lot of effort to determine just what the large z behavior was of this new Q function on the 0,1L sheet. My conclusion was this:

$$\lim_{z\to\infty} e^{-i\pi\mu}Q_\nu^\mu(z)_{0,1L} = -i\sqrt{\pi}\;2^\nu\;[\Gamma(\nu+1/2)/\,\Gamma(1+\nu-\mu)]\,z^\nu$$

For the Q_1 function this predicts $-i\sqrt{\pi}\,2^1\Gamma(3/2)z^1 = -i\sqrt{\pi}\,2^1\,(\sqrt{\pi}/2)\,z^1 = -i\pi z$ which I knew was the right answer. So here we see the general Q function blowing up as z^ν and ruining my SL efforts in ζ. My only exit door [?] was to require a spectrum to get poles in $\Gamma(1+\nu-\mu)$ which is to say $\nu = \mu-1,\mu-2....$ I had to make a few assumptions here about lower terms, but it is clear that only such a spectrum would kill off the offending z^ν term.

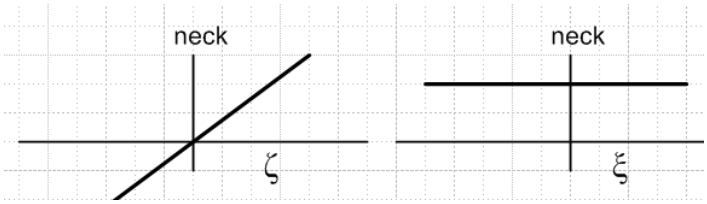
[One fact I did learn is this: all those formulas which appear in Bateman are only for the 0,0L Legendre functions. They all get modified for 0,0R functions -- we just saw an example above. Remember that the 0,0R functions are functions of general z, same as the 0,0L are, and are not the same as Bateman's special "on the cut" functions which are defined only on real x in (-1,1).]

I then suddenly looked up and saw another ""Wrong Way, Dead End" sign staring me in the face. I was back at **Wrong Turn #3** for the jigger problem. With this spectrum I just found for the 0,0R Q functions, the products $P_n^m(\xi)_{0,0L}\,Q_n^m(j\zeta)_{0,0R}$ all vanish! The only allowed spectrum for Q was where P = 0. It's either this, or don't restrict the spectrum and have Q blow up and SL not fly.

So after consuming some very major energy figuring out the 0,0R sheet Q functions, the conclusion was that you cannot make it work!

Comment on the better parameterization of coordinates ζ and ξ .

Up to this point, I never even got to the issue of using the better "bloid style" ζ and ξ parameterization. Had the $Q_{0,0R}$ functions been well behaved, I would have activated the alternate parameterization in which ζ changes sign through the hole, and ξ is the same for both upper and lower bloid labels. The corresponding picture would have been this, which can be compared to the one shown above



This way to handle the coordinates is obviously much more friendly "in the hole region". As noted earlier, you would never use this for a spheroid problem where you want the entire spheroid to have the same ζ. Here we are in effect putting two signs on each spheroid.

New Current Status Feb 1, 2010.

As I think about the jigger problem (walking to Smith's) I can sort of see the problem with trying to get the $Q_n^m(j\zeta)$ functions to decay far from the neck, and then come in for a landing with $Q_n^m(j0+) = 0$. This is sort of like asking r^{-n-1} to come in and be finite at the origin in a spherical coordinates problem. So one idea I got was to put another boundary surface at $\zeta = \zeta_0$. This would be a spheroidal surface. Above it you would really have just Q functions, and below it you would try a linear combination of Q and P functions.

But then in the upper bloid we have now 4 regions to worry about, and two sets of matching to do. I could imagine doing a lot of work in this direction and then finding in the end that it won't fly.

Idea of using "the cone method". Then on that same walk today, I got what I think is the right idea (but of course it could another Wrong Way idea). I think my Smythe form really is non-viable. This form uses a certain linear combination trick to get V = 0 on the bloid surface, similar to the trick used in the spheroid problem. We have integer m, and then the spectrum of n is controlled by the ζ problem. But in my form, I could not get a viable spectrum! I remembered how I did the cone Green's problem in spherical coordinates, and I see now that that is probably the right thing to do here. You then take your n spectrum from the $P_n^m(\xi)$ functions by requiring n to be the zeros of $P_n^m(\xi_1) = 0$. Although our shape here is not a cone, this equation makes V = 0 on the bloid, the difference is ξ versus z. I guess I missed this point. The bigger reason for this approach is a winner: somehow as you move the Green's charge farther away, the jigger solution has to morph into the cone solution! And in the cone solution we used this method of getting the n spectrum.

So I am basically proposing that we "start over" and find a new Smythian form that uses this method of getting V = 0 on the metal bloid, then process that and see where it leads. Basically, my original Smythian form was "not capable of" providing a solution to this problem. It was a linear combination of the correct atoms, but I could not make it meet the BC's. So see " Green's Function for an Oblate Bloid using cone method.doc"

Comments Feb 3, 2010. I have a new cone method document and am about to try the problem there. Assuming if works there, something needs be said about why one form works and another does not.

So what was "wrong" with my bloid-surface-boundary Smythian forms?

$$V_o(\zeta,\xi,\varphi) = \Sigma_{nm} E_{nm} [P_n^m(\xi_1) Q_n^m(\xi_0) - Q_n^m(\xi_1) P_n^m(\xi_0)] P_n^m(\xi) Q_n^m(j\zeta) e^{im\varphi}$$

$$V_i(\zeta,\xi,\varphi) = \Sigma_{nm} E_{nm} [P_n^m(\xi_1) Q_n^m(\xi) - Q_n^m(\xi_1) P_n^m(\xi)] P_n^m(\xi_0) Q_n^m(j\zeta) e^{im\varphi}$$

If I used $Q_n^m(j\zeta)_{0,0L}$, I could not find a SL problem in ζ which would give orthogonality, so I could not solve for the E coefficients. Furthermore, if I chose an n,m spectrum such that this Q was continuous in the neck, then the Q spectrum was orthogonal to the $P_n^m(\xi)$ spectrum giving no spectrum at all. When I later tried $Q_n^m(j|\zeta|)_{0,0L}$, I rescued continuity in the neck, but then continuity of the ζ derivative led to the same "no spectrum" result. When I tried, $Q_n^m(j\zeta)_{0,0R}$, then Q blew up on lower bloid rays. So I think the main problem with this approach is with the $Q_n^m(j\zeta)$ function having conflict at ∞ and 0. Of course m was integers in the first place, and then all my SL approaches in ζ required n have certain integer values as well. Then the $P_n^m(\xi)$ function was sort of the "cathode follower" just taking the n values it was handed by the SL ζ problem.

What is the moral of the story here? Just because you assemble a set of "atoms" and meet some of the BC's, that does not mean this assembly is the right assembly. My form was designed mainly to meet the V=0 BC on the bloid wall, and to meet the Gauss pillbox "BC" around the Green's point charge. I thought I was meeting the decay BC at large ζ, but that kept interacting with the various continuity BC's at the neck where the coordinate ζ = 0. You have to keep in mind that certain "boundary conditions" are that the potential be continuous, and that its gradient be continuous where there is known to be no surface charge! These are sort of "hidden boundary conditions" that you don't at first think about, whereas the V=0 on the wall, and decay at infinity BC's are always highly "visible".

All my efforts to "find a spectrum" for n,m were forced to hinge on making certain Gamma function arguments vanish, and that always led to an n spectrum as a subset of the integers, and that in turn always led to a contradiction so there was "no spectrum". If the cone solution is the right method, then we learn that the n spectrum is NOT integers, but is an infinite set of strangely spaced real values. So my underlying building block "atoms" were more or less right, but the spectrum of n was very wrong. I was never going to *find* such a spectrum by messing with the gamma functions.

So here is little comparison:

My original form: (inside and outside in ξ direction)

$$V_o(\zeta,\xi,\varphi) = \Sigma_{nm} E_{nm} [P_n^m(\xi_1) Q_n^m(\xi_0) - Q_n^m(\xi_1) P_n^m(\xi_0)] P_n^m(\xi) Q_n^m(j\zeta) e^{im\varphi}$$
$$V_i(\zeta,\xi,\varphi) = \Sigma_{nm} E_{nm} [P_n^m(\xi_1) Q_n^m(\xi) - Q_n^m(\xi_1) P_n^m(\xi)] P_n^m(\xi_0) Q_n^m(j\zeta) e^{im\varphi}$$

The cone method form: (inside and outside in ζ direction)

$$V_o(\zeta,\xi) = \Sigma_{nm} A_{nm} [Q_n^m(j\zeta_0) - \{ Q_n^m(j0+)/ P_n^m(j0+)\} P_n^m(j\zeta_0)] Q_n^m(j\zeta) P_n^m(\xi) e^{im\varphi}$$
$$V_i(\zeta,\xi) = \Sigma_{nm} A_{nm} [Q_n^m(j\zeta) - \{ Q_n^m(j0+)/ P_n^m(j0+)\} P_n^m(j\zeta)] Q_n^m(j\zeta_0) P_n^m(\xi) e^{im\varphi}$$

In the first form, we have $Q_n^m(j\zeta)$ as our "common" factor which led to so much trouble. In the second form, we have $P_n^m(\xi)$ as the common factor, which all by itself will make V = 0 on the walls.

If I succeed with the cone method, I will come back here and make some final comments.

[12.9.10. My guess now is that if you did the correct ip-1/2 method, then closed the p contour, you might end up with the cone method where the strange n values arise from pole residues of the deformed contour]

Review of the Cone Method Work (this review text is being written on 2.14.10)

I found docs called "Green's Function for an Oblate Bloid using cone method. doc" v1,v2,v3. I studied them all and concluded that v1 and v2 are OBS and made them so, so only v3 is valid.

In this document:

I first review the motivation for "the cone method" which still seems very good. I then constructed a fancy (*) and a simple (**) Smythian form in this new method. I then noticed that the simple form cannot work for the jigger problem since we need V = 0 on the disk and the simple form does not allow that. But the fancier 2-coefficient form might work.

I then have a "note added 2.10.10" where I comment to regard all the preceding P and Q as the new "real" p and q function forms. From this point on, I was planning on using the p and q functions. But I think I gave up on this plan after just a few equations, so rest of doc is still in P and Q notation.

I then get underway on the open bloid problem. A picture, and a form. At this point, I decided to change from $e^{im\varphi}$ to the cos(mφ) form setting $\varphi_0 = 0$. My starting form is called (***). I realized that I must be using the 0,0L Q function to get decay up and down. F_{nm} up, and G_{nm} down. Doing all the continuity checks turns up $G_{nm} = F_{nm}(-1)^m$. I remember being happy that this solution for G passes all the checks.

I then go into the pillbox section. I find the F_{nm} and therefore also the G_{nm}. I then have a Final Result, and I was again pretty pleased at this point. I comment that the n-spectrum comes from S-L in ξ.

I take the cone limit and am extra pleased that it matches Smythe's cone result! So then I write up a little Summary of my spanking new bloid solution! This is a long summary, sort of little internal META review of how I did it.

Then comes the fateful question as the next heading: what makes this potential be real? I then realize for the first time that the terms in my sum are definitely NOT real, and I am in big trouble. Maple confirms.

My first reaction is an obvious one: maybe just take the real part of the result. While pondering this, I derive two little theorems:

Theorem A: If f(z) solves ODE Lu = λu and L is "real", then the Re(f) and Im(f) also solve Lu = λu .

Theorem A1: If f(iz) solves ODE Lu = λu and L is "real", then if L includes a first derivative, we will find that Re[f(iz)] and Im[f(iz)] do NOT solve Lu = λu.

I then apply this at once to show that you canNOT just use the Re or Im parts of the atoms. I quote:

Example: We know that $L_\zeta\, Q_n^m(j\zeta) = n(n+1)\, Q_n^m(j\zeta)$ where L_ζ is real. Whether this is an EV problem which determines the spectrum of n , or whether it is just a cathode follower and n is some real non-integral value, in either case we will find that $Re[Q_n^m(j\zeta)]$ and $Im[Q_n^m(j\zeta)]$ do NOT satisfy the equation $L_\zeta u = n(n+1)u$. Therefore, these things are not factors in possible oblate "atoms".

The thing I have that is not real is this: $j\, Q_n^m(j\zeta_>)\, P_n^m(j\zeta_<)$

```
I * LegendreQ(2.3,3,1*I)* LegendreP(2.3,3,3*I);
                                  -62.36240814 - .1209166466 I
```

I then try a few idea to rescue this problem. Another huge "Dead End" sign loomed at me! The Maze!

I then have a little section on "the complex nature of j $Q_n^m(j\zeta_>)$ $P_n^m(j\zeta_<)$". Just confirming that we really do have a problem. This would have to be real in each partial wave! I thought about Q/Q ratios, but to no avail. So "reality" is a new "hidden boundary condition".

I then observed a little paradox about my $(-1)^m$ factor if you think about a point on the neck disk. Near the point potential has to be the same above or below it!

The next section is titled "hole in plate limit", ie, limit of our bloid solution. Somehow, that weird n spectrum has to become the spectrum of $P_n^m(\xi_1 = 0) = 0$, and I know all about this spectrum. I try then to write down exactly what the n spectrum is. This leads to my historically famous Legendre S-L problem on the interval (0,1) [instead of the traditional (-1,1)]

I then take this limit of my fancy complex solution. That is to say, this limit is the Green's Function of an iris, a problem that got me started on all this stuff in the first place. I then move the Green's charge down into the hole. And then we get the final hammer blow that, in this case, V = 0 everywhere! The reason is this: the spectrum is now set by $P_n^m(0) = 0$. But the solution has a factor $P_n^m(j0+)$ in our limit, and then this is 0 (on cut, off cut does not matter, it is the gamma functions doing it).

I think I took this limit and then went back earlier in the document when I realized the solution was complex and therefore invalid.

And so we come to the end of "Green's Function for an Oblate Bloid using cone method v3. doc"

At this point, I decided to try to find some p and q functions that were real, independent solutions of the ζ equation, and use them instead of the P and Q. This turned out to be much harder than I thought, and then it led to another dead end, but let's review it here. I started a doc called "Reality of P(iz) and Q(iz).doc" and that is where I searched for and found, somewhat to my amazement, some p and q functions which I thought would fill the bill. This p and q document kept getting longer as I added more and more properties of the p and q functions which I found. I shall now review this " P and Q" doc a little bit

In the P and Q doc, I first did my "due diligence" and looked in Smythe, MF, and then the web for functions p and q that would be real solutions of the ζ ODE, but I found nada. People use the $P_n^m(i\zeta)$ and $Q_n^m(i\zeta)$ forms only. My "solution" to finding real p and q functions was to supply appropriate phases to knock out the non-real parts of these functions. It turned out that by so doing, I created functions p and q that were *even* in ζ, and this later turned out to be another fatal flaw. But I did not realize this for quite a long time.

I then rehashed the problem that my bloid solution was complex. I then wanted to look at Smythe's various problems to see why HE never had this complex potential problem. His only real problem with m terms was the "Neumann Spheroid" problem, and I showed that in that case, the factors really are real.

So I then "constructed" my q and p functions. I then summarized, added more properties, summarized again, and so on. My last " New summary of Everything So Far" near the end is pretty complete (it could of course have errors, but I think it is pretty much correct".)

Now, while this P and Q doc was being written, at the same time I was trying to solve the bloid in separate documents. I would take a running start, jump, and then smash into a Maze passage-ending wall, as usual.

In "oblate bloid with p and q attempt 1.doc" I used the "bad parameterization" for ξ and ζ and this at once led to a crash, since this includes $\xi = -1$, so you have the n-spectrum conflict. Also you have to then make $P_n^m(\xi_1) = 0$ on the top, and $P_n^m(-\xi_1) = 0$ on the bottom, and these are not in general compatible. So this was a horrible start and I will never look at it again.

In "oblate bloid with p and q attempt 2.doc" I use the correct parameterization. I propose a nice Smythian form with my little p and q friends in it, using F_{nm} up and G_{nm} down. I fold over the negative m

sum, I talk about the n spectrum, and then I take a look at "the neck" region. "We can tell for sure that $F_{nm} \neq G_{nm}$ because if they were the same, the upper and lower potentials would be exactly the same, but we know that cannot be true since the Green's charge is only in the upper part." But then when I require continuity of V at the neck, I get F = G! Dead end.

I then "bounce back for more" by trying a more general Smythian form having A,B and C coefficients up and A',B',C' down. The upper Vo/Vi boundary match eliminates A, the lower A'. I then look at the neck, and suddenly this document ends abruptly! I think I decided to reorganize the order of things, and that let to attempt 3. So I won't look again at attempt 2.

In "oblate bloid with p and q attempt 3.doc", which cleans up attempt 2, I very carefully do everything. Fancy section headings, very organized, you name it! When the dust settles, I have a form that meets ALL my boundary conditions, but still has two full degrees of freedom which is impossible. I then realize that my form is no good for a new reason: it makes $\partial_\zeta V_i = 0$ on the neck plane. Obviously if you moved the point charge close to this area, that would not be the case. You would definitely have some electric field normal to the neck plane. I then realized the cause of this problem: because my p and q functions are symmetric by construction, their derivatives at $\zeta = 0$ are zero [wrong! see below] , and this means that however you try to construct your V from the p and q atoms, you will always find that $\partial_\zeta V_i = 0$ on the neck plane, so the Smythian form is NO GOOD. "Dead End, Go Back Again" says the Maze.

So I then went back to the P and Q document and wrote some text at the end. I wonder if there is some way to construct p and q without killing off the "odd" parts of the P and Q functions.

That, then, is where I find myself at 8 PM on Valentine's Day Feb 14.

"spheroidal wavefunctions"

While cleaning up the desktop, I noticed a paper on **"spheroidal wavefunctions"**, and I see that MF and AS talk about such things. Do these have some connection to what I am doing? I think these are the *Helmholtz* solutions when $k^2 \neq 0$, and when k = 0 you reduce to the Laplace equation and these functions reduce to the associated Legendre functions! So these are those generalizations I was once wondering about. In a sphere you have r^n and $Y_{\ell m}$, but if you add a k^2, it becomes Bessel j "spherical Bessel functions" that give you the modes of the sphere. I think here, we have this same idea but applied to a spheroid instead of a sphere. There will be some fancy "radial" solutions. From a PDF:

What are spheroidal wave functions?

Spheroidal wave functions are generalizations of Legendre functions and spherical Bessel functions for spheroidal coordinates rather than for the spherical polar coordinates in which the latter functions usually occur. The literature on spheroidal wave functions is often in the context of specialized applications, but two applied-mathematics monographs are especially useful: those by Julius Stratton and colleagues[1] and by Carson Flammer.[2] Josef Meixner and colleagues have derived analytical results,[3,4] and summaries of many results appear in *Higher Transcendental Functions*, edited by Arthur Erdélyi,[5] and in the *Handbook of Mathematical Functions*, edited by Milton Abramowitz and Irene Stegun.[6] For extensive discussions of computational methods—with programs in Mathematica, C, and Fortran—see my atlas of mathematical functions.[7]

So OK, these are wave equation things, more general than I need for my little Laplace problems. But I do wonder if they somehow reduce to the real functions I am looking for? [I was later to see these animals again in Moon and Spencer.]

Update Feb 22, 2010

A lot has happened since the last entry a week ago.

One thread was this: I thought my p and q bloid failure was due to these functions being symmetric. I knew that $Q_{1,0L}(i\zeta)$ was symmetric and $Q_{1,0R}(i\zeta) = 1\text{-}\zeta\cot^{-1}\xi$ was asymmetric, and I knew this comes about by rotating the log cut at z=1 from left to right to expose a clear z-plane allowing $z = i\zeta$ a free run top to bottom. So I wanted to find asymmetric forms of the p and q function in hopes that this would break the theorem (false it turned out) that symmetric p and q => no Green's bloid solution.

My first hope was the built-in Maple "CTR" (cut to the right) versions of the P and Q functions were what I needed. After several days, I had to conclude that Maple CTR stuff is "ill defined" and not usable. It did not match their help claims. (Feb 18). [I since learned how to look at Maple code, so if I wanted I could figure out what these CTR functions are, but no reason to do that now.]

Next, if I was going to rotate cuts to the right, I needed a perfect understanding of $(z^2\text{-}1)^\alpha$. I first took the wrong interpretation, got strange answers, and finally saw Bateman's statement that you take this as a product of two factors in ALL formulas for all exponents. I derived the three little p 123 "Bateman rules".

I then made Maple graphs of the convergence regions for all Bateman Legendre forms, just for fun, but also in hopes of finding one of these forms that would have my cut to right capability and resulting asymmetry. I noticed that Q (43) had a nice hourglass region that nicely included the entire imaginary

axis for $z = i\zeta$. I had high hopes for Q (43) and "did the form" very carefully, and was able to in this way generate a "new" all-real q function. It appeared to be antisymmetric, but then when I fixed my usual algebraic errors, it turned out to be the same old symmetric q. So this big Q (43) push did not pan out.

Along the way, I directly verified that Q (43), when manually entered using the hypergeom function, perfectly matches the built-in Q, and I did this for a few other formulas. This caused me to actually believe that hypergeom really does know how to analytically continue as needed, and therefore so do P and Q. This "search for asymmetric p and q" then ended after more rotation of power cuts to the right when I realized that in the Q_1 case, it is the log cut inside the hypergeom F that you need to rotate, not the powers outside. I made a valiant attempt to actually rotate the F cut, but it led nowhere. In retrospect, since Maple knows how to continue hypergeom, all the Bateman forms really are "the same" in terms of evaluation, and you are not going to find something new by picking one versus another in that regard.

At this point, just for completeness, I decided to make Maple plots of the p and q functions, just to verify the symmetry that was killing me. My 3D plots looked odd, so I made 2D ones. I then realized that the q function, though symmetric, *did not* have zero slope at $\zeta = 0$. I saw this on the graph and was then led to derive the exact slope-at-0 formulas which were easy to do, and this verified the fact. While doing this, I found a few errors in my Leg props doc and fixed them.

This non-zero-slope-of-q I realized could break the failure theorem on my Smythian bloid form, so I wrote attempt #4 on the bloid Green's and this time it went through! I have not checked the result yet, wanted to get this all written up first. It will likely collapse like all my other "solutions", but for the moment I seem to have made it all the way through the Bloid Maze which I entered on Jan 28, about 24 days ago. A long time to be in a Maze and I suspect I am not really out yet.

Update Feb 26, 2010.

The bloid solution still stands, but has not been challenged yet. I instead worked on some of the supporting math problems, many new documents now exist. Here are some of them:

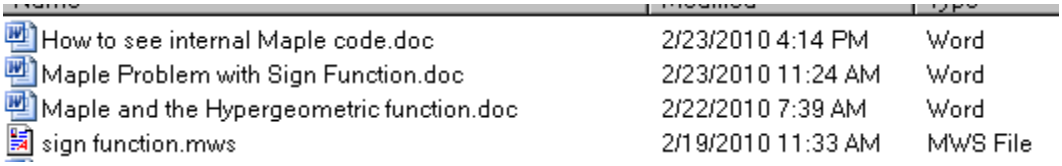
In ODE/Legendre:

Reality of P(iz) and Q(iz).doc	2/26/2010 6:42 PM	Word
batemanpq.mws	2/26/2010 6:42 PM	MWS File
Why are p and q symmetric.doc	2/26/2010 6:41 PM	Word
why are p and q SCRAPS.doc	2/26/2010 6:36 PM	Word
A Maple study of Bateman Q (40).doc	2/26/2010 9:45 AM	Word
Deriving the Bateman P and Q forms.doc	2/26/2010 7:38 AM	Word
Zeros and Poles of P and Q in n,m and Q(43).vsd	2/25/2010 8:22 PM	Visio
bateman p 111 (3).mws	2/25/2010 8:05 PM	MWS File
a study of Q1(z) using Bateman Q (40).doc	2/23/2010 6:24 PM	Word

In complex:

complex ODEs.doc	2/24/2010 12:36 PM	Word

In Maple:



How to see internal Maple code.doc	2/23/2010 4:14 PM	Word
Maple Problem with Sign Function.doc	2/23/2010 11:24 AM	Word
Maple and the Hypergeometric function.doc	2/22/2010 7:39 AM	Word
sign function.mws	2/19/2010 11:33 AM	MWS File

I learned now to see the Maple internal code and learned much about the hypergeom function it uses, really an evaluation routine only. I had trouble with the sign function, also with signum, made my own. I then kept wondering of the two terms in the Bateman Q (40) table entry were my sought-after "even and odd functions" and I had only found the even one so far. This extremely confusing issue has not been completely cleared up in the "What are p and q symmetric" doc. Along the way, I figured out how all the Bateman table entries can be derived, "Deriving the Bateman P and Q..". So I now have a complete answer to the question I once asked: what are a pair of good real solutions to the flipped ζ Legendre equation. So at this point, I plan to continue on with limits of the attempt #4 bloid solution and see where it takes me. I ought to give the jigger problem a shot along the way, I think it will be easy.

Update Feb 27, 2010

I took the cone limit of my Attempt #4 bloid solution and got the right answer. I then took the neck limit, and then the iris limit, then both at once. I was then able to write a formula for the charge density on the surface of an iris given a Green's charge in the hole. I got an answer, but it is of course a tangled double sum of $P_n^m(\xi_0)\ q_n^m(\zeta)\ \cos(m\varphi)$ with messy coefficients. It is a far cry from the simple answer Smythe claims for this charge density in his problem 38 which he gets obviously by some other method.

In any event, my bloid solution held together. In the neck limit, it came out perfectly symmetric up and down, this being the result

$$V(\zeta,\xi) = \Sigma_{nm} B_{nm} [\sqrt{\pi}\ 2^m\ h(n,m)]\ q_n^m(\zeta)\ P_n^m(\xi) \cos(m\varphi) \qquad \text{//upper or lower}$$

There are only a few more things I want to do in bloid land, then I will sign off. First, I want to compare my real-functions answer with the complex-functions answer and see if they have some simple connection. Then I want to take a shot at "the jigger problem" because I think it will be simple.

Update Feb 28, 2010.

Today I tied off some loose ends.

(1) restated the final result for the bloid Green's problem at the end of the attempt 4 document.

(2) found an error in the Smythian form that led to my "complex solution", removing that solution from existence, and thus removing the paradox of having a complex solution to a real problem. See comments at the top of " Green's Function for an Oblate Bloid using cone method v3.doc"

(3) did the jigger problem and the result is extremely similar the upper bloid solution.

So I think this concludes my Trip through the Bloid Maze. I have come out the other end!