Choice C Bowl Attempt PhL 1.16.10

This is a proposed method for solving the charged bowl problem.

Overview (1 page, 9.28.10) **1**
1. The Problem with Choice A and Choice B Plans. **2**
2. Description of the Choice C Plan. **2**

Overview (1 page, 9.28.10)

Choice C is a bowl/disk inversion option not considered in the previous A and B documents. This option was motivated by my reading of Kelvin and of those Smythe problems (not yet solved as of this doc).

The idea is to put a uniform sticky sphere of charge just outside a charged bowl in R space. The charge on this sticky sphere is chosen to make its interior constant potential exactly cancel that constant potential of the bowl, leaving the metal bowl at V = 0. The disk in R' space then will also be at V = 0. The goal is to try to solve the resulting R' space problem, map that solution back to R space, de-superpose the sticky sphere, and end up with a solution for the charged bowl.

I note that this method is similar to Choice B but instead of using an *infinite* surrounding sphere in R space (which creates an on-axis point charge in R' space) we use a *finite* sphere (which creates a plane of sticky charge in R' space just to the left of the disk plane). I note that this Choice C plan is a superposition of two problems which passes my Red Flag validity test.

So, what is this R' space problem we want to solve? Before adding the sticky sphere, we had some charge densities (different) on the inner and outer surface of the charged bowl, which map to a single effective sticky charge on the disk (at the disk location shall we say). None of these charge densities is known. When we add the sticky sphere in R space, this adds an entire plane of sticky charge in R' space which is known. The outer part of this is the most interesting because on the disk we have V = 0. Thus, in R' space we have a Mixed BC problem: a known sticky charge density outside the disk (Neumann part, the image of the sticky sphere in R space) and a known potential on the disk of V = 0 (Dirichlet part).

I conjectured that this problem, being *planar*, would be easier than a direct solution of the bowl problem, which is also a Mixed BC problem. But I was unable to solve this planar problem at the time of this doc, and that is how I left things.

I note that if you rotate the position of the charged bowl, the bowl maps into an iris instead of a disk, and you then have a similar R' space Mixed BC problem: V = 0 on the iris, σ = image of sticky sphere on the inside of the iris.

Later on I learned various ways to solve this problem. One is based on the Smythe problem sequence, where you start by figuring out the Green's function for a disk with point charge in its plane. You then integrate that into a ring, and then you would integrate that ring into your desired planar sticky-sphere σ image outside the disk, and in that way you could solve the R' space problem and then carry out the above program. I never carried out this plan, but it would work. Another solution method would be a Sneddon solution of the Mixed BC problem in R' space probably in cylindrical coordinates which I think would be a dual integral equation problem.

This Choice C really is the plan followed by Kelvin and Smythe, but their batting order is a little different, and Smythe I understand better. Smythe figures out a known R space configuration that creates

a ring of uniform charge on the R space cap. (R' has an iris with a ring of charge inside, but this could also be a disk with a ring of charge outside). He then does an integration on both sides such that the cap ends up with a uniform σ on it, and figures out the R' space situation to make this work. Then finally he adds the sphere in R space to cancel this cap of charge, and ends up with the charged bowl. I do this all in great detail elsewhere.

In some sense, I was hoping that by using a *finite* sticky sphere in R space, I was avoiding having to deal with an induced point charge and thus I was avoiding having to do a Green's function problem in R' space. But in the Smythe path just outlined, you still have to do a Green's function along the way, and a lot more.

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1. The Problem with Choice A and Choice B Plans.

Now that I have looked at both Kelvin and Smythe on the bowl problem, I think there may be another way to use "inversion" to assist in solving this problem.

The idea is to use a *finite* sphere (around the bowl in R space) instead of an infinite one to achieve "potential shifting" to cause a piece of metal to have its constant potential shifted to ground. Both Kelvin and Smythe "do this" in their approaches (at some point).

My approach *so far* has been to assume a global constant potential -A on one side and model it as a point charge at the origin of size q = -aA on the other side (the two inversion sides, R and R' space). An infinite charged sphere with internal potential -A on one side maps into a point charge at the origin q = -aA on the other side. This follows from the rule

$$\varphi'(\mathbf{r}\,;\, q'_i,\, \mathbf{r}'_i) \;=\; (a/r)\,\varphi(\mathbf{r}'\,;\, q_i,\, \mathbf{r}_i) \qquad \text{// LHS = RHS}$$

so if φ = -A, then φ' = -aA/r which is a point charge q = -aA at the inversion origin.

So when I offset the potential of a charged metal object by adding a global constant -A on one side, it creates a point charge on the other side, and then I always end up with a Green's function situation that I don't want. [this is "the problem with Choice A and Choice B"]

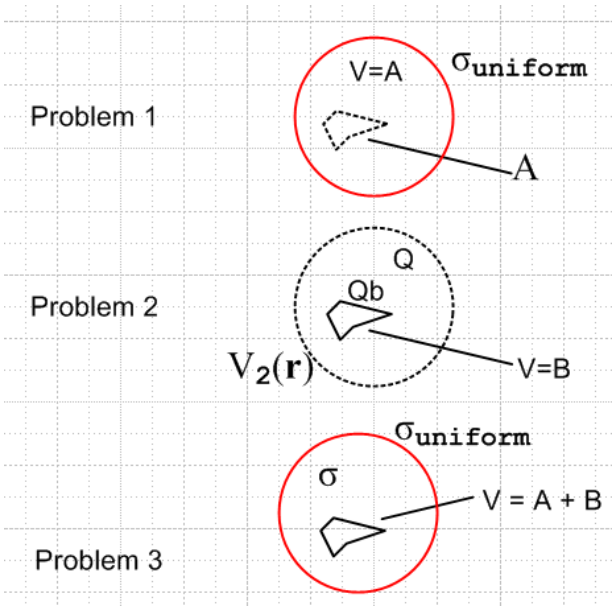
Also, one of the negative aspects of doing things this way is that a global potential is not physical. It causes the potential at infinity to be non-zero. So if you use it to offset a metal's potential to get V = 0 on the metal, you have this unphysical situation where V = 0 on the metal, but V = -A at infinity. So it is a little unclear what you mean by a "grounded " piece of metal in this case (but the method works).

2. Description of the Choice C Plan.

The Kelvin / Smythe idea I think is to do the metal potential offset using a *finite* sphere of charge instead of an infinite one. This is a sphere of uniform sticky charge. Of course for a finite sphere, you have to figure out where the image sphere will lie. You know from the general rule what the (variable) density will be on the image sphere.

Suppose then in R space you have a charged metal object with potential B (problem 2). You can surround it with a sticky charge sphere (problem 1) which creates potential -B inside that sphere. If that sphere has radius R, then you know that the total charge will be Q = -RB (Q = CV). Outside this sticky charge sphere, its contribution to the total potential drops off as Q/r where r would be distance from the sphere center.

In my doc " Superposition in electrostatics.doc" I show this superposition Example 3B:



Here Problem 2 is the initial metal object (perhaps a metal bowl at V = B) and Problem 1 is the added sticky sphere. You can superpose and you end up with Problem 3 as shown, where the metal object can still be metal. It has potential A+B which we make be 0. In this picture, bowl has initial potential B in problem 2, and in problem 3 it is "grounded". If we tried to use a charged *metal* sphere for Problem 1, in Problem 3 that sphere would have to be non-metal and would have some varying Dirichlet function on it, as I show in Example 3 of my superposition doc. This is not what we want to do, we want the red sticky sphere.

Now, for the bowl problem, we pick an enclosing sticky charge sphere that lies just *outside* the bowl sphere, and so our Example 3B analysis above applies. In R space we then achieve a bowl at V = 0 which really is grounded since ∞ is also V=0. In R' space this sticky sphere maps into a "plane" just to the left of the our disk plane. We know which side, because we know the outer surface charge on our bowl (in Problem 2) maps to the left side of the disk from our Inside Outside Theorem in our Jackson inversion META notes.

Our Problem 3 then is this: In R-space we have our metal bowl at V=0, and we have an envelope of uniform sticky σ around the entire bowl sphere. In R' space, we have a metal disk at V = 0, and we also have an infinite plane of non-uniform sticky σ' which is the image of the sticky bowl-enveloping sphere in R space.

Now suppose we can somehow figure out "everything" in R'-space. We then know everything in R-space. We then reverse-superpose in R space to get back to Problem 2. This means that we take our Problem 3 charge densities on the inside and outside of the bowl, and we subtract our uniform σ only from the outer one to get our final Problem-2 σ densities. Before doing this, we will likely find that the inner and outer bowl charges are the same. So after doing this, the inner and outer charge densities will differ by a constant. I have seen exactly this "differ by a constant" result reported in both Kelvin and Smythe! So this makes me think I am on the right track.

The R' situation at the Problem 2 level is that we have a certain sticky surface charge on the disk and the disk is not at constant potential. We don't know this surface charge, nor do we know the R-space corresponding surface charge. [I think we would slice the R'-space charge in half and give each half to a surface of the bowl, after transforming.] The Problem 1 level is a thin charge covering the entire disk plane which is the image of the surrounding R-space charge. We know this charge. Although it goes to infinity, I think it causes a physical potential which is 0 at ∞. Now we do the superposition in R' space and we end up with (1) the disk having the same sticky charge it had in Problem 2. (2) the disk also has the central portion of the spherical Problem 1 charge image. (3) outside the disk going to infinity we have the rest of the image of the problem 1 charge. (4) the disk as now at 0 potential because the metal in R space is at 0 potential.

So it is this Problem 3 R'-space problem we need to solve. It is a mixed BC problem. On the disk we have a prescribed Dirichlet potential V = 0. Off the disk we have a prescribed Neumann surface charge. The problem is very similar to the mixed bowl problem we start with in R space: bowl has some constant potential V on the bowl part, and a Neumann condition σ = 0 on the non-bowl part of the bowl sphere. But the advantage hopefully is that we have a flat geometry in R' space which is simpler.

In Smythe's problem development of this problem, he puts the metal bowl at the other pole. The image of the bowl is then an iris, and then we have a sort of inverted Dirichlet/Neumann problem in R' space. We have V = 0 on the infinite iris (plane with hole), and we have our varying σ image of the enveloping charge in the hole. Smythe makes it seem that it is not too hard to solve this problem! I assume it is just as easy to solve in either bowl pole position.

So let's comment on this problem for the way I set things up. Smythe suggests we first solve the problem for a point charge outside the disk, then superpose that for a ring of charge outside the disk, then superpose those to match your image σ. He does not give the Green's Function, only the charge on the disk, which is what we need to know. Off hand, this seems a less symmetrical problem than the on-axis Green's Function I have the messy oblate coordinates answer for (and messy charge distribution expression for). But maybe if you just want the charge density, there is a trick.