Overview of all docs in the bowl folder PhL 9.28.10

There are bowl-related docs in other folders such as the inversion folder and elsewhere. I will try to review the 8 docs in this "bowl folder" in the order in which they were created, which was over a 10 month period Oct 2009 to July 2010. As the physics work flow doc shows, I did very many of other things in this time period, and this bowl stuff was just one small but very motivating thread.

1. potential of half-spherical shell v1.doc 37 pages created 10.15.09

This is a my first and also a failed attempt to **solve the charged half-sphere bowl problem using the Stakgold integral equation method** $\int_\sigma E(s|\xi)I(\xi)dS_\xi=1$. In this equation E ~ 1/R and I = σ_{tot}, the sum of the (unknown) charge densities on both sides of the charged bowl on which V=1. The bowl radius is 1. On sort of a lark, I tried to ***imitate*** the way this method is applied to solve the *full* sphere problem, but I took a *subtle* wrong turn. In a careful side-by-side review of the steps in these two problems, this wrong turn led me to write my "**Legendre My Problem**" doc sequence (in the Stak folder) in which I concluded that $\int_0^1 dx\ P_n(x)\ I(x) = \delta_{n0}$ has no solution I(x). The wrong turn was not realizing that the equation $\Sigma_{n=0}^{\infty} P_n(z_s)\ I_n = 1$ has solutions *in addition to* $I_n = \delta_{n,0}$ on the interval z_s in (β,1) when β ≠ -1. For the half sphere problem, the correct I_n solution will solve this equation *and* will also give I(z) = 0 on the cap of the bowl, where $I(z) = \Sigma_n\ (n+1/2)^{-1}\ I_n\ P_n(z)$ for all z in (-1,1). These are in fact the "**dual series equations**" treated in Canonical and Sneddon for this problem. If you look at just the first of these equations and pretend the other doesn't exist, there will be lots of solutions for I_n, all but one of which are wrong, including δ_{n0}. It was while writing this doc that I found the Canonical pdf paper. Note that I am really solving for the bowl's charge density, not the potential as the title suggests. Of course the potential can be obtained from $\int_\sigma E(s|\xi)I(\xi)dS_\xi=V(s)$ once I is known. The overview is quite long at 6 pages, mainly because the doc is long and I do many things. Nothing was successfully solved in this doc.

2. potential of half-spherical shell v2.doc 14 pages created 11.1.09

This is my second failed attempt to solve the charged half-sphere bowl problem using the **Stakgold integral equation method** $\int_\sigma E(s|\xi)I(\xi)dS_\xi=1$. Basically I show that this can be written as an integral equation **involving K,** $(16\pi/\sqrt{2})\ \int_0^1 dz\ I(z)\ d^{-1/2}\ K(\sqrt{2b/d}) = 1$ (where b and d are certain functions of z) but I don't know how to solve this integral equation for I(z). [It does have similarities to a Polyanin integral I used in "the charged disk problem.doc".] I arrive at this result by **doing the dφ integration** which is part of dS_ξ which is really $d\Omega_\xi$. The required azimuthal integral was $\int_0^\pi dx(1/\sqrt{a \pm bcosx}) = (2/\sqrt{a+b})\ K(\sqrt{2b/(a+b)})$ a > b > 0, but I could not find this anywhere and had to derive it for myself. I later re-encountered this integral and found it first on the Wolfram Alpha site, then I found it as a recent addition in GR7, but the K was shown as E. This led to an **email exchange with Dan Z** who confirmed at least that "something" was wrong. I sent Dan some supporting work but have not heard anything since then. I also have some comments on $k(z_s,z) \equiv (1/2)\ \Sigma_{n=0}^{\infty} P_n(z_s)\ P_n(z)$ [which appears in Canonical] and

made a failed numerical attempt to confirm or disprove the above K equation. Again, I am trying to solve for the charge density, not the potential. The overview is 3 pages long. Nothing was successfully solved in this doc. [I think this integral is also a $Q_{-1/2}$ function, the famous toroidal series as in integrals folder.]

3. Choice A bowl attempt + Kelvin.doc 14 pages created 1.15.10

This is the first of three related docs in which I try to use the **method of inversion to solve bowl problems**. I use **inversion pair Choice A** which relates the **charged disk** by inversion to a certain bowl Green's function problem. I know the charged disk result from Jackson, and I am then able to compute σ on either side of the bowl in this Green's function problem for the bowl (the two σ's are equal). But what I really want is the charged bowl solution, not a Green's function solution. I ponder ways to get from one to the other, but find none. Knowing that Kelvin solved both problems, I went off and did some "parsing" of his paper. I quote Kelvin's result for σ on the charged bowl (the two σ's differ by a constant), and quote a similar result from the Canonical doc. But at this time I am simply unable to follow Kelvin's fancy geometric discussion. At least I know the answer, if not how to solve the problem. In this doc I "invent" the notion of a sticky-charge surface. The overview is 2 pages long.

4. Choice B bowl attempt.doc 4 pages created 1.16.10

I continue trying to use the **method of inversion** to solve bowl problems. This time I use **inversion pair Choice B** which relates the charged bowl to the **on-axis Green's** function for the disk. I know this Green's function and the related charge density on the disk from my work with oblate spheroids in oblate coordinates. The results for σ_{disk} are pretty messy, involving a sum Σ_n which includes $P_n(\pm\sqrt{1 - (\rho/b)^2})$ and other factors. I don't see how this complicated result will get any simpler when I invert it back to the charged bowl problem, so I conclude that this is not a good method to try. I then start looking elsewhere for a solution. I notice Smythe having a set of problems 38-42 which seem related (and which I later solved). M&F have nothing, but I am aware of the Canonical doc's dual series approach and later I learned more about this in Sneddon. The overview of this short doc is only 1/2 page.

5. Choice C bowl attempt.doc 4 pages created 1.17.10

I continue trying to use the **method of inversion** to solve bowl problems. This time I use **inversion pair Choice C** which relates a **superposition of 2 problems** in R space to a superposition of 2 problems in R' space. In R space I have a charged bowl surrounded by a sticky-charge sphere which makes the bowl have V = 0. In R' space I have a disk at V=0 therefore, and a coplanar iris with a certain charge density which is just the image of the sphere. The goal is to try to solve the resulting planar R' space problem, map that solution back to R space, de-superpose the sticky sphere, and end up with a solution for the charged bowl. But alas, the R' space problem is a dual integral equation one, and I did not know at the time how to deal with such things. I do note that this Choice C really contains the essence of the Kelvin and Smythe solutions. The overview is 1 page.

6. Bowl Green's Function Attempt using Toroidals.doc 4 pages created 5.2.10
[this file is down one level now]

When writing this, I had done only a very slight study of toroidal coordinates. I knew that the bowl was a "hole free" surface of constant η and that is why toroidals might be very appropriate. Here I merely

outline how one *might* solve either the charged bowl problem or the bowl Green's function problem using a Smythian form which involves a sum of $a_n \cos(n\eta)\, Q^m_{n-1/2}(\cosh\mu_>)\, P^m_{n-1/2}(\cosh\mu_<) \cos(m\varphi)$ type atoms. I was unaware of Mehler-Foch at this time but knew something must exist like that. I then scanned what my usual authors had to say about toroidals, found little, then went off to work more on my general toroidal coordinates doc. Overview is 1 page.

7. Kirk on bowl problems including a Kelvin review.doc 6 pages created 7.3.10

By this time I had done the Smythe problems 38-42 and I found this Princeton paper which focused on solutions of bowl problems (aka orifice in sphere). The author Kirk first talks about approximation methods for small holes. Kirk starts to review Green's approx solution but stops. Then he gives a **review of Kelvin's work** and states the famous result for σ on the charged bowl. I **learned** from Kirk (and I guess Kelvin therefore) of **the coplanar disk-to-disk inversion scheme** I had not thought of before and which I put to use later. The inversion of one disk relates to the Green's function of the other disk, so this provides a "planar Green's function for disk" method. This was **a lucky find** and shows the benefit of "looking around" at things on the web. Kirk's attempt to review Kelvin is not as clear as I would have liked it to be, but I do understand. Kirk next **mentions toroidals** and put me on to **Lebedev's book** which I then obtained. The final section is a review of Kelvin's method for finding σ on a charged **ellipsoid**, and then this gives the disk charge in a limit. I think I was able to do this Kelvin derivation on my own. The list of references is good and very classic. The TOC is the "overview" for this short doc. [Note that I owe Kirk thanks for two things: (1) the planar inversion scheme; (2) the Lebedev book.]

8. Brief comments on the charged bowl problem.doc 1 page created 7.9.10

This is really a nothing document. I just state that I know of **three methods** of doing bowl problems: toroidal coordinates, inversion, and Sneddon's dual series. I comment that, just as the charged ellipsoid has a fairly simple potential (F and trig functions, I quote it), the charged bowl *ought to have* a similarly simple form stated in toroidal coordinates. As of 9.29.10, I don't think I know what this result is.

9. Kelvin and Smythe Bowl Review docs -- both very short, just the basic steps.