Bipolar Doc Questions PhL 11.27.15

Question 1: Look at the lines of constant u. On the top imagine there is some $0 < u_1 < \pi$ value u_1 and on the bottom there is some $\pi < u_2 < 2\pi$ value. How are u_1 and u_2 related if they both lie on the same circle?

Answer: What do we know? Both constant-u curves lie on the same circle, which is this circle

$$x^2 + (y- y_c)^2 = R^2 \qquad y_c = a/\tan u \qquad R = a/|\sin u| \ . \qquad (2.6)$$

The equation $R = a/|\sin u|$ gives one clue. It says this

$$|\sin(u_1)| = |\sin(u_2)|$$

For the upper half where u_1 lies in $(0,\pi)$ we know that $\sin(u_1)$ is positive.

For the lower half, where u_2 lies in $(\pi,2\pi)$, we have $\sin(u_2)$ is negative. This our equation is

$$\sin(u_1) = - \sin(u_2)$$

But we can write

$$\sin(u_1) = - \sin(u_1 \pm \pi)$$

so this suggests that

$$u_2 = u_1 \pm \pi.$$

Since u_1 is in the lower range, it must be the plus sign. So

$$u_2 = u_1 + \pi.$$

$$u_2 - u_1 = \pi \qquad \text{// answer to the question.}$$
