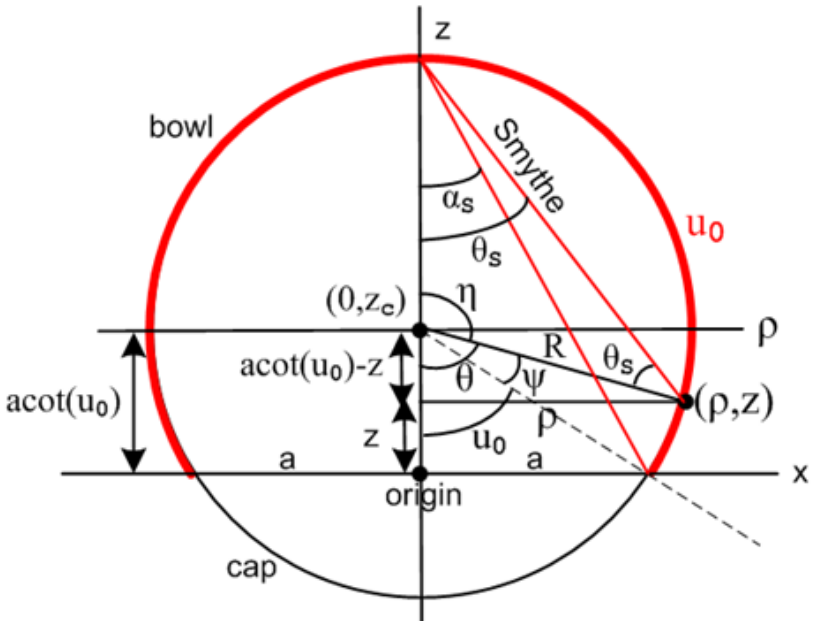
Comparing Smythe's potential on cap with G.25 for the same thing PhL 12.24.15

A recent doc, has my new bowl picture, and the plot is now in Appendix G. 12.31.15

First my picture,



$$x^2 + y^2 + (z-z_c)^2 = R^2$$
$$\rho^2 = x^2+y^2$$
$$z_c = a\cot(u_0)$$
$$R = a / |\sin(u_0)|$$

Smythe says this for potential on the cap

$$V(\theta) = V_0(2/\pi)\sin^{-1}[\cos(\alpha/2)/\cos(\theta/2)], \qquad (D.7)$$

where

$\theta = \theta$ in my picture above, so θ runs from 0 to u_0 on the cap
$\alpha = u_0$ I claim below (D.7).

Meanwhile, I get this fancy other result

$$f_2(\theta) = (1/\pi)\ \Sigma_{n=0}^{\infty}\{\ \sin(n\theta_0)/n\ + \sin[(n+1)\theta_0]/(n+1)\ \}\ P_n(\cos\theta)\ . \qquad (G.25)$$

Here we have θ being the θ of (E.14), which is my angle η in the above picture, the angle measured from the bowl center. And θ_0 is the lip measured the same way. So

$\theta = \eta = \pi-\theta$ runs from $\theta = \theta_0$ to π on the cap.
$\theta_0 = \pi-u_0$

So if I want to compare these two expressions, I have to get them in terms of the same variable! So start with

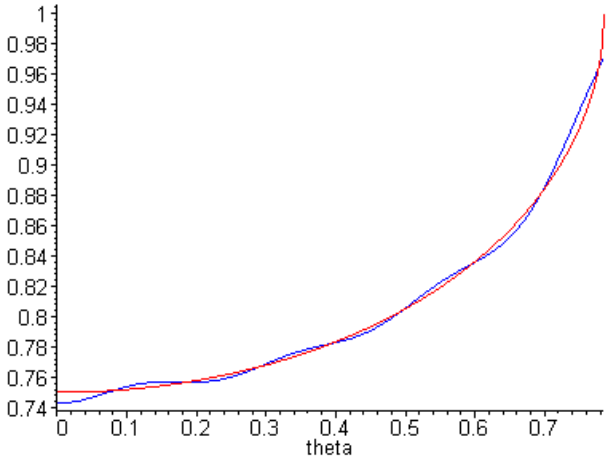
$V(\theta) = V_0(2/\pi)\sin^{-1}[\cos(u_0/2)/\cos(\theta/2)]$ using my θ and u_0

$f_2(\theta) = (1/\pi)\ \Sigma_{n=0}^{\infty}\{\ \sin(n\theta_0)/n\ + \sin[(n+1)\theta_0]\ /(n+1)\ \}\ P_n(\cos\theta)$

$= (1/\pi)\ \Sigma_{n=0}^{\infty}\{\ \sin(n[\pi-u_0])/n\ + \sin[(n+1)[\pi-u_0]]\ /(n+1)\ \}\ P_n(\cos[\pi-\theta])$

When I plot these two babies, they match! Here with 30 terms

```
plot([V,f2],theta = 0..u0,color = [red,blue]);
```



So let's maybe write them the other way

$f_2(\theta) = (1/\pi)\ \Sigma_{n=0}^{\infty}\{\ \sin(n\theta_0)/n\ + \sin[(n+1)\theta_0]\ /(n+1)\ \}\ P_n(\cos\theta)$. (G.25)

$V(\theta) = V_0(2/\pi)\sin^{-1}[\cos(\alpha/2)/\cos(\theta/2)]$,

$\alpha = u_0 = \pi-\theta_0$

$\theta = \pi-\theta$

Then the Smythe form becomes

$V(\theta) = (2/\pi)\sin^{-1}[\cos(\alpha/2)/\cos(\theta/2)]$,

$= (2/\pi)\sin^{-1}[\cos([\pi-\theta_0]/2)/\cos([\pi-\theta]/2)]$,

$= (2/\pi)\sin^{-1}[\cos(\pi/2-\theta_0/2)/\cos(\pi/2-\theta/2)]$,

$= (2/\pi)\sin^{-1}[\sin(\theta_0/2)/\sin(\theta/2)]$,

So I am now claiming this amazing sum

$$(1/\pi)\ \Sigma_{n=0}^{\infty}\{\ \sin(n\theta_0)/n\ +\sin[(n+1)\theta_0]\ /(n+1)\ \}\ P_n(\cos\theta)\ =\ (2/\pi)\sin^{-1}[\sin(\theta_0/2)/\sin(\theta/2)]$$

or

$$\Sigma_{n=0}^{\infty}\{\ \sin(n\theta_0)/n\ +\sin[(n+1)\theta_0]\ /(n+1)\ \}\ P_n(\cos\theta)\ =2\sin^{-1}[\sin(\theta_0/2)/\sin(\theta/2)]$$

This must exist somewhere! I cannot find it anywhere! It is a Legendre transform of a certain function.

I will add this at the end of Appendix. Start with