**Edit log for charged bowl** PhL 11.30.11

**Now 2011: Various PDF export programs and how integrals appear** ..... 1
**Update 11/26/15: A very major update effort with 1/5/16 release (50 pages of edit log! )** ..... 2
Question from Rob Maier. ..... 6

**Now 2011: Various PDF export programs and how integrals appear**

I never had an edit log in the original push. So today I start one.

Changes needed:
ref to (10.3a) in Sec 2(e)
Stak Ref

I don't see any reference to Stak, so not clear why I would add a reference to same. Today I added the email address at the front, changed the date, and did the first fix above and one other detail.

I will now save the old PDF and create a new one with Acro 8 in my usual fast way.

The lower integral endpoints ARE too low for some reason. The uppers are OK. Why does it do this?

Let's make a test document and see if a fix can be found. PDF995 does not have the problem my notes claim. Confirmed! I made a little test2.doc and it shows the problem at once. Someting related to the way Acrobat does its conversion.

$g(y) = \int_0^\infty d\tau\, P^m_{-1/2+i\tau}(y)\, f(\tau)$ // expansion

// above is Word

$g(y) = \int_0^\infty d\tau\, P^m_{-1/2+i\tau}(y)\, f(\tau)$ // above is pdf995

$g(y) = \int_0^\infty d\tau\, P^m_{-1/2+i\tau}(y)\, f(\tau)$ // expansion

// above is Acro 8.0

I guess the fix is to just use pdf995 for this document?? But why do they differ??

Web has nothing on "integration endpoint" and PDF. etc

Let's just do this one with PDF995! It only takes 2 minutes so find. I could eventually find some other way to do the Word integral thing, but there is no need as longas PDF995 works for the bowl doc.

On **Nov 30, 2011** this PDF was put up after the 995 generation. The only problem is that the TOC, though live links, does not appear as bookmarks, a feature of the Acro 8 generation. So I have a choice of bad integral endpoints or no bookmarks. I take no bookmarks!

**Update 11/26/15: A very major update effort with 1/5/16 release (50 pages of edit log! )**

I got an email regarding the bowl paper, and I am wondering why the last release is 11/30/11, but the current active doc has date 12/29/13. I must have made some changes, but this log says nothing and there is no errata log history. Well OK, errata log says this

I had no errata doc before today 12.29.13. I am doing a read through due to Agassi email. // I did a read through, adding **punctuation** mostly, then **updated the title date**. **No major errors** were noted.

So just cosmetics, fine. So that is the file into which I will make further changes during my next edit session. For now I will do things in the error log. I will have to review the 12.20.13 doc instead of the pdf. I might as well just fix the cosmetics rather than log them.

Reviewing live here: I do trivial punctuation fixes with no errata log entry, such as periods after equations.

1. intro - OK
2. 
   (a) OK, review of the ellipsoid case with its simple result
   (b) OK, various atomic forms, very good
   (c) OK, Smythian form for the bowl with two unknown functions of parameter $\tau$
   ( possible errata noted for this section in the errata log)
   (d) OK, uses MF transform to find $V(\xi,u)$ = potential everywhere with bowl $u_0$ at $V_0$

So at this point I have solved the bowl in toroidal coordinates, title of the doc. Plot excellent.

   (e) OK, validate integral (2.6) from GR7 statements.

3. 
   (a) bug found:

Equation (3.3) seems wrong, What are the P and Q there?? Very bad error! Where is my source for this equation. This paper was before I derive everything in line. Source is "charged bowl in toroidals 1_11". The answer given there is

$$ch(\pi\tau)\, A(\tau) = V_0 \sqrt{2}\, Qch[\tau(\pi-u_0)] \;/\; [Qch(u_0'\tau) + P\, sh(u_0'\tau)]$$
$$ch(\pi\tau)\, B(\tau) = V_0 \sqrt{2}\, Pch[\tau(\pi-u_0)] \;/\; [Qch(u_0'\tau) + P\, sh(u_0'\tau)]$$

$$P = \{ V_2\, ch(u_0'\tau)\, ch[\tau(\pi-u_2)] - V_0 ch(u_2\tau)\, ch[\tau(\pi-u_0)] \} =$$
$$Q = \{ V_0 sh(u_2\tau)\, ch[\tau(\pi-u_0)] - V_2 sh(u_0'\tau)\, ch[\tau(\pi-u_2)] \}$$

which explains the P and Q. Looking at the denominators, I think Q=C and P = D. Then I should have

$$ch(\pi\tau)\, A(\tau) = V_0 \sqrt{2}\, C\, ch[\tau(\pi-u_0)] \;/\; [C\, ch(u_0'\tau) + D\, sh(u_0'\tau)]$$
$$ch(\pi\tau)\, B(\tau) = V_0 \sqrt{2}\, D\, ch[\tau(\pi-u_0)] \;/\; [C\, ch(u_0'\tau) + D\, sh(u_0'\tau)]$$

$$D = \{ V_2\, ch(u_0'\tau)\, ch[\tau(\pi-u_2)] - V_0 ch(u_2\tau)\, ch[\tau(\pi-u_0)] \}$$
$$C = \{ V_0 sh(u_2\tau)\, ch[\tau(\pi-u_0)] - V_2 sh(u_0'\tau)\, ch[\tau(\pi-u_2)] \}$$

Then reorder the last two equations

$$ch(\pi\tau)\, A(\tau) = V_0 \sqrt{2}\, C\, ch[\tau(\pi-u_0)] \;/\; [C\, ch(u_0'\tau) + D\, sh(u_0'\tau)]$$
$$ch(\pi\tau)\, B(\tau) = V_0 \sqrt{2}\, D\, ch[\tau(\pi-u_0)] \;/\; [C\, ch(u_0'\tau) + D\, sh(u_0'\tau)]$$

$$C = \{ V_0 sh(u_2\tau)\, ch[\tau(\pi-u_0)] - V_2 sh(u_0'\tau)\, ch[\tau(\pi-u_2)] \}$$
$$D = \{ V_2\, ch(u_0'\tau)\, ch[\tau(\pi-u_2)] - V_0 ch(u_2\tau)\, ch[\tau(\pi-u_0)] \}$$

So my fix is to set Q→C and P→D in what my doc now shows

$$ch(\pi\tau)\, A(\tau) = V_0 \sqrt{2}\, Q ch[\tau(\pi-u_0)] \;/\; [C\, ch(u_0'\tau) + D\, sh(u_0'\tau)]$$
$$ch(\pi\tau)\, B(\tau) = V_0 \sqrt{2}\, P ch[\tau(\pi-u_0)] \;/\; [C\, ch(u_0'\tau) + D\, sh(u_0'\tau)]$$

where (3.3)

$$C = \{ V_0\, sh(u_2\tau)\, ch[\tau(\pi-u_0)] - V_2\, sh(u_0'\tau)\, ch[\tau(\pi-u_2)] \}$$
$$D = \{ V_2\, ch(u_0'\tau)\, ch[\tau(\pi-u_2)] - V_0\, ch(u_2\tau)\, ch[\tau(\pi-u_0)] \} \;.$$

Fix is done in line. This was a bad one because P and Q look like Legendre functions!

Resume reading

3.
(a) OK, major bug found and fixed (noted above with P and Q)
(b) OK, bowl with flat lid, I love it, agrees with Lebedev
(c) OK sessile and Hu, but Hu quote seems odd. But I checked, only one there is.

4.
(a) OK, comparing my result with Lebedev and Kelvin with quotes from both!
(b) STOP

What is the meaning of "a" ? Below (4.1) I say that a is the lip circle radius and this gets into $h_u$. But then below (4.7) I note that Lebedev's "a" is the bowl radius which I call R. Then "a" reappears in (4.12) with no definition of "a". So I will now go look this up. I finally found the derivation in toroidal coordinates.doc located miles away in my curvilinear math folder. Yes, a is the bipolar coordinate focal distance which is the bowl lip radius.

Source for cap of bowl? Smythe has this

**40.** Invert the preceding problem, apply Green's reciprocation theorem, and show that the potential at a point $P$ on a sphere, part of whose surface is occupied by a thin conducting bowl at potential $V_0$, is $(2V_0/\pi) \sin^{-1} (\cos \alpha/\cos \theta)$ where $\theta$ is the angle, measured at the bowl end of the rotational symmetry axis, between this axis and $P$ and $\alpha$ that between this axis and the edge of the bowl.

**41.** Starting with the preceding result find the first term in the spherical harmonic expansion for the potential of a freely charged spherical bowl all elements of which are at a distance $a$ from the origin where its radius subtends an angle $\beta$. From this show that its capacitance is $4\epsilon a(\beta + \sin \beta)$.

There is the result, but then you have to decode it. Find another source! Now, I see now that $\beta = \pi - u_o$ if you just draw a picture. A very shallow bowl has a small Smythe $\beta$, Then his answer is

$$C = 4\varepsilon a(\beta+\sin\beta) = 4\varepsilon a(\pi-u_0 + \sin[\pi-u_0]) = 4\varepsilon a(\pi-u_0 - \sin[u_0-\pi]) = \varepsilon a(\pi-u_0 + \sinu_0)$$

$$\rightarrow \quad \varepsilon R(\pi-u_0 + \sinu_0)$$

But why do I have $(R/\pi)\{\pi - u_0 + \sin(u_0)\}$ ?? This is one of my most important results!

If you multiply my result by $4\pi\varepsilon$, you get his result. So I must be in cgs units! Where do I make this cgs decision?

$$\sigma_+ = -(1/4\pi)(1/h_u)\,\partial_u V(\xi,u)|^{u=u0} \qquad 1/h_u = (ch\xi - cosu)/a$$

Yes! This is cgs. In my bipolar coords page 33 I say that $n = \varepsilon E = -\varepsilon\partial_x V$ so you have to take my result for charge and mult by $4\pi\varepsilon$ to get the SI units. I need to add this somewhere.

**Units**. This document uses cgs for charge and capacitance, as determined by (4.1). To get results for charge or capacitance in SI units, multiply our stated results by $4\pi\varepsilon_0$.

OK, I worked this in at end of intro with no pagination change. And gave Smythe reference.

Resuming now my reading.

4.
(a) OK, comparing my result with Lebedev and Kelvin with quotes from both!
(b) OK after changes made

5.
(a) OK, added a plug for bipolar doc
(b) OK, added another plug since doing lots of basis bipolar type equations
(c) OK, the red line pictures showing u labels

(d) STOP

What is my reference to "section (a) " ?? Made fixes, and I resume

5.
(a) OK, added a plug for bipolar doc
(b) OK, added another plug since doing lots of basis bipolar type equations
(c) OK, the red line pictures showing u labels
(d) OK, strange pix of u(ρ,z) just to show weirdness of toroidal coordinates.
(e) OK, another weird coordinate picture
(f) OK, yet another weird picture, leave as is it is OK.

6 OK, little text, all plots of the potential

7. Doing Mehler integrals
(a) OK, I tell how to do them, and list off some I have done.
(b) OK, a particular integral (4.2) I needed earlier to get bowl charge densities
(c) OK, where to find Mehler integrals, and I note various errors

8. OK, Evaluate to elementary functions

Question: I guess I did the plots from these elementary function forms for the potential??

9. OK, shows exactly how I compute u and then do the potential plots.

So yes, I am in fact using my elementary form results. It is the only result I have other than the integral given in (2.8).

I just set formats on all equation numbers so they align on the right edge, there are many.

10. The toroid
(a) OK, here I get the potential and show M&F have an error in their result
(b) OK, capacitance compared with surrounding sphere. Fat and thin limits.
(c) OK, σ computed and plotted on a toroid.

The rest is appendices. So I have actually done a lot for the toroid as well as the bowl.

A. Maple code Just text code so reader can cut and paste into Maple
B. It is OK, that funny number for degenerate toroid capacitance
C. OK, a Kelvin review. Rough sailing, I will not try to improve it.
D. OK, a complicated review of a set of Smythe problems relating to the bowl
E. OK, very fancy explanation of dual equation solving Snedden style using my matrix idea
F. OK, gives several forms of the Abel transform (matrix inverter for App E)
G.
(a) painful stuff, solving the bowl with dual integrals and Abel transforms
(b) OK, I show intermediate result $g_1$ (total charge on bowl) agrees with Ch 4 method

(c) OK, bowl by matrix method, enough!!! Checks are built in, so I think OK.

OK, this concludes my review today 11/26/15 (Thanksgiving) that was triggered by an inquiry from some guy. I did perhaps 30 edits, and one big one so this thing needs a new release!

**Question from Rob Maier.**

Here is the guy's question:

Greetings. I recently found your "Charged Bowl in Toroidal Coordinates" preprint on the web, and read it with much interest. no comment on the quality or lack thereor in my pdf

I am interested right now in the question of a charged toroid, which you treat in Chapter 10. As you say, "the potential must be periodic in u with period 2pi, and this fact causes quantization to integers of the parameter n appearing in the atomic form". That is, P_{n-1/2} appears, i.e., the degree is a half-odd-integer.

Have you by any chance encountered, or considered, the case when the domain does not include the full range of the coordinate u, so that the quantization of n is not to integers? I have looked in standard places such as Smythe's book, but have not found any examples of this.

The reason I am asking is that I have just discovered some formulas expressing such unusual functions as P_{1/3}, P_{1/4}, P_{1/6} in terms of P_{1/2}; and the same, with integer displacements of the subscripts. It would be nice to apply them in electrostatics problems.

Thanks,
Rob Maier

I have a feeling he did not read my thing and is just interested in doing something with his discovery as he says that $P_{1/3}$, $P_{1/4}$ and $P_{1/6}$ are related to $P_{1/2}$. He is wondering if these would apply to any electrostatics problems.

I have NOT made a PDF or done a release, just holding.

But the guy's question above I think I can deal with right now. I say this,

$$\xi \text{ in } (0,\infty) \qquad u \text{ in } (0,2\pi) \qquad \varphi \text{ in } (0,2\pi)$$

$$\begin{array}{lllll} & & \text{expo} & \text{osc} & \text{osc} \\ (1) & \sqrt{ch\xi - cosu} & [P_{n-1/2}^{m}(ch\xi), Q_{n-1/2}^{m}(ch\xi)\,] & [\,\sin(nu),\cos(nu)] & [\,\sin(m\varphi),\cos(m\varphi)] \end{array}$$

$$\begin{array}{lllll} & & \text{osc} & \text{expo} & \text{osc} \\ (2) & \sqrt{ch\xi - cosu} & [P_{i\tau-1/2}^{m}(ch\xi), Q_{i\tau-1/2}^{m}(ch\xi)\,] & [\exp(\tau u), \exp(-\tau u)\,] & [\,\sin(m\varphi),\cos(m\varphi)] \end{array}$$

Now suppose we had a double-bowl where the two bowls are at different potentials and we somehow insulate them from each other at the lip. Suppose V = 0 for one bowl $u_0$ and V = V for the other bowl at $u_2$. Then a Smythian form might be

$$V(\xi,u) = \sqrt{ch\xi - cosu}\ \Sigma_n\ P_{n-1/2}(ch\xi)\ A_n \sin(nu + \varphi_n)$$

if we can drop the Q part, hold on that. Now to meet the BC's we need

$$\sin(nu_0 + \varphi_n) = 0 \quad \text{for all n}$$
$$\sin(nu_2 + \varphi_n) = 0 \quad \text{for all n}$$

What does this say about n?

$$nu_0 + \varphi_n = N\pi \qquad N = \text{any integer}$$

$$nu_2 + \varphi_n = M\pi \qquad M = \text{any integer}$$

Then we have

$$n(u_0 - u_2) = (N-M)\pi$$

$$n = (N-M)\pi/(\Delta u) = N(\pi/\Delta) \qquad \text{where N must be an integer}$$

So the spectrum of n is now multiplies of $(\pi/\Delta)$ so you would say

$$V(\xi,u) = \sqrt{ch\xi - cosu}\ \Sigma_{n=N(\pi/\Delta)}\ P_{n-1/2}(ch\xi)\ A_n \sin(nu + \varphi_n)$$

which brings in $P_s(ch\xi)$ for $s = N(\pi/\Delta) - 1/2$ for $\Delta u$ ranging from 0 to $2\pi$ really.

Now $1/2 \le |\pi/\Delta| < \infty$

Then $s_{min} = 0$ $s_{max} = \infty$

so it seems you could have his fractional values. Let's try

$$s = 1/J$$

$$1/J = N(\pi/\Delta) - 1/2 \qquad 1/J + 1/2 = N(\pi/\Delta)$$

$$(\pi/\Delta) = (1/J + 1/2)/N = [(J+2)/2J\ ]/N = [(J+2)/(2JN)]$$

For example, suppose J = 3.

How about an example:

$\Sigma_{N=0}^{\infty} P_{N(\pi/\Delta)-1/2}(ch\xi)$

Suppose we want

$1/3 = (\pi/\Delta) - 1/2 \qquad (\pi/\Delta) = 1/3+1/2 = 5/6$

$\Delta = (6/5)\pi$ which is possible. Then

$\Sigma_{N=0}^{\infty} P_{N(5/6)-1/2}(ch\xi) = P_{-1/2} + P_{1/3} + P_{7/6} + ....$

This is fine, but later I tried it again and could not figure how to get V = 0 on a custom modified toroid and some negative constant V = -V at infinity. But the above shows the general idea of n getting quantized to non-integer values.

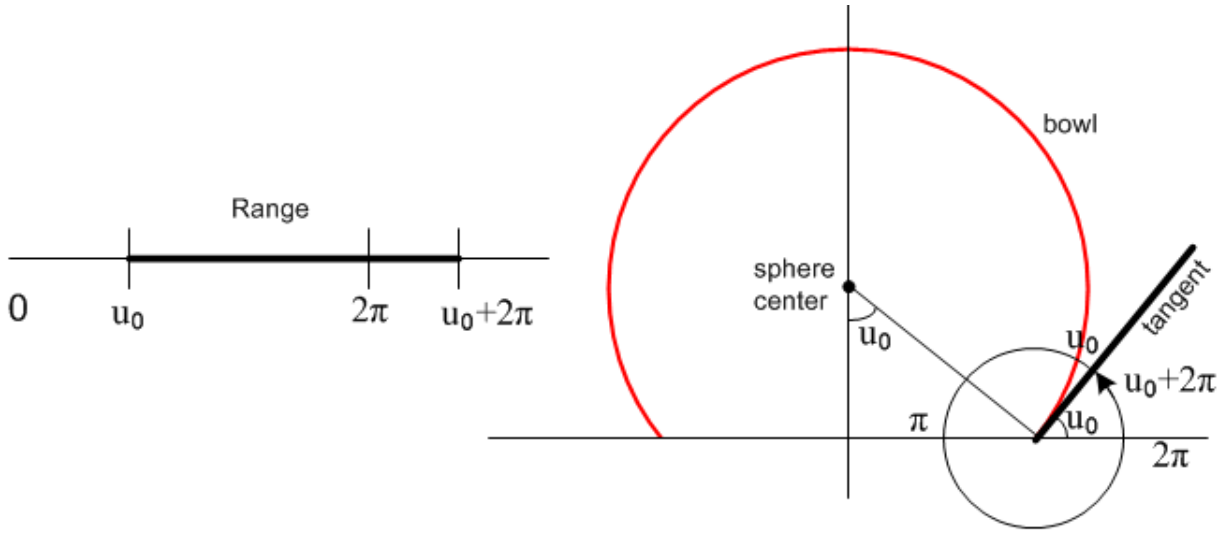
**Nov 27, 2015**

**Limit for σ as bowl goes to a disk**

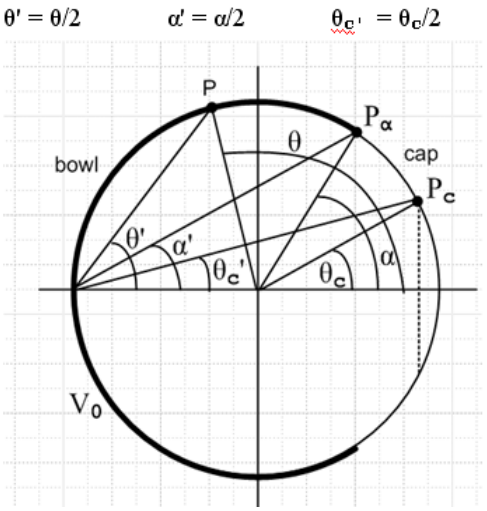
I decided to test my charge density σ results and I think they are wrong! The limit going to the flat disk is garbage. I tried to find a quick fix, but I am confused and this will take a long time to understand. I already wrote two bug analysis docs on this question on Fri 11/28/15 and could not resolve it.

**Nov 28, 2015 Sat**

The limit issue is still unresolved, but I have another side question about interpreting coordinate $u_0$. Well, here it is




I failed in bipolar doc to show that $u_0$ has the trivial interpretation shown. I need to update bipolar doc to show this!! I gave some other weird interpretation of $u_0$ that seems more complex than what is shown above. Also I now show the 0 point on the left. Now consider my bowl doc picture

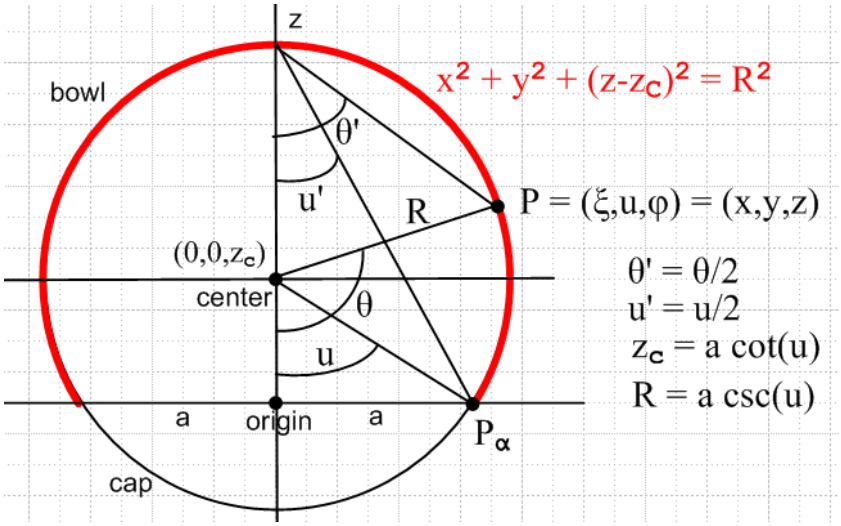



$P_\alpha$ $\theta' = \theta/2$ $\alpha' = \alpha/2$ $\alpha = u_0$

$P = (\xi,u,\varphi) = (x,y,z)$ $(\rho,z)$ a cot(u) acotu-z

Where is this figure?? Similar to a figure in bowl toroidal.vsd, but not the same. Fact: I looked through all VSD files in both the bowl and toroidal curvilinear areas, it is not there. I will have to rebuilt it. The only starting point is the picture on page 1 of bowl toroidal.vsd. But no, THAT is a screen clip! I will now try harder to find the original. This is what happens when you abandon a paper without making an index of code and figures, you PAY later! Can I search for the word "bowl" in vsd files? Ransack searches the entire D drive and finds only my screen clip version. And nothing on the C drive. Where did I read the Kelvin stuff? It is right there in D:\Work\My Interests\Physics\E&M\Electrostatics\bowl . My Kelvin doc has lots of fancy vsd drawings, but they all seem to be lost! No, those pix are in my inversion theory folder in electrostatics, they are not lost. And I just found something close to the above in the inversion folder vsd file, and I will now reconstruct from that.

Here is my new picture where I orient more favorably and replace α by u. β



There is no doubt that the angle α shown in this picture is my $u_0$, at least for $0 < u_0 < \pi$ which I might comment on somewhere. On scratch I just verified the half-angle formulas which are all really just one.

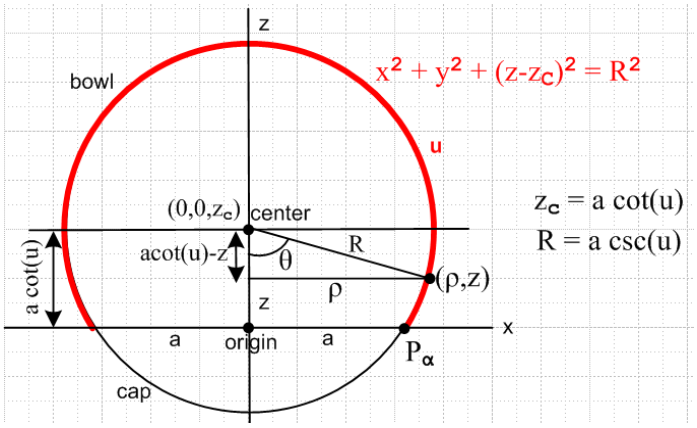
Next: I claim with no support at all that

$$\sqrt{\cos\alpha - \cos\theta} = \sin\alpha / \sqrt{ch\xi - \cos\alpha}$$

where $\alpha = u$ for the bowl shown. So rewrite this in terms of u

$$\sqrt{\cos u - \cos\theta} = \sin u / \sqrt{ch\xi - \cos u} \qquad (*)$$

Proof: Consider this picture (from method of inversion support)




From the thin triangle we get

$$\sin\theta = \rho/R = a\,[sh\xi/(ch\xi - \cos u)] * \sin(u)/a = \sin u\, sh\xi /(ch\xi - \cos u)$$

which is a result I got on scratch a while ago. But this triangle also gives this result

$$\cos\theta = [a\cot u - z]/R = [a\cot u - a \sin u/(ch\xi - \cos u)]* \sin(u)/a$$

$$\cos\theta = \sin u\, [\cot u - \sin u/(ch\xi - \cos u)]$$

$$= \sin u\, [\cos u/\sin u - \sin u/(ch\xi - \cos u)]$$

$$= \cos u - \sin^2 u/(ch\xi - \cos u)$$

$$\cos u - \cos\theta = \sin^2 u / (ch\xi - \cos u)$$

$$\sqrt{\cos u - \cos\theta} = \sin u / \sqrt{ch\xi - \cos u} \quad \eta \qquad u_0$$

$z_c = a /\tan(u_0) = a\cot(u_0) - z$

$R = a / |\sin(u_0)\}$

Ref:

$x = a\cos\varphi\ sh\xi/(ch\xi - cosu)$ $\rho = a\ sh\xi/(ch\xi - cosu) = \sqrt{x^2+y^2}$

$y = a\sin\varphi\ sh\xi/(ch\xi - cosu)$ $h_\xi = h_u = a/(ch\xi - cosu)$

$z = a\ sinu/(ch\xi - cosu)$ $h_\varphi = a\ sh\xi/(ch\xi - cosu)$ (5.1)

$\rho^2 = x^2+y^2$ ρ

**I will now try to prove this**, since I can find no proof easily. // I have spent 2 hours trying to prove this and have still not succeeded. I need to find it proved in one of my docs. I *am* able to show this from the above picture

$\sin\theta = sh\xi\ sinu / (ch\xi - cosu)$

which then relates θ to the toroidal coordinates ξ and u. This equation then says

$\sin\theta/sh\xi = sinu / (ch\xi - cosu)$ (**)

Now if (*) and (**) are both valid, then it must be that

$\sqrt{cosu - cos\theta} = \sin\theta/sh\xi$ ?

$cosu - cos\theta = \sin^2\theta/sh^2\xi$ ?

Now here are my toroidal equations,

$x = a\cos\varphi\ sh\xi/(ch\xi - cosu)$ $\rho = a\ sh\xi/(ch\xi - cosu) = \sqrt{x^2+y^2}$

$y = a\sin\varphi\ sh\xi/(ch\xi - cosu)$ $h_\xi = h_u = a/(ch\xi - cosu)$

$z = a\ sinu/(ch\xi - cosu)$ $h_\varphi = a\ sh\xi/(ch\xi - cosu)$ (5.1)

Where is the x,y,z origin for these equations? It is at the cat's eye center, the point between the foci, it is NOT at the center of the bowl. The equation of the bowl-sphere is this (red), where

$x^2 + y^2 + (z- z_c)^2 = R^2$ $z_c = a /tanu$ $R = a/|sinu|$ . $u = \alpha$ (2.6)

In the above picture, the Origin is between the bowl edges, and the z axis goes from there to the left. Let's just verify this

$[ a\cos\varphi\ sh\xi/(ch\xi - cosu)]^2 + [ a\sin\varphi\ sh\xi/(ch\xi - cosu)]^2 + [a\ sinu/(ch\xi - cosu) - a /tanu]^2 = [a/|sinu|]^2$

$$[\cos\varphi\ \mathrm{sh}\xi/(\mathrm{ch}\xi - \cos u)]^2 + [\sin\varphi\ \mathrm{sh}\xi/(\mathrm{ch}\xi - \cos u)]^2 + [\sin u/(\mathrm{ch}\xi - \cos u) - 1/\tan u]^2 = [1/|\sin u|]^2$$

$$[\cos\varphi\ \mathrm{sh}\xi/(\mathrm{ch}\xi - \cos u)]^2 + [\sin\varphi\ \mathrm{sh}\xi/(\mathrm{ch}\xi - \cos u)]^2 + [\sin u/(\mathrm{ch}\xi - \cos u) - 1/\tan u]^2 = [1/|\sin u|]^2$$

Combine the first two terms

$$[\mathrm{sh}\xi/(\mathrm{ch}\xi - \cos u)]^2 + [\sin u/(\mathrm{ch}\xi - \cos u) - 1/\tan u]^2 = [1/|\sin u|]^2$$

Mult through by $(\mathrm{ch}\xi - \cos u)^2$

$$\mathrm{sh}^2\xi + (\mathrm{ch}\xi - \cos u)^2[\sin u/(\mathrm{ch}\xi - \cos u) - 1/\tan u]^2 = [(\mathrm{ch}\xi - \cos u)^2/|\sin u|]^2$$

$$\mathrm{sh}^2\xi + (\mathrm{ch}\xi - \cos u)\sin u - (\mathrm{ch}\xi - \cos u)^2/\tan^2 u = [(\mathrm{ch}\xi - \cos u)^2/|\sin u|]^2$$

$$\mathrm{sh}^2\xi + (\mathrm{ch}\xi - \cos u)\sin u - (\mathrm{ch}\xi - \cos u)^2 \cot^2 u = (\mathrm{ch}\xi - \cos u)^2 \csc^2 u$$

How on earth is this an identity?

How do you derive (*) ? For a point on the sphere, you know that $\rho^2 + z^2 = R^2$ so then you know

$$a^2\ \mathrm{sh}^2\xi/(\mathrm{ch}\xi - \cos u)^2 + a^2 \sin^2 u/(\mathrm{ch}\xi - \cos u)^2 = R^2$$

or

$$a^2[\mathrm{sh}^2\xi + \sin^2 u]/(\mathrm{ch}\xi - \cos u)^2 = R^2$$

STOP! A point on the sphere is a point on the bowl and has a constant value of u. The value of ξ should not be in any way restricted, but the above equation says the ξ is a function of u! Let's make sure the above is just an identity, so continue on.

But I also know from bipolar doc that $R = a/|\sin u|$ so the above becomes

$$a^2[\mathrm{sh}^2\xi + \sin^2 u]/(\mathrm{ch}\xi - \cos u)^2 = a^2/\sin^2 u$$

$$[\mathrm{sh}^2\xi + \sin^2 u]/(\mathrm{ch}\xi - \cos u)^2 = 1/\sin^2 u$$

$$\sin^2 u[\mathrm{sh}^2\xi + \sin^2 u]/(\mathrm{ch}\xi - \cos u)^2 = 1$$

$$\sin^2 u\ [\mathrm{sh}^2\xi + \sin^2 u] = (\mathrm{ch}\xi - \cos u)^2$$

$$\sin^2 u\ \mathrm{sh}^2\xi + \sin^4 u = \mathrm{ch}^2\xi - 2\mathrm{ch}\xi\cos u + \cos^2 u$$

$\sin^2 u\,(\mathrm{ch}^2\xi - 1) + \sin^4 u = \mathrm{ch}^2\xi - 2\mathrm{ch}\xi\cos u + \cos^2 u$

$\sin^2 u\,\mathrm{ch}^2\xi - \sin^2 u + \sin^4 u = \mathrm{ch}^2\xi - 2\mathrm{ch}\xi\cos u + \cos^2 u$

$\sin^2 u\,\mathrm{ch}^2\xi + \sin^4 u = \mathrm{ch}^2\xi - 2\mathrm{ch}\xi\cos u + 1$

$\sin^4 u = \mathrm{ch}^2\xi\,(1-\sin^2 u) - 2\mathrm{ch}\xi\cos u + 1$

$\sin^4 u = \mathrm{ch}^2\xi\cos^2 u - 2\mathrm{ch}\xi\cos u + 1$

$\sin^4 u = (\mathrm{ch}\xi\cos u - 1)^2$

$\sin^2 u = \mathrm{ch}\xi\cos u - 1$

This does NOT look like an identity to me. Maybe it is a clue to what I am blinded from seeing.

**Nov 28, 2015**

I have cleaned up the full-sphere and disk limits of the potential and all is well, and all is added to bowl doc in great detail. This amounted to a rewrite of the entire tail end of Section 4. The first part of this section still has very major problems in terms of quality of presentation.

I guess I am now committed to a full upgrade of bowl doc. This was an early doc and it just did not get the quality infusion that later docs got. I want to always do in-line calculations, not quote results from notes I have that I can never trace.

In any event, the limit mystery is solved, just do it right and things work out. I can now respond to the emailer without worrying about my results being wrong. I will do that manana. Now 9:30 and sign off.

**Sun Nov 29, 2015**

Today I rewrote most of Chapter 4 using my more recent "style" of showing all calculations and having good equation/figure numbers. The entire bowl paper needs to be redone in this style and that is going to take a lot of effort. I don't want to do that right now because I am in the middle of other duties, so this bowl paper update is officially now put on hold. I did not find any disasters that are wrong in it, so no need to "pull it" from the web.

Well I did lots of other things and I am back at 6 PM same day, doing a bowl paper free edit pass.

2.1 Ellipsoidals and a dashed hope

I like this section, the hope was to have just a single function as the answer like it is for an ellipsoid.

2.2 About the atomic forms. No details yet of my toroidal coordinates.

I was sailing along, then took a multi-hour hit thinking I could trivially solve for $A(\tau)$ and $B(\tau)$ with Maple or by hand. I realize now that the solution I give is extremely tricky to obtain, and I will have to add Appendix H to do this fancy calculation which is critical to the whole business. Tomorrow I will write that appendix and integrate it in. Maybe I will put other small calculations in there as well.

**Mon Nov 30, 2015**

I really have to do something about my non-mention of the coordinates early-on in this doc.

OK, I have rewritten Section 1 to include a bipolar section and then a toroidal section where I have derived everything from scratch and have gathered it all up in one subsection. I will now review Section 2 and fix references so they point to this new Section 1.

2.1 ok
2.2 ok
2.3 ok, explained why Q must be rejected.
2.4 ok, here is where I get the solution to the bowl.

Well, this is going to be a very long road. I got things much improved today but there are dozens of miles yet to travel, and things only get harder as I move deeper into this monster.

**Tues Dec 1, 2015**

I am unable to show that the disk limit of my bowl potential matches Jackson's disk potential. Started a separate doc on that, it almost seems to be untrue. But I also have a problem with my $\eta_1$ and $\eta_2$ signs, so now is a good time to review Chapter 8 where these signs first appear!

$f(z) = (a+u)^{1/2} = (a + \cos(z))^{1/2}$ $\qquad$ $f(z) = \sqrt{a+\cos(z)}$

OK, this led to my writing Appendix I detailing out this very trick subject. Improved the picture, it is all good, much better than my passing quick comment with the weird unexplained picture.

So I am fully up on this $\eta$ sign factors, and I can resume tomorrow on the mystery of why my bowl potential limit does not seem to match Jackson's limit. This is a limit I never considered when doing this paper originally.

**Weds Dec 2, 2015.**

Found a trivial way to get rid of those $\eta$ sign factors AND to get rid of Appendix I. This appendix explains an important idea, but it is not needed in the bowl paper, so it will rest aside but intact. I checked the bowl plots with the new code and it looks the same of course.

I have now completed my edit of Section 8, it is MUCH better and simpler than before. Put a box around the final result.

Might as well do Section 9 right now. Done.

So recently I have edited **Section 8** and **Section 9** and it is all very good, constant simplifications.
I then did a full edit of **Section 7** on the Mehler integrals, added references, cleaned it up.

**Section 6** is also just fine, reviewed it just now.

Now we can return to the mystery of why the disk limit does not give the Jackson potential!

**Sat Dec 5, 2015.**

I have been busy. Just rewrote Section 4.4 on the bowl capacitance, this time "showing all the work" including an Appendix H.4 integral evaluation that neither Maple or GR7 could do directly. It is all there, I added the sphere and disk limits, and everything is fine there.

Before that effort I spent a lot of time trying to get my "evaluated" bowl potential to produce the right capacitance. This effort is now in "finding the large r behavior". One of the two terms in C is just not appearing, despite lots of checking on my part! I just see now that I faced this problem as well in the document "capacitance of the toroidal bowl v1 ".

I finally solved this problem and it is written up in " large r behavior ... v2".

OK I now have lots of "potential limits" in Section 2.5 to verify my trig potential form every which way. I am very confident now that it is correct.

I would like to work in the Cartesian coordinates version of the potential somewhere, an Appendix. I have this written up here: "capacitance of the toroidal bowl v3" . Here is what I wrote there

$$V(\xi,u)= (V_0/\pi)\sqrt{ch\xi - cosu}\ *$$
$$\{ (1/\sqrt{ch\xi - cosu})\quad \cot^{-1}[(-1)^{n1}|\sqrt{1+cosu}|\ /\sqrt{ch\xi - cosu}]$$
$$+\ (1/\sqrt{ch\xi - cos(2u_0-u)})\ \cot^{-1}[(-1)^{n2}|\sqrt{1+cos(2u_0-u)}|/\sqrt{ch\xi - cos(2u_0-u)}]\ \}$$

where

$$Q \equiv 1/\sqrt{[z^2 + (a-\rho)^2][z^2 + (a+\rho)^2]}$$
$$ch\xi - cosu = 2a^2Q$$
$$ch\xi = (a^2+\rho^2+ z^2)Q$$
$$\cos u = (a^2-\rho^2- z^2)Q$$

I think the sign is wrong here!

$$\sin u = 2az\ Q$$
$$a = R\sin u_0$$

only now I would use // disk wants $\pi < u < 3\pi$

$$V_{disk}(\xi,u) = (2V_0/\pi)\ \cot^{-1}[\frac{-\sqrt{2}\cos(u/2)}{\sqrt{ch\xi-cosu}}]\ . \qquad (2.5.4)$$

Let's just see if this flies. I need

$$\cos^2(u/2) = (1 + \cos u)/2 = (1/2)(1 + (a^2-\rho^2- z^2)Q )$$

$$\cos(u/2) = -\sqrt{(1 + (a^2-\rho^2- z^2)Q )}/\sqrt{2}$$

That ratio looks pretty ugly,

$$\frac{-\sqrt{2}\cos(u/2)}{\sqrt{ch\xi-\cos u}} = \frac{\sqrt{(1 + (a^2-\rho^2- z^2)Q )}}{\sqrt{2}\, a\sqrt{Q}}$$

Trying to get

$$V_{\mathbf{disk}}(\xi,u) = (2V_0/\pi)\ \sin^{-1}\left[\frac{2a}{\sqrt{(\rho-a)^2+z^2} + \sqrt{(\rho+a)^2+z^2}}\right]$$

So the claim is that

$$\cot^{-1}\left[\frac{\sqrt{(1 + (a^2-\rho^2- z^2)Q )}}{\sqrt{2}\, a\sqrt{Q}}\right] = \sin^{-1}\left[\frac{2a}{\sqrt{(\rho-a)^2+z^2} + \sqrt{(\rho+a)^2+z^2}}\right]$$

Write

$$A^2 = z^2+ (\rho-a)^2$$
$$B^2 = z^2+ (\rho+a)^2 \qquad\qquad Q = 1/AB$$

$$A^2 + B^2 = 2z^2 + 2\rho^2 + 2a^2 = 2(z^2+\rho^2+a^2)$$

$$-(A^2 + B^2)/2 = -z^2-\rho^2-a^2 = (-z^2-\rho^2+a^2) - 2a^2$$

$$(-z^2-\rho^2+a^2) = 2a^2 -(A^2 + B^2)/2$$

Then want to show

$$\cot^{-1}\left[\frac{\sqrt{(1 + [2a^2 -(A^2 + B^2)/2]/AB )}}{\sqrt{2}\, a\sqrt{1/AB}}\right] = \sin^{-1}\left[\frac{2a}{A+B}\right]$$

or

$$\cot^{-1}\left[\frac{\sqrt{(AB + [2a^2 -(A^2 + B^2)/2] )}}{\sqrt{2}\, a}\right] = \sin^{-1}\left[\frac{2a}{A+B}\right]$$

On scratch this is very close! I have some signs wrong. Suppose my cosu sign is wrong. Then we would want to show instead that

$$\cot^{-1}\left[\frac{\sqrt{(AB - [2a^2 -(A^2 + B^2)/2] )}}{\sqrt{2}\, a}\right] = \sin^{-1}\left[\frac{2a}{A+B}\right]$$

Then draw a triangle and it works perfectly!!!

Status 8 PM. I wrote Appendix J showing how to convert anything to Cartesians, then I used the disk as an application of these equations. Maybe I am now done with appendices. Just picking up all the loose ends.

**Dec 7, 2015**

Wrote Appendix H.1 to solve the equations for $A(\tau)$ and $B(\tau)$ as needed in Chapter 2. Done.

Before starting yet another review, let's clean up Chapter 5. // OK, I spent lots of hours doing this, it is very much better, elaborate pictures and clean explanation, last time things were vague. And of course the toroidal primer is removed since show that in Section 1. Got Section 5 fully indexed as well.

What's next? I went through Chapter 6 and added an opening comment about electric fields and the plots, took a while to get it right So Ch 6 seems OK.

At this point I am generally OK with Ch 1-7, but I have done nothing yet with Ch 10 and the toroid, not so mention most of the appendices.

I added a "tiny hole" $\sigma_{\mathtt{in}}$ analysis after the comparison with other people's $\sigma_{\mathtt{in}}$ expressions.

I can see that there is still much cleanup needed even before the toroid is dealt with.

**Dec 8, 2015**

Cleaned up yesterdays small hole presentation and noted it in the TOC as "plots". Mentioned the earlier capacitance result in the later calculation. Trying to make things have continuity, so to speak, not to sound like each section never heard of the other sections.

Reread Ch 5, it is very good, added a wire frame of the u surface, which really explains its shape!

Let's now take a stab at Chapter 10.

I am removing this sentence after the comparison

(lost it, claiming that sphere gives lowest C for some reason), maybe OK sentence after all.

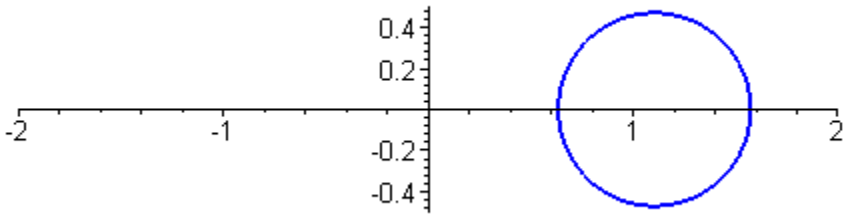
For the first time, I am trying to plot the toroid potential, it is a rough thing to do since have a sum of terms for each point. But I got it started and I at least see something on 1D plots, but something is wrong as well.

So I am well underway into the toroid chapter 10!

**Dec 9, 2015**

I opened today by adding spherical coordinate surfaces to Chapter 9 to try and demystify the meaning of my plots. I think it helps.

Now back to plotting toroid potential. I select $\xi_0 = 1.5$ and a = 1 so we have this




which shows the general scale of things.

**Dec 11, 2015**

Yesterday I worked on the toroid section up to the capacitance. Got the "number of terms" figured out, have Q evaluation problems and documented all that, got nice plots of the toroid potential. Then started into the capacitance section.

Today was OB at 3:30 AM! I worked on the bowl index and got it done up to this capacitance stuff, that took many hours of hunting around for each of 50 items in vsd and mws files. So that is done.

Puzzle: Why did I have trouble with the Q function yesterday, but I did not have trouble when doing the sum rule check of Appendix B?

Today I tried to index App B but found that key files are missing. So I recreated the sum rule code in a brand new file called the Q sum rule.mws. Was not hard to do.

**Question**: The sum rule is supposed to add up to 2.22 but my stable sum adds up to 2.18. Why is that?

The sum rule comes from here

$$1/\sqrt{a - b\cos(x)} = (1/\pi)\sqrt{2/b}\ \Sigma_{n=0}^{\infty}\ \varepsilon_n\ Q_{n-1/2}(a/b)\cos(nx) \qquad \text{// expansion}$$

$$\int_0^{\pi} dx\ \cos(nx)/\sqrt{a - b\cos(x)} = \sqrt{2/b}\ Q_{n-1/2}(a/b) \qquad \text{// projection} \qquad (10.1.8)$$

You set b = 1 and x = 0 a = z and the first line says

$$1/\sqrt{z - 1} = (1/\pi)\sqrt{2}\ \Sigma_{n=0}^{\infty}\ \varepsilon_n\ Q_{n-1/2}(z) \qquad \text{// expansion}$$

Then the sum rule is this

$$\sqrt{z-1}\ \Sigma_{n=0}^{\infty}\ \varepsilon_n\ Q_{n-1/2}(z)\ =\ (\pi/\sqrt{2}\ )\ =2.221441469 \qquad \text{// sum rule}$$

and this should be true for any z! Can rewrite as

$$\sqrt{ch\xi-1}\ \Sigma_{n=0}^{\infty}\ \varepsilon_n\ Q_{n-1/2}(ch\xi)\ =\ (\pi/\sqrt{2}\ )\ =2.221441469 \qquad \text{// sum rule}$$

MAJOR BREAKTHROUGH!!!! (noon)

While reviewing the Bateman Q function options, I tried out form (45) in place of form (36) for the Q function. My reason for doing this was just because it seemed to have a nice convergence picture in my document "convergence of Bateman forms.doc" . But then I realized that picture was wrong, I fixed it, and there is still a branch point at z = 1 so it is not ideal. BUT, I just tried it anyway in those sum rule formulas and it cleaned ALL of them up, no more rising tails for large term counts!! I am absolutely astounded at this piece of luck I just had!!

I cleaned up the end of Section 10, but not quite done with capacitance and charge density.

Skipping now to Appendix B, I want that cleaned up.

I am rebuilding the table in Appendix B

| z | T(z) | terms used | $-\log_{10}(z-1)$ |
|---|---|---|---|
| 1.1 | 2.8458184043445624867 | 15 | 1 |
| 1.01 | 2.5638505479853129010 | 140 | 2 |
| 1.001 | 1.6894173781562543317 | 300 | 3 |
| 1.0001 | .83988995411338681764 | 600 | 4 |
| 1.00001 | | 1200 | 5 |
| 1.000001 | | 4000 | 6 |

(B.5)

STOP. What am I trying really to do here. I start with this

$$C(z)\ =\ (2R/\pi)\sqrt{z^2-1}\ \ \Sigma_{n=0}^{\infty}\ \varepsilon_n\ [Q_{n-1/2}(z)/\ P_{n-1/2}(z)]\ . \qquad z=ch\xi\ =\rho_c/R\ . \qquad (B.1)$$

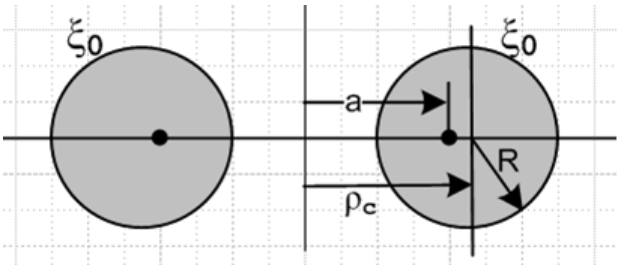
There are two parameters here R and z for capacitance. You can say

$$R\sqrt{z^2-1}\ =R\sqrt{(\rho_c/R)^2-1}\ =\sqrt{\rho_c^2-R^2}$$

Then

$$C=(2/\pi)\sqrt{\rho_c^2-R^2}\ \ \Sigma_{n=0}^{\infty}\ \varepsilon_n\ [Q_{n-1/2}(\rho_c/R)/\ P_{n-1/2}(\rho_c/R)]$$

Look at the picture:




I really want $\rho_c$ fixed and I want then to vary R from 0 to $\rho_c$. The makes the toroid start as a wire and end up as a degenerate. The is the "video" I want to talk about. So set $\rho_c = 1$ to get

$$C(R) = (2/\pi)\sqrt{1^2 - R^2}\ \ \Sigma_{n=0}^{\infty}\ \varepsilon_n\ [Q_{n-1/2}(1/R)/\ P_{n-1/2}(1/R)]$$

$$T(R) = \sqrt{1^2 - R^2}\ \ \Sigma_{n=0}^{\infty}\ \varepsilon_n\ [Q_{n-1/2}(1/R)/\ P_{n-1/2}(1/R)]$$

This is the function I should talk about, not some C(z) thing. I will then be interested in the limit of this thing as R→ 2. So forget T(z) and just use this function as you see it here.

I am now doing R from .99999 to 1 with 1000 terms in the series. It is taking a long time, so I want to carefully preserve the result here if it is a good result. Each time I shrink the range, I get to see more closely to the limit R = 1, and I get the limit of C more closely. It is really cranking!

Well this is wasteful, I don't need a smooth curve with 500 points, I just need maybe 10 points to see what is happening. I tested and if you say numpoints = 10, that is how many plot makes. But it still has to cank a long time. Let's drop to .9999 and do that.

OK, I can present my main result in a single calculation where I do 8 little points approaching R = 1 starting at .99999 and I show both the C and T values.

Then I can do the wronskian trick and do it with a different series and see what that says.

```
\
>
> Rmin := 0.99999;
```

$$Rmin := .99999$$

```
> Nterms := 4000;
```

$$Nterms := 4000$$

```
> for i from 0 to 7 do
    R := Rmin+(1-Rmin)*(i/8);
    r[i] := R;
    sumterms := add(term(n),n=0..Nterms);
    C[i] := evalf(2/Pi)*sqrt(1-R^2)*sumterms;
    T[i] := sqrt(1-R^2)*sumterms;
    #print(i,"     ",R, "     ",C);
    printf("i=%d  R=%f6  C=%f10  T=%f10\n",i,r[i],C[i],T[i]);

  od:
i=0  R=.9999906  C=1.74137010  T=2.73533810
i=1  R=.9999916  C=1.74137110  T=2.73534010
i=2  R=.9999936  C=1.74137310  T=2.73534210
i=3  R=.9999946  C=1.74137410  T=2.73534410
i=4  R=.9999956  C=1.74137510  T=2.73534610
i=5  R=.9999966  C=1.74137610  T=2.73534810
i=6  R=.9999986  C=20.75712610  T=32.60521710
i=7  R=.9999996  C=14.32191410  T=22.49681010
```

With the Wronskian method, I get the following stunning results,

```
Rmin := 0.999999;

Nterms := 40000;
```

```
i=0  R=.99999900  C=1.74138042  T=2.73535397
i=1  R=.99999913  C=1.74138029  T=2.73535376
i=2  R=.99999925  C=1.74138013  T=2.73535351
i=3  R=.99999938  C=1.74138064  T=2.73535431
i=4  R=.99999950  C=1.74138056  T=2.73535419
i=5  R=.99999963  C=1.74138049  T=2.73535407
i=6  R=.99999975  C=1.74138041  T=2.73535396
i=7  R=.99999988  C=1.74138034  T=2.73535384
```

OK, I am now writing a new Appendix B and will continue tomorrow.

**Dec 12, 2015**

Did more math work in App H.5 and H.6 to support the thin wire limit. Then started into the thin wire part of the new Appendix B. Found the factor ln(8z) factor of 8 which is important. A day of just plugging away.

**Dec 13, 2015.**

Did more work in the thin-ring toroid limit in App B.1. Added two toy analyses that give the correct result for capacitance. Rejected the Jackson potential of a thin ring on the axis idea as a test limit of the general potential formula. Since this is $\xi_0 \to \infty$, it just repeats what I have already done.

I am ready now to start the horn toroid Section B.2 which is my Big Deal.

**Dec 14,15 2015**

Worked on Appendix B, supporting Appendix H, examples of series doing non-uniform convergence.

**Dec 16**

Hoping to get Appendix B finished today. First step is updating the index for what was done in last several days. // Did that for App B v2 first two sections, today is section 3.

Pause to get Appendix H finalized and installed. Why do I have a hole for H.4, what was that supposed to be??? I don't know! I think I was saving that for something. I don't see anything written yet. OK, I will leave H.5 as is and deal with this later. I may have to relocate everything in the end. Do not install now.

Reviewed Section B.1

At 9:15 AM I am finally ready to start on Section B.3.

At 7:30 I am done with Appendix B, finally, and have it indexed as well. Appendix H keeps getting new sections as I continue to provide a trace for everything claimed.

Tomorrow I want to integrate this Appendix B with the Section 10 main text.
Then comes the toroid charge density calculation which will require more work to support.

**Dec 17, 2015**

Am proofing Chapter 10 again from the start

10.1 OK
10.2 OK
10.3 OK
10.4 STOP.

Found a huge bug in my semilogplot which was showing false knee in Appendix B. I was wondering why it was a straight line but my old similar plot was curved. This is a bug in the log plot thing which I noted in lines doc, and which now has a descriptive doc in the Maple folder area! So I repaired Appendix B, and then used the plot there in Section 10.4. So we can now continue

10.1 OK
10.2 OK
10.3 OK
10.4 OK

Section 10.5 is the charge density calculation which I STILL have not gotten to as of 4:30 PM !!! Ready to start it, but have to do other things first.

My σ derivation is in the toroidal cap function. doc, but I have not external verification! I really need to find someone else who does this problem. Perhaps Smythe? M&F? Lebedev?

Lebedev: He gives the capacitance result, but no go on σ.

Smythe: No, I searched for "toroid"

M&F: no.

Web search

I found a Wilson paper which has these results:

$$\psi(\xi,\eta) = \frac{4E_0 c}{\pi}\sqrt{2\cosh\xi - 2\cos\eta}\cdot\sum_{n=1}^{\infty} n\frac{Q_{n-1/2}(\cosh\xi_0)P_{n-1/2}(\cosh\xi) - Q_{n-1/2}(\cosh\xi)P_{n-1/2}(\cosh\xi_0)}{P_{n-1/2}(\cosh\xi_0)}\cdot\sin n\eta. \qquad (14)$$

which seems oddly different from my form

$$V(\xi,u) = V_0\frac{\sqrt{2}}{\pi}\sqrt{ch\xi - cosu}\ \Sigma_{n=0}^{\infty}\varepsilon_n P_{n-1/2}(ch\xi)\frac{Q_{n-1/2}(ch\xi_0)}{P_{n-1/2}(ch\xi_0)}\cos(nu)\,. \qquad (10.1.11)$$

This result can be transformed to Smythe's (1968, 1974) answers using Whipple's identity (Cohl et al., 2000). Charge density on the surface $\xi = \xi_0$ is

$$\sigma(\eta) = -\varepsilon_0 \frac{\partial}{\partial n}\psi(\xi_0, \eta) = \frac{\varepsilon_0}{h_\xi}\frac{\partial \psi(\xi_0, \eta)}{\partial \xi} = \frac{\varepsilon_0}{c}[\cosh \xi_0 - \cos \eta]\frac{\partial \psi(\xi_0, \eta)}{\partial \xi}. \qquad (15)$$

Differentiation of the total potential gives, with the Wronskian (Lebedev, 1972, 7.7.2),

$$\sigma(\eta) = \frac{2\varepsilon_0 E_0}{\pi}\frac{[2\cosh \xi_0 - 2\cos \eta]^{3/2}}{\sinh \xi_0}\sum_{n=1}^{\infty}\frac{n}{P_{n-1/2}(\cosh \xi_0)}\sin n\eta. \qquad (16)$$

As $b/a \to 0$, $\xi_0 \to \infty$ and the $n = 1$ term dominates, so that the surface charge density on a thin toroid is

$$\sigma(\eta) \xrightarrow[b/a \to 0]{} 2\varepsilon_0 E_0 \sin \eta. \qquad (17)$$

This is the classical solution for the charge density on an infinitely long circular cylinder in the corresponding $\mathbb{R}^2$ potential problem.

I see now that Smythe has a torus potential solution in a problem

**118.** The torus generated by the circle of radius $b$ given by $u_1 = u_0$ in the last problem is charged to a potential $V_0$. Show that the potential outside it is

$$2V_0(\cosh u_1 - \cos u_2)^{\frac{1}{2}}\sum_{n=0}^{\infty}\frac{(-2)^n(2n+1)P^n_{-\frac{1}{2}}(\coth u_0)P_{n-\frac{1}{2}}(\cosh u_1)}{(2n+1)!!(\sinh u_0)^{\frac{1}{2}}P_{n-\frac{1}{2}}(\cosh u_0)}\cos nu_2$$

where $\cosh u_0$ is $c/b$, $c$ being the distance from the center of $u_0$ to the line $u_1 = 0$.

which compare to my result

$$V(\xi,u) = V_0\frac{\sqrt{2}}{\pi}\sqrt{\mathrm{ch}\xi - \cos u}\ \Sigma_{n=0}^{\infty}\varepsilon_n P_{n-1/2}(\mathrm{ch}\xi)\frac{Q_{n-1/2}(\mathrm{ch}\xi_0)}{P_{n-1/2}(\mathrm{ch}\xi_0)}\cos(nu)\ . \qquad (10.1.11)$$

It would seem that I can translate his result to my notation taking $u_1 \to \xi$, $u_2 \to u$. Then I would have to show that this strange fact was true

$$(-2)^n(2n+1)\,P^n_{-1/2}(\coth u_0)\,P_{n-1/2}(\mathrm{ch}\xi) / (2n+1)!!\,\sqrt{\mathrm{sh}(u_0)}\,P_{n-1/2}(\mathrm{ch}u_0)$$

Stop. The Smythe result is very ugly for many reasons:

(1) it has three variables he calls $u_0$, $u_1$ and $u_0$ whereas I have only two.

(2) one of his P functions has two indices!

(3) there are lots of messy factors.

I think the previous author says there is a Whipple connection.

Back up to the M&F results. Where did they make their mistake (that I claim they make)? I cannot see where their thing is derived! It is just quoted after a discussion of something unrelated. But maybe I can get a hint. I quote this sum rule

$$1/\sqrt{a - b\cos(x)} = (1/\pi)\sqrt{2/b}\ \Sigma_{n=0}^{\infty}\ \varepsilon_n\ Q_{n-1/2}(a/b)\cos(nx) \qquad \text{// expansion}$$

If I set a = chμ and x = η and b - 1 this would say

$$1/\sqrt{ch\mu - \cos(\eta)} = (1/\pi)\sqrt{2}\ \Sigma_{n=0}^{\infty}\ \varepsilon_n\ Q_{n-1/2}(ch\mu)\cos(n\eta) \qquad \text{// expansion}$$

whereas M&F say

$$\frac{1}{\sqrt{\cosh\mu - \cos\eta}} = \frac{\sqrt{2}}{\pi}\sum_{n=0}^{\infty} Q_{n-\frac{1}{2}}(\cosh\mu)\cos(n\eta) \qquad (10.3.79)$$

so THIS is where they are missing their $\varepsilon_n$ factor which is 2 for the n=0 term.

So I have other support on my sum rule (of course numerically I have shown it is correct! )

Aha! Deriving it is supposed to be my Missing Appendix H.4. I should do this, and also quote other people on the sum rule to convince myself that M&F are really wrong.

So where is my derivation of the transform shown as bowl (10.1.8)? Its in temp1.doc! But my proof is not really very strong. A quick web scan turns up little. I will roll my own derivation using an integral representation for Q.

AT 5 PM I have finished deriving the transform. It is now in Appendix K. I added all known appendices, and there is a hole in Appendix H which I leave for the time being. The Q stuff was too big to fit into that hole.

So now it is time to derive the σ density for the toroid. Then I will search for verification. Meanwhile, where is my existing σ derivation? toroidal cap function doc.

I have a result for σ and I am trying to integrate σ over the surface to verify the result, cannot even get the surface area of a toroid! Resume manana. A full day on σ, really my first day on this.

**Dec 19, 2015**

Worked on the σ situation, was able to integrate to get the right answer, it was a huge effort. Very complicated, but it worked. Then I started writing 10.5 on σ. I did the thin-ring limit and got a good result there. Realized the horn limit has the same uniformity issue so can't really get a simple limit there. So pieces are coming together finally.

**Dec 21, 2015**

I finished Chapter 10, Maple Visio doc and everything, and have it indexed. Learned how to search for text in Visio files.

I am not happy about the appendix ordering. Appendix B is out of place because it is on toroid. Here is what I now have. On the right I show where referenced from:

| | | |
|---|---|---|
| A | Maple code for the **bowl**. | Chapter 9 |
| B | **Toroid** capacitance two limits, wire and horn | Chapter 10 |
| C | Kelvin and **bowl** | Overview |
| D | Smythe and **bowl** | Overview |
| E | Dual Equations and **bowl** | Overview |
| F | Abel transforms (needed in G) | Overview |
| G | Solving **Bowl** with double Abel | Overview |
| H | Support, integrals and P Q limits, a grab bag, and H.4 is blank! | Mostly Chapter 10 |
| I | The behavior of $f(z) = \sqrt{a+\cos(z)}$ as an analytic function | a comment in Chapter 8 |
| J | Converting toroidals to cartesians | Chapter 2 |
| K | Fourier Cosine Series Transform and Apps | Chapter 10 |
| L | Integration of **toroid** surface charge density | Chapter 10 |

I think the need for Appendix I went completely away! I previous Chapter 8 I had the little picture and the phases $(-1)^n$. The reason is that now I have gotten rid of things like $\sqrt{1+\cos(2u_0-u)}$ and replaced them by half angle things were there is no ambiguity! I worked in a reference to Appendix I, but I just think it is NOT RELEVANT and should be removed and put in a safe place. That puts a hole at App I.

Here is an idea for changing the above list

| | | |
|---|---|---|
| A | Maple code for the **bowl**. | Chapter 9 |
| J | Converting toroidals to cartesians | Chapter 2 |
| C | Kelvin and **bowl** | Overview |
| D | Smythe and **bowl** | Overview |
| E | Dual Equations and **bowl** | Overview |
| F | Abel transforms (needed in G) | Overview |
| G | Solving **Bowl** with double Abel | Overview |
| H | Support, integrals and P Q limits, a grab bag, and H.4 is blank! | Mostly Chapter 10 |
| B | **Toroid** capacitance two limits, wire and horn | Chapter 10 |
| K | Fourier Cosine Series Transform and Apps | Chapter 10 |
| L | Integration of **toroid** surface charge density | Chapter 10 |

Then I back up the last two to be J and K.

Impacts:

1. All appendix references need to be checked
2. The index needs to be updated.

So I will now undertake this reorg.

1. Save off Appendix on f(x) = $\sqrt{a+\cos(z)}$ DONE, went to separate archive doc.
2. Include its index data at the top of that doc, remove from index file.
   There was no index data yet generated for this appendix.
3. Remove referencing comment and place at the start of that doc.
   OK. Appendix I refs no longer exist in bowl doc. And no (I. refs either.

4. Move Appendix B to a scratch temp doc.
5. Change all B. to I.
6. Edit the index document in the same way, done.
7. Install this new Appendix I in the right place. done
8. Repair all references by searching the doc for (B. or Appendix B. done
9. Delete the temp doc.

10. Move the converting toroidals J into a temp doc.
11. Change all J. to B.
12. Install this into bowl doc as Appendix B.
13. Redirect all refs to J. to B., and Appendix J to Appendix B. done
14. Delete that temp file.
15. Update index file. But nothing there yet, so nothing to do.

16. Move Appendix K to a temp file.
17. Change all K. to J.
18. Copy this back to bowl doc.
19. Redirect all refs to K. to J., and Appendix K to Appendix J. done
20. Update index doc, nothing to do.
21. Delete the temp file.

22. Update Appendix L in place, changing L. to K. done.
23. Change all refs to L. to K., and Appendix L to Appendix K. done
24. Update index file. Nothing to do.

OK, here then is where we stand with appendices

| | | |
|---|---|---|
| A | Maple code for the **bowl**. | Chapter 9 |
| B | Converting toroidals to cartesians | Chapter 2 |
| C | Kelvin and **bowl** | Overview |
| D | Smythe and **bowl** | Overview |
| E | Dual Equations and **bowl** | Overview |
| F | Abel transforms (needed in G) | Overview |
| G | Solving **Bowl** with double Abel | Overview |
| H | Support, integrals and P Q limits, a grab bag, and H.4 is blank! | Mostly Chapter 10 |
| I | **Toroid** capacitance two limits, wire and horn | Chapter 10 |
| J | Fourier Cosine Series Transform and Apps | Chapter 10 |
| K | Integration of **toroid** surface charge density | Chapter 10 |

I still have a hole in H.4 to deal with. I have lots of bad refs to this hole!

Task:  clean up all refs to App H equations. Done.

Task:  fix all ∞ symbols which are in Courier font.



φ

I think that picked them all up. Most are out in the open, it is the superscripts I wanted to fix and did.

Decision:  Do I have Chapters or Sections?  Lets have Chapter 1, but Section 1.1?
When you have a book, they are chapters. When you have a short paper, they are NOT chapters, they are sections. So do I have a book or a paper?  It is 147 pages. Web says 20 pages is an article.

In Bipolar I called them Sections, not Chapters. Tensor doc has Chapters, is 400 p.

No, this is a shorter thing, let's use Sections, topics are not separated enough to be Chapters.

Nothing to do, I already call things Sections. That is how it will be.

**Task**:  Do the index for all appendices I have so far reviewed.

I found section H.4 hiding inside the Appendix L document, it is now installed.

I think the final structure is now in place. I have of course to review appendices I have so far ignored.

**Dec 22, 2015**

**Appendix C** on Kelvin. Doing an edit pass, added a quip on inversion theory. It was confusing, I think now much less confusing.  This is in fact a very good appendix IMHO!!

**Appendix D.** Did a pass on this, new eq nums, fixed references to main text. The point here is how Smythe solves the bowl problem, but then I pile on extra stuff comparing the iris and the disk. Done.

**Appendix E:**

In Sneddon's treatment, the $P_n(\cos\theta)$ are replaced by Jacobi polynomials $P_n^{(\alpha,\beta)}(\cos\theta)$ so the bowl problem is seen to be a special case with $\alpha = \beta = 0$. [ remove this sentence. ]

OK, I have reviewed App E several times, fixing small bugs, it is better and it is good and relevant.

**Appendix F:** reviewed and OK, it states various forms of the Abel transform, then had very obscure added material which I sort of like.

**Appendix G:** This is very heavy duty indeed.

opening OK
(a) OK
(b) OK
(c) OK
(d) OK

I renamed the sections, but maintained the old equation numbers. What a painful calculation!

Still undone

Appendix A update the code **DONE**
Maybe add the toroid code there as well? **DONE**
update the entire overview summary section.
add references in red
maybe there are other references as well?
check for red text in the main body, links not closed

**Dec 23, 2015**

What should I do about inconsistency in arguments of arctan2Pi in terms of rise run?
Bipolar uses arctan2Pi(rise,run) whereas Bowl uses arctan2Pi(run,rise) = arctan2Pi(x,y).
Answer: leave as is. Stay with the x,y form in Bowl.

Need to change Bateman Q from (36) to (45) in Chapter 7. Here is the old

$Q_\nu(z) = 2^\nu\,[\Gamma(1+\nu)]^2\,(z+1)^{-\nu-1}/\Gamma(2+2\nu) * F(1+\nu,1+\nu,2+2\nu,\, 2/(1+z))$ // Bateman (36)
$Q(\nu,\xi) = Q_\nu(ch\xi) = 2^\nu\,[\Gamma(1+\nu)]^2\,(ch\xi+1)^{-\nu-1}/\Gamma(2+2\nu) * F(1+\nu,1+\nu,2+2\nu,\, 2/(1+ch\xi))$ (7.4.2)

I have edited Chapter 7, but now I have to fix up lots of code to reflect this new function. This update will begin

(7.4.4) is fixed and graphs look the same with the new (45) Q function

Where is the next place I display P and Q in bowl doc? I don't! I always refer back to (7.4.4), so that was lucky. No more fixes to do on this.

Appendix A is updated with both bowl and toroid code, and I did tests of both. Also did a test directly from the PDF. So this appendix stays in place.

OK, the plotting stuff and App A is now all OK, had to do some more repairs and cleanups. Maple 2015 really does have a mws to mw converter, I checked.

Easy to search for red text! Put * in box and select font color red! Found a problem below (3.4), something that needs to go into Appendix H.1 !!! There are in fact three references here which I need to support, perhaps in a new H section.

List of fixes needed:

| | |
|---|---|
| below 3.4 (three occurrences) | will take effort to fix |
| below (E.14) | this is fixed, just a reference to (G.4) |
| below J.3.5 | done |

I fixed the three items noted on the first line above. This resulted in a large expansion of Section H.1 to include double-bowl support. All done and all fine.

Now search for ** occurrences. Found several items, all fixed. There are no more!

These items remain from the AM list:

Appendix A update the code **DONE**
Maybe add the toroid code there as well? **DONE**
update the entire overview summary section.
add references in red **DONE**
maybe there are other references as well?
check for red text and/or ** in the main body, links not closed **DONE**

| u | $\sigma_c$ = sign(cos(u/2)) | $\sigma_s$ = sign(sin(u/2)) |
|---|---|---|
| 0-π | + | + |
| π-2π | - | + |
| 2π-3π | - | - |
| 3π-4π | + | - |

**Dec 24, 2015**

I think the next step is to proof a section, then write its little overview for the summary. I will start with appendices.

DO FULL equation reference checks please!!!
DO FULL eq seq number checks please!!!

Appendix A: OK.

**Appendix A** contains copy-and-paste Maple code for generating plots of the potential of a charged bowl and charged toroid.

Appendix B: (change title from Cartesian to cylindrical
B.1 OK
B.2 OK

**Appendix B** shows how to convert expressions from toroidal to cylindrical coordinates, with application to the potential of a charged disk.

Appendix C Kelvin, OK

**Appendix C** reviews Lord Kelvin's approach to the charged bowl using the theory of inversion.

Appendix D. OK, did various edits and fixed up the picture a bit.

**Appendix D** summarizes a sequence of Smythe problems which use the theory of inversion to explore the properties of a charged bowl by considering the Green's function of a conducting iris with a point charge located in the iris hole.

Appendix E:

**Appendix E** presents a simple matrix theory for solving "dual equations". The theory is then applied to the charged disk in cylindrical coordinates and the charged bowl in spherical coordinates.

Appendix F. S short appendix, nothing is derived!

**Appendix F** states the generalized Abel transforms in several forms and comments on their connection to Legendre and Bessel functions.

Appendix G
opening material OK
G.1 this is awesomely tedious but I guess OK
G.2 I do this check to verify my $g_1$ result and the Vin error. I should send them an email?

G.3 OK, given $g_1$ I compute the $a_n$
G.4 I added a piece to this which makes contact between G and Smythe in D, an amazing result.

**Appendix G** applies the dual equation matrix approach of Appendix E and the Abel transforms of Appendix F to find the bowl potential in spherical coordinates using a double Abel transform .

This concludes my straight shot through appendix A through G which are somewhat related and I was glad to do them all in the same day.

**Dec 25, 2015**

Updated the index for App A thru G.

Appendix H
H.1 read it all, fixed things up, it is good, all fine.
H.2 added GR7 quotes, just fine integral derivation
H.3 just fine, did not clip GR7 for simple integral.
H.4 (a) OK (b) OK, I like the Maple verification.
H.5 very good, $n\to\infty$ limits
H.6 just fine, $x\to\infty$ limits
H.7 just fine, $x\to 1+$ limits
H.8 just fine, no changes.

**Appendix H** contains supporting mathematical details that would further clutter the main text were they not placed here. Equations are solved, integrals are evaluated, and limits are derived. Maple is often used to verify results.

Appendix I.
I.1 the thin wire limit, cleaned up gamma ratio, all OK
I.2 these are two excellent warm-ups for the degenerate toroid limit !!
I.3 well done! Right to the point.

**Appendix I** first examines the thin-wire limit of a toroid and shows how capacitance lingers even as the wire is made extremely thin. In I.2 the theory of series non-uniform convergence is reviewed and applied to two simple series examples. Then in I.3 this theory is applied to determine the capacitance of a horn toroid to 8 decimal places.

Appendix J.
J.1 review of reg bv theory, this is an A+ section, no charge to reader.
J.2 does the fourier cosine series in particular
J.3 just fine.
J.4 the Q sums,.

**Appendix J** reviews the general notation of a transform, and shows how the Fourier Series Cosine Transform fits into that framework. This last transform is then stated for the specific case that the transformed function is $f(x) = 1/\sqrt{a-b\cos x}$ .This result is then used to state the free-space Green's function

in cylindrical coordinates. The final section derives some (perhaps) novel sums of Q functions that are needed in Appendix K.

Appendix K
K.1 done and OK
K.2 done and OK

**Appendix K** does warmup integrations to compute the toroidal circumference and area using toroidal coordinates. The main act then is integration of the toroidal charge density σ to get the total toroid charge Q and from that the capacitance C. This calculation is a check on result (10.5.10) for the toroid surface charge density and makes use of the Q sums of Appendix J.

Done with appendices! I moved the summaries to bowl doc opening section for the moment.

Now we come to the main text.

Chapter 1
1.1 bipolars OK
1.2 toroidals OK

**Chapter 1** reviews bipolar and toroidal coordinates.

Ready for Chapter 2, but day has ended. Did some cleanup of capitalization in the TOC. Went with a lower case motif.

$u_0 \approx \pi/4$ $u_0-\pi = \beta$ $4\pi$ $u = 3\pi$ $\beta = \pi-u_0 = \beta$

**Dec 26, 2015**

Found and fixed errors in Fig (1.2.18) and earlier partner.
Reread Chapter 1 a few times, replaced toroid with torus everywhere, toroids with tori. More precision.

Chapter 2
2.1 OK
2.2 OK
2.3 OK
2.4 OK
2.5 OK after I rewrote some of it, it was a mess before.

I am done for this day, now 8 PM and Trek time. No much work today due to Star Wars and lunch at the T&G. Ready to start Ch 3 tomorrow. Sections, not Chapters!

**Dec 27, 2015**

Working on Section 3, added $V_0 \neq V_1$ exterior solution, renumbered, will now proof

Chapter 3

3.1 done and OK, after much editing to include the interior solution and a reader exercise.
3.2 OK
3.3 OK, I keep editing all the time.

Section 4

4.1 OK as is
4.2 OK
4.3 has two parts
the full sphere limit: OK with some edits
the disk limit: OK as is, fixed some obs references.

I ask this question at the start of Section 4.3,

and why is there no iris limit for everything as well !

Answer: The disk limit is commonly known, Jackson p 92, but the iris limit is much less well known so not useful to compare against. Continuing

Section 4

4.1 OK as is
4.2 OK
4.3 has two parts
the full sphere limit: OK with some edits
the disk limit: OK as is, fixed some obs references.
4.4 done and OK

Section 5, several sections

the u coordinate: OK
the $\xi$ coordinate: OK
sphericals r and $\theta$: OK, explains the funny plots. I like this Section.

Section 6

OK, little intro then pictures with no eq nums.

Section 7

7.1

STOP. Am now doing Appendix L on the Mehler integrals.

**Dec 28, 2015**

Finished Appendix L, it took a LOT of work. I will now review it all

L.1 OK
L.2 OK, short and easy

L.3 OK
L.4 OK, this is the long one
L.5 OK, this has the extra Maple verification
L.6 OK

It is done! Now back to Section 7.

Section 7
7.1 OK and edited to refer to Appendix L
7.2 OK showing how to compute integral X
7.3 OK
7.4 OK, little Baffin island at the end

Section 8
all OK

Section 9
all OK, very short, the plotting code for the bowl.

Section 10 this will take some time, very recent, lots of info.
10.1 OK, we have the potential as a sum.
10.2 OK and just fine
10.3 OK, short and good plots I think
10.4 OK, review of both limiting cases
10.5 OK, the torus σ, no verification so do two checks
10.6 OK and fini!

Status: I first reviewed all the appendices, and then I reviewed the main document. I still owe some overview paragraphs.

**Section 1** reviews bipolar and toroidal coordinates.

**Section 2** discusses the notion of atomic forms and Smythian forms. A Smythian form for the charged bowl is obtained and its coefficients computed, resulting in a solution for the bowl potential for which a preliminary Maple plot is displayed. The Mehler-Fock Transform is encountered. The potential is checked in various limiting cases such as the disk and large-r limit.

**Section 3** addresses the problem of two bowls having a common but possibly insulated lip. Special cases of a bowl with a flat lid and a sessile droplet are considered.

**Section 4** uses the bowl potential obtained in Section 2 to compute the surface charge densities on the two sides of the bowl. The results are compared with those of other workers including Kelvin. A quick look is taken at the case of a bowl with a very small opening. The limiting cases of a full sphere and flat disk are studied. The large-r limit is then examined to obtain the bowl capacitance.

**Section 5** studies graphically the nature of the u and ξ toroidal coordinates.

**Section 6** contains a selection of bowl potential plots for various bowl labels $u_0$.

**Section 7** discusses Mehler integrals and evaluates one as a detailed example. Some of important integrals are stated but their derivations are relegated to Appendix L. The short list of available sources for Mehler integrals is reviewed and some errata in these sources are noted. The Mehler functions $P_{i\tau-1/2}(ch\xi)$ and $Q_{i\tau-1/2}(ch\xi)$ are then plotted in several ways to reveal their oscillatory nature. The notion of regional analytic continuation of functions is briefly addressed.

**Section 8** evaluates the integral-form bowl potential into a relatively simple expression involving only elementary functions.

**Section 9** describes the Maple code which plots the bowl potential.

**Section 10** basically repeats all of the above for a torus instead of a bowl. Whereas the initial bowl potential is an integral over continuous variable $\tau$, the torus potential is a sum of terms involving the discrete index n. The sums are generally reasonably convergent allowing for truncation. Maple code to plot the torus potential is presented and plots are shown. The torus capacitance is obtained, and the surface charge is computed, checked in several limits, and plotted with more Maple code. Detailed discussion of the thin-wire limit and the horn toroid limit is deferred to Appendix I.

OK, I think the time has come for Production! Spell checks, visual alignment, pagination and all that good stuff. Hurray. We are now 163 pages.

**Dec 29, 2015**

I started into production work. Fixed the TOC update problem, divided into sections. I then decided I wanted to say more about Mehler Fock since it is so critical to the paper. I did this in J.5. I took a shot at deriving the transform, but learned that it is a big Sneddon type mess and decided to back away. That is the subject of some other paper some day. Found a nice thesis on the subject, student of O at Oregon in about 1960, and he did a proof, but it is just a horrible mess filling a whole thesis. So tomorrow I will resume production work with this new section J.5 installed.

**Dec 30, 2015**

Reviewed my attempts to verify M-F orthog and completeness, but still could not come up with the right factor. Every attempt ends in integrals I cannot do. So I let this go with some comments in J.5, at least I did find someone who had a $\delta$ function ortho statement, and I quoted it.

**Appendix J** Final Review

J.1 OK
J.2 OK
J.3 OK
J.4 OK
J.5 OK

**Dec 31, 2015**

Added section on kinds of Legendre functions based on what I learned yesterday.

Now back to production? I think and hope I am done adding things. Maybe some file cleanup would help in that regard. Got the ordering iτ-1/2 the same everywhere and added note in overview.

I reviewed all docs today, and then combined directories and cleaned up the bowl in toroids directory. Tiny changes made to bowl doc. Looked at all the collected PDF files. Cleaned up the Black desktop and sent email folder.

Just reread **Appendix G**, just because I know it is the toughest one. Seems OK.

Just reread the entire **Overview and Summary**, tiny edits.

Just reread **Appendix F** about those Abel transform forms and mysticism about Legendre and Bessel.

Just reread **Appendix E** and did tiny edits.

Just reread **Section 5** with those fancy plots of u and ξ, I like it all!

Just reread **Section 6**, it is killer good, both the intro and the plots.

Just reread **Section 4**, lots and lots of math work going on in this long section. Core of paper? Maybe a co-core with Section 2.

Just reread **Section 3**, the double bowl stuff, all OK. DNA gets honorable mention.

stopping at 9 PM this New Year's Eve. Recalling Jennifer in the Trolley basement restaurant.

**Jan 1, 2016**

Just reread **Section 1**, decided to add nice plots of a bowl and a torus, realized Maple can do these, got a light model going, all good. Made lots of tiny improvements.

Just reread **Section 2** about atoms and Smythian forms and solving the bowl. This paper is very good!

Just reread **Section 5**, it is excellent, the graphics make all the difference.

Just reread **Section 7**, always making tiny edits.

Just reread **Section 9**, short and sweet, minimal code gets beautiful plot.

Just reread **Section 8** about eval bowl pot into elem functions. Boring but fine.

Proofing **Section 10**

10.1 OK
10.2 OK, repaired fig 10.1.2 to remove left side saying $-\xi_0$. I like this "engineering" section.
10.3 OK, added two new plots for thinner tori.
10.4 OK, added comments about work.
10.5 OK, very dense detail here
10.6 OK but I am disturbed by that minimum effect at the end.

I spent a few hours working on those minima and I think they are real and I try to explain why.

Section 10 repeats the entire earlier doc for toroids, so it is a huge section, 20 long pages.

Status: I have now recently reread all the Sections and intro, and some of App E,F,G,J I think they are all OK. I will maybe do a few now.

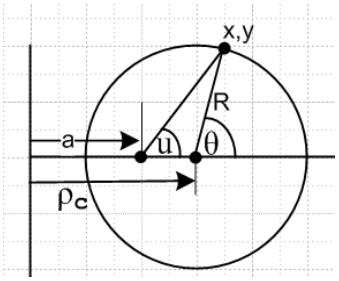
Appendix H; brutal detail. I browsed through it, very painful to any reader. But if they want to know how something works, there is the answer. $\rho_c = 1$

I am too tired now to do more proofing. Manana.

**Jan 2, 2016**

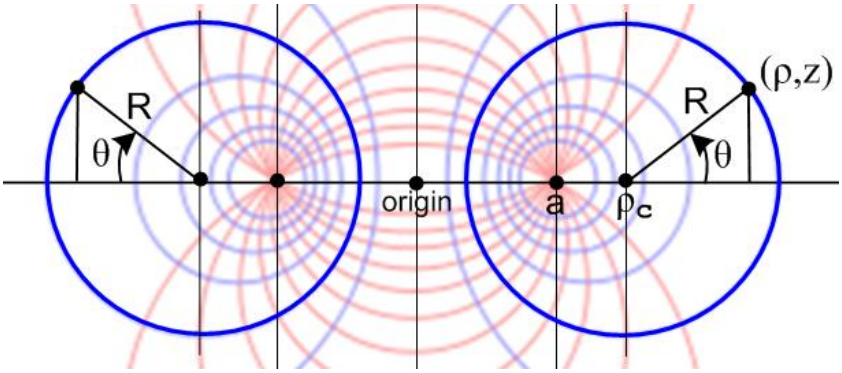
Found the bug with σ on toroid, took about 6 hours to get it all cleaned up. I am now very confident that my σ on toroid is correct, and I see no one else who states the result. There is no anomaly minimum or anything like that.

Discovered that my Fig (10.6.1) is probably wrong




This does NOT appear in bipolar. It does not say where the origin for (x,y) is, and maybe those should be (ρ,z) anyway. Very bad! I don't use the picture, but I would like to fix it if possible.

(ρ,z)




**Chapter 10** final review: went through it quickly, it is much better than a week ago!

I recently (last 3 days or less) did App E,F,G,J I, so I will look at some of the other appendices now.

**Appendix H.** First, lets check all refs TO App H from outside App H. DONE, there are about 20 references into Appendix H! Now do a read through.

H.1 OK, brutal endless calculations, good to not be in the main body, Maple verifications all.
H.2 OK, an integral
H.3 OK, another integral
H.4 OK,
(a) arghh, brutal brutal integrals
(b) another cosine series transform integral!
H.5 OK, limits
H.6 OK, more limits
H.7 OK, more limits and the $T(1) \le \pi$ inequality as application
H.8 OK, the alternate T(z) series near z = 1.

Stop!! Remember that $\varepsilon_n = 1$ for the first term and 2 for other terms since $\varepsilon_n = 2 - \delta_{n,0}$ .

Quick reading of **Appendix I**, it is good, I hope somebody reads it some day.

Casual quick review of **Appendix J**, all OK and good.

Casual quick review of **Appendix K**, all OK and good.

Casual quick review of **Appendix L**, workman brute force on all 6 Mehler integrals.

**Appendix A**, not much to say. I tried it once, won't try it again.

**Appendix B**, easy reading, OK.

**Appendix C**, easy reading, OK.

**Appendix D**, easy reading, OK. Lots of extra stuff I wanted to stuff in here. It is all related.

OK, I am done proofing once again! Just reread the entire intro and summary.

Next step:  spell checking by doing partitioning, since doc is too large.
Then: pagination, centering of figures, look for thrown eq nums.

**Jan 3, 2016**

Decided to add my analytic function appendix as Appendix M. I had to edit it to make it "fit", and I think it is a good addition!  Better there than wasting away in some folder!  Now 199 pages.

Update the index for this addition: had to recreate the big picture in Ahlfors area.

Time for **spell check**: Do this 50 pages at a time in a scratch doc.

first shot:  start thru (5.14). But spell check won't come on! This method won't work.

Copy out through end of section 5.  DONE

Copy out Section 6 through App C DONE

Copy out App D through App I  DONE

Copy out App J through end. DONE

That completes the spell check, I found about 8 errors.

I noticed lots of ∞ and → signs that I want to fix. maybe all → signs.
The problem is that these get into Courier in sub and supers and I want them TNR.
DONE, only the arrows needed fixing, about 10.

Center all drawings, more or less: DONE.

Check all EQ nums for sequence AND position;  DONE!

Reviewed the References, removed commas where not needed before (pub, date).

OK, it is finally time for pagination.

I will not do section headings, too much work! The eq nums show people what section they are in.
And lots of my sections have weird long names which don't really help much.

Are there any old errata? No, all are red.

Pagination:

TOC OK
Overview PK
Section 1 done
Section 2 done
Section 3 done
Section 4
- 4.1 done
- 4.2 done
- 4.3 done

Section 5 done
Section 6 done
Section 7 done
Section 8 done
Section 9 done
Section 10
- 10.1 done
- 10.2 done
- 10.3 done
- 10.4 done
- 10.5 done
- 10.6 done

Appendix A done
Appendix B
- B.1 done
- B.2 done

Appendix C done
Appendix D done
Appendix E done
Appendix F done
Appendix G done
Appendix H
- H1,2,3,4 done
- H5,6,7,8 done

Appendix I
- I.1 done
- I.2 done
- I.3 done

Appendix J
- J.1 done
- J.2 done
- J.3 done
- J.4 done

Appendix K
- K.1 done
- K.2 done

Appendix L done

| | |
|---|---|
| Appendix M | done |
| References | done |

Let's next just browse through looking at eq num alignment.

Ready to try a PDF on Alta using USB drive.

I think Alta has some kind of problem, something clicked a lot inside during boot. But the thing is at least running. But it came a cropper on its first attempt, so I will try gain (usually fails 1 of 3 tries)/ Second try looks better, creating pages. Success. The bookmarks all look good.

Now just peruse things looking for graphic/glyph problems!

p 4 tab over the capital Q line
p 19 the lower integral end points are pretty bold at 135% but are OK

It is the *difference* between the bold 0 and the non-bold ∞ that is annoying!

(2.4.10) align

$\int_{\mathbf{0}}^{\infty}$ I could globally fix this, the lower end is Bold for some reason.

Should run a test. The lower bold looks OK when the upper π or whatever is also bold as in (E.18)

(F.1) and nearby, the endpoints are in different fonts. Letters a and b are different fonts.

I wish I had a bolder ∞ symbol as in (F.10).
Look below G.13 at more endpoints.
(G.21) endpoints look very good.

(H.6.16) thrown eq num!!

(M.3) thrown eq num!

OK, I browsed the entire doc.

**Question**: Is there a bolder ∞ symbol. It is light like all the Symbol font letters. Bold or not it looks about the same. Character is F0A5.




∞ ∞ dec = 8734 hex = 221E

This ∞ symbol is 221e unicode. ∞ Too small and too dark if enlarged. $\int_0^\infty$ The holes are too small, but not bad. ∞ ∞

$\int_0^\infty$ ∞

∞**Cambria**

∞**century** $\int_0^\infty$

Courier ∞ $\int_0^\infty$ a ∞

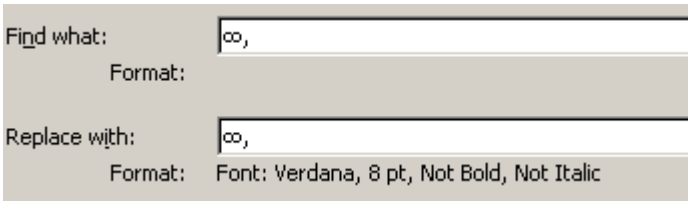
∞ **Franklin GH** $\int_0^\infty$ // not too bad but uneven original: $\int_0^\infty$

∞ **Garamond** $\int_0^\infty$ // but uneven when bolded

∞ Lucida Sans Unicode $\int_0^\infty$ dx ∞

∞

I just tried this global change on a test1 bowl doc with the fields all open,




I will fix the other problems and do a cycle with this test1 doc. Just want to get a global impression of what this changed ∞ symbol looks like.

I like it! Here is what it looks like in Word,

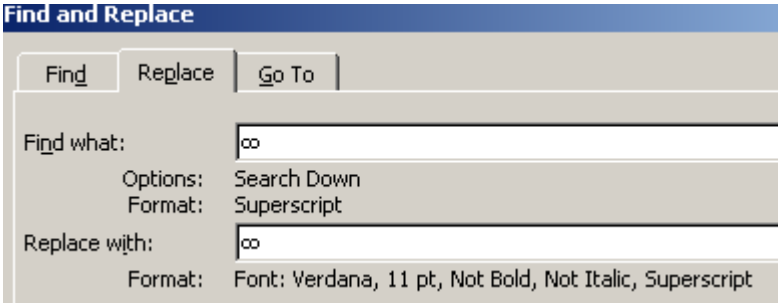
original: $\int_0^\infty$ Verdana 8 no bold: $\int_0^\infty$

I changed it in symbols doc.

OK, save original as bu, and now the current doc and pdf are ready to ship!

Upper endpoints of sums should be Verdana 11 unbold! I tried it. I will fix all the sums tomorrow in this regard. Just replace superscript ∞ everywhere in this way. Too late today.

**Jan 3, 2016**




When you copy and paste, funny things happen. For example, if I copy out one of my equations which has a Verdana superscript size 11 ∞ symbol, the ∞ changes to size 12 without asking me!! Why is it doing that do you suppose?

$V_0/\sqrt{ch\xi_0 - cosu} = \Sigma_{n=0}^{\infty} P_{n-1/2}(ch\xi_0)\, A_n \cos(nu)$

One argument: text takes the format of the recipient document. All my docs use 12 point Courier for superscript, so that 12 is applied to the ∞ symbol.

Exper shows that the 12 actually looks good so I can use that for my symbol file. Let's go replace that in bowl right now but then have to recheck thrown eq nums.

Ready now for another PDF cycle. Done.

BUG: all on Alta, (J.4.6) is OK in the doc but it is thrown in the PDF! I had this kind of problem once before, I now have to find notes! But this time it is not a difference in the Word, it is a difference in just the PDF!! This IS an equation that has my new infinity.

I see nothing wrong in Word on Alta. I note that *previous* eq num has a tab AFTER the eq num. I delete that tab in Alta word and try another cycle, am not hopeful. // Did not fix the problem.

So how do I fix this one line? Find those old notes please. A tough search. Probably happened when I was doing an update of some doc on the site. Let's try Lagrange. Well it would be in an edit log! Found it, in the tensor doc edit log near the end.

Back to problem. I decided to make a small doc with equation (J.4.6) in it, to get a faster PDF cycle time. But when I paste from Word to this test doc. the equation is not thrown in bowl doc but it IS thrown in the test doc!! I do a side by side comparison and I see the problem! The tab spacing control is the same, but in the test doc, there are 5 tabs before the (J.4.6) while in the bowl doc there are 4 tabs! I did this again: copy the line from bowl doc to a new doc. It creates a new tab. No tab seems right on the edge.

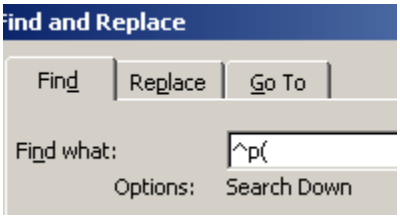
Everything I can detect looks the same (think Colombo spotting the party invitation on the video).I see no difference.

I think there might have been a control character in the tab flow. I deleted and rebuilt the tabs on the right, and now the problem of copying to a new doc is fixed. No longer adds a tab.

Test: Copy out the new right-looking J.4.6 line from bowl to test3.doc. Alta definitely has a problem with one of its hard drives. There is a huge delay and the red disk light comes on for a long time and I hear a click. I create test3.pdf, and is not thrown. So maybe I fixed this problem.

I now do a new Alta PDF cycle with this repaired bowl doc. Current bowl doc is on Alta desktop right now. The PDF cycle takes about 2 minutes. // Problem is fixed. I suspect a control character in the set of tabs at the end.

**Aside**: How can I search fast for thrown eq nums? Wildcards in find? Can turn them on and off. You cannot do wild cards in a PDF find operation! But in Word do this




This finds lines that begin with a paren, which is usually what a thrown eq num looks like. This picks up other things but not too many. // But on a test doc this does not detect wrapped lines! I don't see any way to detect this problem, it is just a wrap like in any paragraph. You have to visually look for eq nums over on the left.

Let's bring back bowl doc and bowl pdf from Alta and make them be the current ones. DONE.
Verify J.4.6 is OK in PDF: it is OK.

Peruse the new PDF. But first working on the Alta bug, a long check disk is now in progress so I cannot do more PDF's right now.

But I can peruse the PDF and make a list of errata.

- my zipcode is wrong in the title block, hardly matters, **repaired, confirmed**

- should say "our Lebedev et al. reference" since I now have two Lebedev references, **repaired, confirmed**

- below (1.1.2) contant should be constant, **corrected, confirmed**

- start of H.8.4 derivative misspelled, **corrected, confirmed**

- at (J.4.5) had and and, **repaired, confirmed**

- page 71 bottom tab over the equation, **repaired, confirmed**

• H.5.5 is thrown in PDF, but not in the doc. Delete all tabs at the end and retype. **, confirmed**

• I.1.3 is thrown! Also thrown in doc, **repaired., confirmed**

• J.5.18 is thrown! Also thrown in doc, **repaired. , confirmed**

Proofing:

| | |
|---|---|
| title block | zip code only error (fixed and checked) |
| TOC | all OK |
| overview | Lebedev ref repair (fixed and checked) |
| section summaries | OK |
| app summaries | OK |

So the above is the main stuff where I don't want to have gaping typos or errors.

2:15 PM. I have fixed about 9 errors above, let's try a new cycle on dying Alta.
Booting alta now, have the flash drive ready to insert there. But on boot I get light blue window saying a 3-stage disk check wants to run! I let it complete, see separate doc. The PDF export seems to still run OK. Filed on first attempt. Second OK, got the file back and shut down Alta to lower drive wear.

• Period at end of 1.2.12
• 2.2.1 word "expo" needs alignment
• page 19 should say "discussed below Fig (1.2.1), word "in" to be deleted
• in 2.5.1 adjust the upper red minus sign
• in 2.5.10 a right paren is missing, maybe space a bit as well.
• in 3.8 remove the period altogether, check pagination effect of fixing.
• 3.9 is totally messed up, will have to investigate!
• 4.1.2 add space after + sign and adjust at alls spacing
• 4.1.8 is thrown
• 4.3.6 first line remove period, last line add period.
• below 4.4.2 the italic H.3.1 is thrown, move in period too.
• Page 49: add comment that second picture shows the "small hole" potential
• above (7.1.1) in the first integral, what is α ? I presume chα = y.
• below header 10.3 should say "that used to plot the bowl *potential* in .."
• In (10.4.11) should have factor of R in the two capacitances!
• in 10.5.2 remove first period
• 10.5.14 needs a period
• below 10.6.4 need space before the word also

More proofing

Section 1
1.1 OK

1.2 OK
Section 2
2.1 OK
2.2 OK
2.3 OK
2.4 OK
Section 3
3.1 OK
3.2 OK
3.3 OK
Section 4
4.1 OK
4.2 OK
4.3 OK
4.4 OK
Section 5
OK
Section 6
OK
Section 7
7.1 OK
7.2 OK
7.3 OK
7.4 OK
Section 8
OK
Section 9
OK
Section 10
10.1 OK
10.2 OK
10.3 OK
10.4 OK
10.5 OK
10.6 OK

Bravo. I did the above proofing with large print PDF and found 18 things to fix! There seems to be no substitute for an actual proofing. So let's get into the appendices forthwith:

- D.8 add a space before $(2/\pi)$
- I don't see where D.8 comes from, fixed this.
- G.12 need better spacing for $\partial_\theta I$
- Above G.15, put the Vino 1.111 into parens,
- G.16 is not what it says it is, where did it come from? $\cos^{-1}$ ?? Needs work.
- I don't like the spacing on G.22, maybe fix all this up at once.
- below H.1.26 extra space before $u_0'$

- below H.1.27 removed period in left margin
- H.2.3 add blank line and check pagination
- above H.4.4 spacing of lines
- H.4.6 eq num not at right margin
- above H.4.9 clean up equal sign spacing
- first line of H.4.12, bracket and - in wrong font level
- remove period after H.4.13
- below H.5.2 say "so for the toroidal functions"
- H.6.2 needs a period
- H.6.9 bad screen clip
- H.6.16 eq num thrown in the doc, try to fix
- above H.7.2 should say as $x \to 1$, not $x \to 0$
- H.7.5 I think is wrong because it comes from the bolded Q function. Has implications!
- H.8.2 change location of the two primes, and fix wherever this is quoted in bowl doc!
- I.1.3 and two places below remove extra parens inside square root
- below I.1.13 add space after $\rho_c$
- I.1.14 is wrong, should say $2\lambda \ln(e^{\pi}/R) \approx 2\lambda \ln(23.1/R)$
- I.1.15 is also wrong. Clean it all up please.
- I.1.17 needs major repair: primes on dr', V(∞)-V(r) etc. Both lines!
- below I.2.11 should say convergent, not continuous
- below I.2.12 should say $\varepsilon > 0$
- below I.2.18: is it true that $c_1$ and $c_2$ depend on k after H.7.5 is fixed?
- below I.3.14 explain what ±1 means
- J.1.2 should say "a Green's function problem for $L_\lambda$ " and end it there.
- replace m by n for sums in J.1.13 and J.1.14
- below J.2.4 bad line spacing
- below J.2.8 delete extra = sign
- below J.2.8 add paren (where we assume uniform convergence! )
- J.2.9 better spacing, same in J.2.10
- J.3.3 make a better screen flip so not blurry
- line below J.3.6 say " from J.2.8"
- J.3.9 fix spacing after = sign
- add "the secret sauce" in comment below J.3.13
- page 161: why does footnote say page 7, but then say page 9 ? Keep pdf!
- J.4.8 spacing on = signs, and need more eq nums here! ??
- below J.5.5 should say Fig 7.4.8 and not 4.4.8.
- add definition of "Sturm Liouville Problem" in Section J.1
- above K.1.8 put comma after as follows
- below K.2.4 should say evaluated past tense
- in L.2 put a colon after the show that phrase so like other 5 sections. NO. remove all the colons!
- in L.1.6 replace IT with ET
- L.5.8 region, better spacing near = signs
- App M, make 1/2 bold in the title.

- M.3 delete space before not shown
- M.4 better spacing, and also in M.5 with period added.
- below M.7. descend not descent

http://gallica.bnf.fr/ark:/12148/bpt6k99615p Neumann reference added in line

Appendix A
OK
Appendix B
B.1 OK
B.2 OK
Appendix C
OK
Appendix D
OK
Appendix E
OK
Appendix F
OK
Appendix G
needs work
Appendix H
H.1 OK
H.2 OK
H.3 OK
H.4 OK
H.5 OK
H.6 OK
H.7 OK
H.8 OK
Appendix I
I.1 OK
I.2 OK
I.3 OK
Appendix J
J.1 OK
J.2 OK
J.3 OK
J.4 OK
J.5 OK
Appendix K
K.1 OK
K.2 OK
Appendix L
L.1 OK
L.2 OK

L.3 OK
L.4 OK
L.5 OK
L.6 OK
Appendix M
OK

I have done lots of edits. I make each edit blue, then red when I get back the PDF to check.

5:30 PM did an Alta PDF cycle, seemed to go OK, I will now go down the above list of blues and turn them red when I confirm each one.

New errors list

- below I.1.5 should have $\rho_c^2$ inside radical
- J.4.8 space the minus sign
- K.2.5 missed second evaluated!
- L.5.8 add period
- M.3 is thrown
- M.5 add period
- make new TOC

Now doing another PDF cycle to iron out the above list of bugs.

- M.3 is thrown

I will try a copy out and back job on Alta to try to fix this, I see no reason for it. // cycle failed to fix it!I found a tiny 1 point tab that behaves differently in a newly made doc on Alta than on bowl doc there. So I edited to remove it and space things differently. That fixed it.

OK, I think I am ready to put it out there! // did local and **xmission release**, tested, it is out there. File may have a private path located in it which I don't like much.

**Alta** is in bad shape, won't even respond right now!

Put bowl onto **researchgate** 1/6/16 and had no problems.

**How long did this edit cycle last?**

I started on Nov 26 according to the above. I was triggered by the Maier email. I finished on Jan 5. Excel says that is 39 calendar days. I worked on bowl doc every single day! So that is 39/7 = 5.5 weeks full time every day. Wow. This really shows just how bad a shape that doc was in!