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> restart;
Construct the charged bowl potential in toroidal coordinates
 A := sqrt(cosh(xi)-cos(u)):
 B := sqrt(1 + cos(u)):
 C := sqrt(cosh(xi)-cos(2*u0-u)):
 E := sqrt(1+cos(2*u0-u)):
 eta1 := floor((3*Pi-u)/(2*Pi)):
 eta2 := floor((2*u0-u+Pi)/(2*Pi)):
 T1 := (1/A)*arccot(((-1)^eta1)*(B/A)):
 T2 := (1/C)*arccot(((-1)^eta2)*(E/C)):
 V := (V0/Pi)*A*(T1+T2);
 V0 := 1:
 a := 1:
 u0 := evalf(2*Pi/8);

Routine arctan2Pi
Given (x,y) somewhere on a circle, return the angle in (0,2Pi) measured CCW from the
xxis.
Warning:  returned result may include unevaluated mulitples of Pi
> arctan2Pi := proc(x,y)
    local q;
    if type(x,numeric) and type(y,numeric) then
      if x = 0 and y = 0 then print("arctan2Pi(0,0) error." ); RETURN(0) fi;
      if x = 0 and y > 0 then RETURN(Pi/2) fi;
      if x = 0 and y < 0 then RETURN(3*Pi/2) fi;
      if x > 0 and y = 0 then RETURN(0) fi;
      if x < 0 and y = 0 then RETURN(Pi) fi;
      if x > 0 and y > 0 then q := 0 fi;
      if x < 0 and y > 0 then q := Pi fi;
      if x < 0 and y < 0 then q := Pi fi;
      if x > 0 and y < 0 then q := 2*Pi fi;
      RETURN(arctan(y/x)+q);
    else
      'arctan2Pi(x,y)';
    fi;
 end:

Routine getu
Given a,u0, rho,z, compute toroidal bowl-label parameter u in range (u0,u0+2Pi)
> getu := proc(rho,z)
   global a,u0;
   local xi,cosu,sinu,t1;
   if type(rho,numeric) and type(z,numeric) then
     xi := arctanh(2*a*rho/(a^2+rho^2+z^2));
     if rho = 0 then
       cosu := (z^2- a^2)/ (z^2+a^2);
       sinu :=  2*a*z/(z^2+a^2);
     else
       cosu := cosh(xi)-(a/rho)*sinh(xi);
       sinu := (z/rho)*sinh(xi);
      fi;
     t1 := evalf(arctan2Pi(cosu,sinu));
     if t1 < u0 then t1 := t1 + evalf(2*Pi) fi;   # get into proper range
     RETURN(t1);
   else
     'getu(rho,z)';
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   fi;
  end:


Plot the potential of the charged bowl (an azimuthal slice)
> xi := arctanh(2*a*abs(rho)/(a^2+rho^2+z^2)):
> u := getu(rho,z):
> plot3d(V, rho = -2..2, z = -2..3,numpoints=2000,axes=BOXED, view=0..1);

> u := getu(rho,z):
> plot3d(u, rho = -3..3, z = -4..4,axes=BOXED,numpoints=2000);
```