Smythe's bowl method PhL 3.26.05

I have now written this up as Appendix D of my toroidal bowl paper.

Problem 38 gets σ on a grounded iris due to point charge in hole.

Problem 39 gets σ a grounded iris due to ring of charge in hole.

Problem 40 then inverts a grounded iris + ring charge in hole to a grounded bowl + ring charge on cap. We know at this point all the charges: σ on iris (same on both sides), σ on bowl (same on both sides), linear density of the iris ring charge, linear density of the bowl cap ring charge. Of course then we know all four total charges as well.

We then use a tricky Green's Reciprocation Theorem to learn about a *different* problem: the charged bowl at potential V_0. We are able to deduce the potential $V(\theta)$ at any point on the cap for this problem. In this theorem application we treat the bowl as one conductor, and the thin ring on the cap as a second conductor.

Problem 41. In the previous situation, we know $V = V_0$ on the bowl and V = something on the cap, so we have a fully defined Dirichlet problem for the potential of a charged bowl. We write the Smythian form and for V outside the bowl and compute just the first term and this gives the bowl capacitance. We have here as a series the complete potential for the charged bowl!

Problem 42. Here we first superpose "a bunch of problem 40 situations" in order to obtain a uniform σ_0 on the cap of the bowl. This results in a certain charge on both surfaces of the bowl (same on both) which we get from our superposition. Call this bowl charge σ_i.

Next, we superpose a spherical shell with $-\sigma_0$ just outside the bowl sphere. This cancels the cap charge, raises the bowl to potential V, and causes $\sigma_{outside} = \sigma_i + \sigma_0$ and explains the constant difference.