Review of Smythe's iris claims PhL 4.23.10

Overview: This little one pager was written the day before my "three Attempts" to apply my Dirichlet Method to the iris Green's problem. I comment on the nature of the Problem 38 claim for σ. Just another doc in my littered path of docs in which I struggled to find a way to do Problem 38. April 2010.

I start by quoting from "Iris Green's Function attempt using the Stak integral equation method.doc":

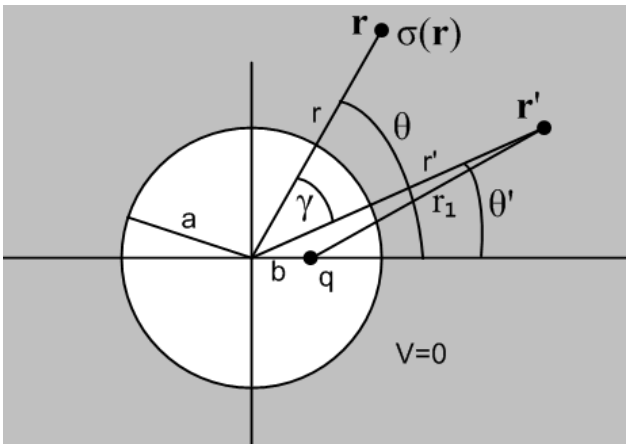
How is it that Smythe gets such a simple answer to this problem (Smythe p 203, Problem 38)

> **38.** A charge q is at a distance b from the center of a circular hole of radius a in an infinite earthed flat conducting sheet and lies in the same plane. Show that the charge density induced on the sheet at a distance r from the charge and c from the center of the hole is $-q(a^2 - b^2)^{\frac{1}{2}}/[2\pi^2 r^2(c^2 - a^2)^{\frac{1}{2}}]$.

I will rephrase this for my picture: "Show that the charge density induced on the sheet at a distance r_1 from the charge and r' from the center of the hole is $-q(a^2- b^2)^{1/2} / [2\pi^2 r_1^2(r'^2 - a^2)^{1/2}]$. "

So he is claiming that the answer is this, for my picture,

$$\sigma(r',\theta') = - (q/2\pi r_1^2) \sqrt{a^2-b^2} / \sqrt{r'^2-a^2} \qquad r_1^2 = r'^2 + b^2 - 2br' \cos\theta'$$



When I first saw this problem, I felt one needed the Green's function for this q placement with V = 0 on the iris, then you could compute σ from that Green's function potential. I went off and did this in oblate spheroidals and got a result, but it is a huge mess of ugly functions, looking nothing like the above. Then I tried doing the disk problem in cylindricals, thinking that problem was similar to the above, but this time I got nowhere using Smythian forms, I could not solve it.

One has the feeling that there is some fairly simple way to obtain the above simple result. I am unable to find any web references for < Smythe "problem 38">. Nor am I able to find this problem anywhere. It is not easy to find a certain problem solved somewhere! [it was in Jackson's problems, more or less...]

The charge density with $1/\sqrt{r'^2-a^2}$ blows up at the edge in the same manner we know for the charged disk in Jackson. But there was no "simple way" to derive that simple result in Jackson! We had to go

through a Smythian form in cylindricals, and then we had to deal with a dual integral equation. Or we had to do oblates which in that case became simple.

Idea #1. We know from "iris Green's function attempt....doc" that

$$V(r',\theta') = q/r_1(r',\theta') + \int_a^\infty r\,dr \int_0^{2\pi} d\theta\ \sigma(r,\theta)/R(r,\theta; r',\theta')$$

$$r_1^2 = r'^2 + b^2 - 2br'\cos\theta' \qquad R^2 = r^2 + r'^2 - 2rr'\cos(\theta-\theta')$$

This is valid for all points **r'**, both in the hole and on the iris. The equation goes with our picture. Well, I was going to take a derivative of V perp to the plane and set this to σ, but this whole equation only exists in the plane! Let's rewrite this thing where point **r'** is not necessarily in the plane:

$$V(\mathbf{r}') = q/r_1 + \int_a^\infty r\,dr \int_0^{2\pi} d\theta\ \sigma(r,\theta)/|\mathbf{r} - \mathbf{r}'|$$

Idea #2. Recall the relationship between a Green's function and a Dirichlet problem. In our case, think of the point charge as creating a certain potential on the iris, which can be written q/R for an in-plane R. So we know this everywhere on the iris. Then think of the z=0 plane as a mixed Dirichlet/Neumann problem.

[This let to the "three attempts" in this folder, but they merely led to non-solvable dual integral equations.]