**Summary of disk green attempts** PhL 7.6.10

These attempts were made April 4-22, 2010. I am trying to find the Green's Function for a disk with the point charge in the plane of the disk. All these attempts use a 3-region Smythian Form which is probably just no good, and that is why I have so much trouble. Eventually I learn (elsewhere) to solve this problem by inversion and get a result in terms of elementary functions for the potential and the induced charge.

**Attempt #1.**

Here I attempt a 3-region Smythian form for the disk (radius a) with point charge in plane Green's problem (distance b from origin). I use cylindricals, so atoms include Bessel functions like $J_m(k\rho)$. Each region is a cylinder, and there are four sets of coefficients. At the point charge boundary I try my usual pillbox thing, but in the z direction I have expo decay so no orthog in z, and I end up stuck:

$$(q/\varepsilon)(1/b)\,\delta(z) =$$
$$2\pi/(2-\delta_{m,0}) \int_0^\infty dk\, k\, e^{-k|z|}\{ B_m(k)\, J_m'(kb) + C_m(k)\, H^{(1)}{}_m'(kb) - D_m(k)\, H^{(1)}{}'_m(kb) \}$$

I then state that the entire rest of the doc is scraps, so I skip it here and move on to the next attempt.

**Attempt #2.**

In order to avoid the above problem, I try again, this time with cos(kz) so z is then an oscillatory direction. This means the Bessel functions are I and K type. As before, we have 4 coefficients. I get down to the point analogous to the above, but now have

$$(q/\varepsilon)(1/b)\,\delta(z) =$$
$$2\pi/(2-\delta_{m,0}) \int_0^\infty dk\, k \cos(kz)\{ B_m(k)\, I_m'(kb) + C_m(k)\, K_m'(kb) - D_m(k)\, K'_m(kb) \}$$

I then digress on the Fourier Cosine Transform (and eventually added it to transforms.doc) and the above becomes

$$(q/\varepsilon)(1/b) = 2\pi/(2-\delta_{m,0})\,(\pi/2)\,k\,\{ B_m(k)\, I_m'(kb) + C_m(k)\, K_m'(kb) - D_m(k)\, K'_m(kb) \}$$

which is more reasonable. I come up with four conditions, this is just one of them. I seem to grind things down so there is only one set of coefficients $c_m(k)$. Somehow I am led here to something like the dual integral equation situation.

I then learn for the first time about **Neumann's Factor (or Number)**, and collect historical information on it (invented by Watson).

I continue on, but am led nowhere good. I suspect that this triple-cylinder Smythian form is just no good.

**Attempt #3.**

Undaunted, I start over again using the cos(kz) basis and the same three cylindrical regions. This time I try harder to find my coefficients $c_m(k)$. I end up with a contradiction (again suggesting the is a bad form).

**Attempt #4.**

Now I am back to $e^{-k|z|}$ for z. This time I am trying to get some dual integral equations. In this and previous attempts I keep arriving at this point, heading toward this dual integral things,

| | | |
|---|---|---|
| $V_1(0,\rho,\varphi) = 0$ | $\rho \le a$ | Dirichlet |
| $\partial_z V_2(0,\rho,\varphi) = 0$ | $a < \rho \le b$ | Neumann |
| $\partial_z V_3(0,\rho,\varphi) = 0$ | $b < \rho < \infty$ | Neumann, except for the Green's charge |

After removing the φ dependence, I end up with something that looks like such things:

| | |
|---|---|
| $\int_0^\infty dk\, A_m(k)\, J_m(k\rho) = F(\rho,m,b)$ | $\rho \le a$ |
| $\int_0^\infty dk\, k\, [\, B_m(k)\, J_m(k\rho) + C_m(k)\, H^{(1)}{}_m(k\rho)] = 0$ | $a < \rho \le b$ |
| $\int_0^\infty dk\, k\, D_m(k)\, H^{(1)}{}_m(k\rho) = 0$ | $b < \rho < \infty$ |

Of course since I have three regions, I guess I have triple integral equations (these are in fact a subject of literature! [ and are in Sneddon ]) I flail a while, then..

I start with new forms where I show the point charge explicitly, so the expansions are then only for the induced charge on the disk. But this too just peters out.

**Here is my closing comment:**

" Somehow, I am just plain approaching this problem wrong. I just don't have any clear view of how a solution is to be obtained. I wanted to try the "Smythian form" approach because that is what Jackson did for his charged disk problem, and I wanted to do this in cylindricals for the same reason. The Smythian forms might be OK, but I just don't know how to solve for the coefficients. There MUST be a better way to go about this problem. Somehow, this problem is harder than all the Smythe problems he and I have done to date.

So I will refer to this series of Attempts (now 1,2,3,4) as my 3-region cylindrical-coordinates Smythian Form method. I tried pretty hard for several weeks at least to make this method fly, but I could not make it fly. "

Since the time of writing these docs in April (it is now July), I have learned how to do this problem by inversion. And I know how to do it in oblates. I think my trouble in these "attempts" is that the three cylinder Smythian form is just plain wrong. Yes, it contains valid atoms, but that does not a valid form make. If I were trying it again, I would break out the point charge, and treat the induced charge on the disk as a single expansion that was Jackson-like for the charged disk. By forcing V = - q/R on the disk, and forcing $\partial V/\partial z = 0$ in the rest of the z=0 plane, a set of dual integral equations would be quickly arrived at. This would be a one-region fit, not a 3 region fit. I basically do this for the iris in my iris folder, and it would be the same idea for the disk. Of course then I don't know how to solve the dual

integral equations, so I get nowhere. But I think there have been modern advances in this subject. [ Sneddon shows how to do it all! ]