Derivation of green Jackson 6.37 and 6.38 PhL 10.13.13
blue Jackson 6.15 and 6.16

I will do this in blue Jackson in SI units, so the equation numbers will be different

1. Statement of problem:

Start with my own transmission lines equations which match Jackson's

curl **H** = ∂**D**/∂t + **J** Maxwell curl H equation (1.1.1)

curl **E** = - ∂**B**/∂t Maxwell curl E equation (1.1.2)

div **D** = ρ Maxwell div D equation (1.1.3)

div **B** = 0 Maxwell div B equation (1.1.4)

B = μ**H** magnetic permeability μ (1.1.5)

D = ε**E** electric permeability ε (dielectric constant) (1.1.6)

J = σ**E** Ohm's Law (σ = conductivity) (1.1.7)

And make use of

B = curl **A** **E** = - grad φ - ∂**A**/∂t . (1.3.1)

and first derive these two equations:

$$\nabla^2\Phi + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}) = -\rho/\epsilon_0 \qquad (6.10)$$

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \nabla\left(\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right) = -\mu_0\mathbf{J} \qquad (6.11)$$

2. Jackson's Derivation.

As a hint, Jackson tells us to use just the two "inhomo" which for him are these

curl **H** = ∂**D**/∂t + **J** div **D** = ρ

I first rewrite these as

(1/μ)curl **B** = ε ∂_t**E** + **J** div **E** = ρ/ε

I then insert **E** = - grad φ - ∂A/∂t and **B** = curl**A** to get

$(1/\mu)\text{curl curl } \mathbf{A} = \varepsilon\partial_t[-\text{grad } \varphi - \partial\mathbf{A}/\partial t] + \mathbf{J}$ $\quad\quad$ $\text{div } [-\text{grad } \varphi - \partial\mathbf{A}/\partial t] = \rho/\varepsilon$

or

$(1/\mu)\text{curl curl } \mathbf{A} = \varepsilon\,[-\text{grad } \partial_t\varphi - \partial_t^2\mathbf{A}] + \mathbf{J}$ $\quad\quad$ $\text{div } [\text{grad } \varphi + \partial_t A] = -\rho/\varepsilon$

or

$\text{grad div } \mathbf{A} - \nabla^2\mathbf{A} = \mu\varepsilon\,[-\text{grad } \partial_t\varphi - \partial_t^2\mathbf{A}] + \mu\mathbf{J}$ $\quad\quad$ $[\nabla^2 \varphi + \partial_t \text{ div } A] = -\rho/\varepsilon$

or

$\nabla^2\mathbf{A} - \text{grad div } \mathbf{A} = (1/c^2)\,[\text{grad } \partial_t\varphi + \partial_t^2\mathbf{A}] - \mu\mathbf{J}$ $\quad\quad$ $[\nabla^2 \varphi + \partial_t \text{ div } A] = -\rho/\varepsilon$

or

$\nabla^2\mathbf{A} - (1/c^2)\,\partial_t^2\mathbf{A} = \text{grad div } \mathbf{A} + (1/c^2)\,[\text{grad } \partial_t\varphi]$ $\quad\quad$ $[\nabla^2 \varphi + \partial_t \text{ div } A] = -\rho/\varepsilon$

or

$\nabla^2\mathbf{A} - (1/c^2)\,\partial_t^2\mathbf{A} - \text{grad } [\text{div } \mathbf{A} + (1/c^2)\,\partial_t\varphi\,] = -\mu\mathbf{J}$ $\quad\quad$ $[\nabla^2 \varphi + \partial_t \text{ div } A] = -\rho/\varepsilon$

Thus I have derived the two equations shown above.

Next step is to use

$\text{div } \mathbf{A} = -(1/c^2)\,\partial_t\varphi$

and these equations then become

$\nabla^2\mathbf{A} - (1/c^2)\,\partial_t^2\mathbf{A} = -\mu\mathbf{J}$ $\quad\quad$ $[\nabla^2 \varphi - (1/c^2)\,\partial_t^2 \text{ div } A] = -\rho/\varepsilon$

and these are what I want,

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\rho/\epsilon_0 \qquad (6.15)$$

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{J} \qquad (6.16)$$

3. An Alternate Derivation

Instead of using the two "inhomo equations", use the two curl equations:

$\text{curl } \mathbf{H} = \partial_t\mathbf{D} + \mathbf{J}$	Maxwell curl H equation	(1.1.1)
$\text{curl } \mathbf{E} = -\partial_t\mathbf{B}$	Maxwell curl E equation	(1.1.2)

Rewrite these as

$(1/\mu)\text{curl } \mathbf{B} = \varepsilon\partial_t\mathbf{E} + \mathbf{J}$

$\text{curl } \mathbf{E} = -\partial_t\mathbf{B}$

or

$\text{curl } \mathbf{B} = \mu\varepsilon\partial_t\mathbf{E} + \mu\mathbf{J}$

$\text{curl } \mathbf{E} = -\partial_t\mathbf{B}$

Now in the first equation replace **B** = curl **A** to get

curl curl **A** = $\mu\varepsilon\partial_t\mathbf{E} + \mu\mathbf{J}$

Then use the same identity used above to write this as

grad div **A** - $\nabla^2\mathbf{A} = \mu\varepsilon\partial_t\mathbf{E} + \mu\mathbf{J}$

Then install **E** = - grad φ - $\partial_t\mathbf{A}$ to get

grad div **A** - $\nabla^2\mathbf{A} = \mu\varepsilon$ [- grad $\partial_t\varphi$ - $\partial_t^2\mathbf{A}$] + $\mu\mathbf{J}$

or

grad div **A** - $\nabla^2\mathbf{A} = \mu\varepsilon$ [- grad $\partial_t\varphi$ - $\partial_t^2\mathbf{A}$] + $\mu\mathbf{J}$

etc.