Jackson Chapter 7 Notes PhL 2.4.03

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7.1 Plane waves in a non-conductor. A very nice description.

We allow there to be ε and μ here, but they are assumed to be constants in space. Notice that these appear in only one place in 7.1, and that is in Ampere's law, if you look back at the general Maxwells which are on page 178. The factor με stays together and of course ends up in the velocity 7.3.

Assuming unit vectors for E and B, Jackson uses the divergence equations to show they are both perp to k, and then with a curl equation he shows that they are also perp to each other and that B = E in magnitude. These then are the main facts that I always just take for granted. He notes that ε and μ might or might not depend on k (we have assumed constant in space already). If at least one is f(k), then v = v(k) and the medium is "dispersive" which means a packet will not maintain its shape, we will do more on this idea later.

Now suddenly on page 205 we have the first appearance of complex conjugation with the Poynting vector. I will digress for a little theorem at this point:

Theorem. Assume that

(1) S = AB is an equation that is *meant* to apply to real quantities A and B

(2) A and B are both complex quantities which have $e^{i\omega t}$ time-dependence.

(3) $\boldsymbol{\mathcal{S}} \equiv (1/2)\ AB^*$ (script S)

Then the claim of this theorem is that the time average of S is: $< S > = \mathrm{Re}(\boldsymbol{\mathcal{S}})$. In other words, we have defined a new complex object $\boldsymbol{\mathcal{S}}$ whose real part is the time-average of S.

Proof: Let $A = a\ e^{i\alpha}\ e^{i\omega t}$ and $B = b\ e^{i\beta}\ e^{i\omega t}$ where a and b and α and β are all real. Our equation (1) really means that $S = \mathrm{Re}(A)\mathrm{Re}(B) = ab\ \cos(\alpha+\omega t)\ \cos(\beta+\omega t)$. If we use the cos cos formula and then time average, we get that $< S > = ab\ (1/2)\ \cos(\alpha-\beta)$ because the other cos term with 2ωt in it averages to 0. Now consider from (3) that $\boldsymbol{\mathcal{S}} \equiv (1/2)\ AB^* = (1/2) ab\ e^{i\alpha}e^{-i\beta}$. Then $\mathrm{Re}(\boldsymbol{\mathcal{S}}) = 1/2\ ab\ \cos(\alpha-\beta)$ QED.

Application: We have $\mathbf{S} = (c/4\pi)\ \mathbf{E} \times \mathbf{H}$ for real fields as our equation (1) .

Then $\langle \mathbf{S} \rangle = \mathrm{Re}\{\ (c/8\pi)\ \mathbf{E} \times \mathbf{H}^*\ \}$, and $\boldsymbol{\mathcal{S}} = (c/8\pi)\ \mathbf{E} \times \mathbf{H}^*$

The idea is that, even though this equation $\boldsymbol{\mathcal{S}} = (c/8\pi)\ \mathbf{E} \times \mathbf{H}^*$ is quadratic in the fields, you can still apply the same "rule" to $\boldsymbol{\mathcal{S}}$ that you apply to E and H, namely, that only the real part of the thing is what you are after.