The traditional scalar form is (9.67) with both terms in there (unlike Sommerfeld). G is shown, the usual thing. Since each component of E and B satisfies the same wave equation as ψ, we get (9.68) and the same for B. Jackson then launches into a whirlwind of vector identities with far surface integrals vanishing and when the dust settles, you have (9.75) and (9.76). I have no doubt he has done this all correctly, my job is to make sure I know how to interpret and use the result.

At this point, it helps to look back at page 236 where he discusses "what happens" at the surface of a conductor. You see that $n \bullet E$ at a surface is a measure of the surface charge σ there. And n x B is a measure of the surface current K (different dimensions from J). In other words, the presence of σ allows or supports a normal E field, while the presence of a surface current K allows a tangential B. The other two items are zero: tangential E and normal B. Inside the metal all four field pieces are 0. These conclusions arise trivially from the $\text{div}E = \rho$ and $\text{curl}B = J$ equations applied to a pillbox and loop straddling the surface. Of course things don't change instantaneously at the surface, so we have picture on page 239 showing that there is some skin thickness to all this activity just below the mathematical surface.

Now back to page 285 and the vector K formulas. The surface S in these formulas is a complete bounding surface in the Green sense. If you assume radiation field types, you can limit S to your finite surface S1, some closed boundary located at some localized spot in the middle of your volume. So this is what (9.77) is saying: he has taken (9.75) and included only the interior surface S1, and has changed the definition of n so it points out of the interior ball and into our "region of interest".

Jackson goes on to "interpret" the terms in (9.75) or (9.77). Since these are surface integrals, for those parts of the surface that are "metal", we can interpret $n \bullet E$ as a surface charge σ that must be there from our general surface rules. And nxB is a surface current. But then we have to interpret the other things like nxE and $n \bullet B$ as magical "magnetic" σ and K. Interpretation is free, what the formula says is what counts.

So, equations (9.77) and its counterpart for B are the equations to use if you are doing diffraction from some 3D localized object (such as a sphere, not necessarily metal). You have to integrate 9.77 over the entire surface of that object.

Next, Jackson wants to flat-adapt this general formula for use in the usual holes-in-metal applications. He imagines on page 286 that the internal surface S1 is made into a pancake. The total internal surface in this picture is both sides, so he has now redefined the symbol S1 so that S1(in integral) = S1 + S1'. You have to integrate over both S1 and S1'. This is done in (9.68) but the two surface sides are so close that the normal vector geometry is the same, so things can be written as in (9.78). This is just a folding of the two sides into an integral over one side, nothing new has been done yet.

Now look at the three difference terms in (9.78) which are integrated only over the right-side surface S1. Think of the actual pancake right surface as metal with holes. The first term will be non-zero only in the holes, since it contains n x E. Jackson wants to *make* the other two terms vanish everywhere on S1. If he can make (9.79) be true, then he achieves that goal. And (9.79) will be true and various required field conditions will also be true if he makes (9.80) be true. These equations are basically assumptions made for fields E' and B' on the *left* side of the pancake, sort of like doing image charges. In fact, we are really doing Sommerfeld's image charge method here, and we will end up with his result! Jackson imagines that currents and charges run around inside the pancake however they must in order to generate the E' and B' fields he is assuming. The pancake has a metal sheet only on the right, I guess, or if not, then he imagines surface charges and surface currents on the left surface of the metal. The idea is that these fields imagined on the left side will affect things on the left side, but we don't care about the left side. The fields there are

just construed to make the computation on the right side be simpler. (Jackson could have clarified this important point I think better than he does.) The end result is then (9.81) which you integrate only over the holes on S1. So this is the vector version of the Kirchhoff formulafor flat surfaces that we most commonly use! He switches the ∇ operator to get (9.82) which seems easier to work with, since G = e(ikR)/R. He points out that this result is EXACT if you really know the tangential E field in your apertures.

Now, let's look carefully at (9.81). Since G = exp(ikR)/4πR where R = |r - r'|, we can compute:

$$\nabla G = (k^2/4\pi)\, e^{ikR} \left[\ i/kR - 1/(kR)^2 \ \right] \hat{\mathbf{R}} \qquad \text{where } \nabla R = \hat{\mathbf{R}}$$

Now we make the usual assumption that we are many λ away at our observation point, so kR >> 1, or R>>λ, so we get

$$\nabla G \approx\ ik\, \hat{\mathbf{R}}\, G$$

Note that $\nabla' G = -\nabla G$, so we pick up a minus sign here! Next, let's put $\hat{\mathbf{n}} = \hat{\mathbf{z}}$ without giving up anything. We then get this exact result, apart from the approximation made above in computing ∇G:

$$\mathbf{E} = -2ik/4\pi \int dA' \ (e^{ikR}/R^2)\ (z\text{-}z')\ \{\ \mathbf{E_t} - \hat{\mathbf{z}}\ (\mathbf{E_t} \bullet \mathbf{R})/(z\text{-}z')\ \} \qquad \mathbf{R} = (x\text{-}x',\ y\text{-}y',\ z\text{-}z')$$

The integration coordinate is r' = (x',y',z') and we might as well set z'=0 right now to get:

$$\mathbf{E(r)} = 1/i\lambda \int dA' \ (e^{ikR}/R^2)\ \{\ z\ \mathbf{E_t}(r') - \hat{\mathbf{z}}\ (\mathbf{E_t}(r') \bullet \mathbf{R})\ \} \qquad \mathbf{R} = (x\text{-}x',\ y\text{-}y',\ z)$$

where $\mathbf{E_t}$ is the *tangential* field in the apertures,

$$\mathbf{E_t} = (E_x,\ E_y,\ 0)$$

and the integral is only over the apertures. The second term is smaller than the first by factors like (x-x')/z and this term can be neglected in the usual diffraction situation where we are far from a small diffractor, this is the z >> d business. If we select our coordinate system of integration such that z' = 0, we then get:

$$\mathbf{E(r)} = 1/i\lambda \int dA'\ \mathbf{E_t}(\mathbf{r'})\ \ (e^{ikR}/R)\ (z/R)$$

and this is **exactly the Sommerfeld I formula** that I like, page 49 of Goodman. Notice that the field direction just passes through. Reminder: this final form is only correct for z >> d. If you want to look closer in (or at large angles away from the z axis), then use the previous result above! A z component of the resulting field should be expected, since the radiated E is perp to the line R.

Of course once you have computed E as above, you can get B from the curl equation,

$$\mathbf{B} = (i/k)\ \nabla \times \mathbf{E} \qquad \text{// as shown as (9.5).}$$