Summary of E&M Units History PhL 1.24.03

Details are in a separate document, here we just want to get the summary ideas.

There are three equations of interest

$F = k1*qQ/r^2$	(1)	/ force between two charges (electrostatics)
$dF/dx = k2*2iI/D$	(2)	/ force between two parallel wires (magnetism)
$k1 = c^2 \bullet k2$	(3)	/ see other doc for how this is shown.
$P = V*I$	(4)	/ power = potential * current

The fact that constants k1 and k2 must be connected is clear since the charge appears in both equations (1) and (2). If we double the unit of charge in both equations, we have to reduce *both* k1 and k2 by 4, so they are proportional. The fact that the constant in (3) is c^2 comes from the place where electricity and magnetism meet -- waves in space.

Now in the beginning there was only electrostatics, before 1800 say, and the cgs-esu system was used where k1=1. This defined a reasonable working unit of charge, the esu = statcoulomb. No one cared about equation (2) because it was not known yet.

After 1800, batteries and currents in wires appeared, and (4) appeared on the scene just from work with DC currents.

Soon, in the 1825 time frame, equation (2) appeared, and people who worked with currents and wires and magnetic fields and such used the cgs-emu system wherein k2= 1. This made $k1 = c^2$ so the unit of charge here, the abcoulomb, was ridiculously large for electrostatic work, but fine for magnetic work. This system of course has an abvolt as well, which turns out to be quite small.

Since batteries typically put out 1.5 e8 abvolts, it was convenient to rescale and define a "volt" such that batteries then put out 1.5 volts, so volt/abvolt = 10^8. Current was still in abcoulombs/sec = abamperes.

Next, people starting liking the mks system with its joules and watts and meters and kg, all being closer to the human scale of things, so more appropriate for building things (engineering). So, instead of using a mixed system of volts and abamperes, a new system was developed called "the practical units".

In this new system, planners wanted to use the normal mks units and the volt. In equation (4) above, since P was watts and V was volts, a current unit was implied which was 1/10 of an abampere. This is simply because joule/erg = 1e7 and volt/abvolt = 1e8, so dividing gives 1/10. This new unit is the ampere, and the corresponding charge is the coulomb. Given this charge unit, and comparing that unit say to the esu, it was then easy to figure out what the right values are for k1 and k2 for this new system. It is easy to show with unit conversions that $k2 = 10^{-7}$, written as $\mu_0/4\pi$ to make $H = \mu_0 B$ in free space:

$dF(\text{dyne}) \sim 1*i(\text{abamp})^2$ => $\text{nt/dyne} = 1/k2 * (\text{amp/abamp})^2$ $1e5 = 1/k2 * 1e\text{-}2$

$dF(nt) \sim k2\ i(amp)^2$

This made $k1 = 10^{-7}\ c^2(mks) = 9 \times 10^9$ written as $1/(4\pi\varepsilon_0)$ so that $D = \varepsilon_0 E$ in free space. The fact then that $k1/k2 = sqrt(1/(\mu_0\varepsilon_0)) = c^2$ is then pretty much a no-brainer.

So this is the SI or mksa or practical system. We end up with volts, amps, watts, Coulombs, Teslas volts/meter, Webers and all that stuff. Everyone is happy, except that B and E have different units which is ugly since they are both really part of the same $F^{\mu\upsilon}$, and we have these two constants floating around everywhere to annoy us. A theorist would rather have c floating around in equations which at least means something and can be set to 1 by rescaling time, compared to c split into two pieces μ_0 and ε_0 each of which mean nothing at all.

One other comment. It is true that E is strong and B weak in normal life as v/c, and this is most clearly seen in the cgs-esu units where the magnetic equation has $k2 = 1/c^2$ quashing magnetic effects. The reason we end up with E and B in different units in mksa is that this makes them similar in size for human life. The natural units (ie, same for E and B) make E much larger than B for humans. However, if we did things with large velocities like c/2 in our normal lives, E and B would be similar in size. This would be as if our natural time scale was nsec instead of sec. Imagine pieces of our equipment that move c/2 relative to each other, we would have similar E and B fields floating around in the natural units.