Effective Area of an Antenna PhL 2.24.03

There are many ideas here that are new to me, so I have tried to number each of the steps to keep them from blurring into each other. I had guidance from:

http://farside.ph.utexas.edu/~rfitzp/teaching/jk1/lectures/node83.html

1. Radiation resistance R_{rad} of a Hertzian Dipole antenna. From Jackson, we know the power and pattern of a "Hertzian Dipole" antenna, a center-fed antenna which is d/2 in length on each side, and where $d << \lambda$. That power is, from 9.29,

$$P = I_o^2 (kd)^2 / 12c = I_o^2/2 \; [\, 2*4\pi^2 / 12\, c * (d/\lambda)^2] = I_o^2/2 * R_{rad}$$

Here, I_o is a peak current, so $I_o^2/2 = I^2_{rms}$ and R_{rad} must then be the radiation resistance of this antenna, of course here in cgs units. Secondly, we know the power dipole radiation pattern is $\sin^2\theta$.

2. Converting R_{rad} to mks units.

1 Coulomb = c(cgs)/10 esu so 1 amp = c(cgs)/10 statamp

1 volt = 1/300 statvolt

1 ohm = 1 volt / 1 amp = 1/300 statvolt / [c(cgs)/10 statamp] = 1/[30c(cgs)] statohms

=> **R(ohms) = 30 c(cgs) R(statohms)**

Thus, for the Hertzian antenna we have

$$R_{rad}\,(\text{statohms}) = [\, 2\pi^2 / 3\, c * (d/\lambda)^2]$$

$$R_{rad}\,(\text{ohms}) = [\, 20\pi^2 * (d/\lambda)^2] = \mathbf{197\ (d/\lambda)^2}$$

3. Model for a receiving antenna. When any antenna is configured for reception, it is modeled as an induced voltage in series with R_{rad}, and your best bet for power into a load is to have $R_{load} = R_{rad}$. If the antenna emf source generates V_o peak voltage, half of that will then be on the load, so the load receives power of

$$P_{load} = (1/2)\,(V_o/2)^2 / R_{rad} = (1/8)\, V_o^2 / R_{rad},$$

where the first factor of 1/2 comes because we want to talk rms power, not peak power. In any matched antenna, therefore, fully **half the received power is re-radiated** into R_{rad} !

4. P_{load} For the Hertzian Dipole antenna. We know that V_o= Eo (d/2) [or twice this for peak to peak] , just from definition of the field, where E_o is the peak electric field amplitude. Thus, from the previous paragraph,

$$P_{load} = (1/8)\ V_o^2 / R_{rad} = (1/32)\ Eo^2\ d^2 / R_{rad}$$

On the other hand, from 2 paragraphs ago we know that for this antenna, in mks units,

$$R_{rad}\ (\text{ohms}) = 20\ \pi^2 * (d/\lambda)^2$$

Therefore, the **power delivered to the matched load of a short dipole antenna** is

$$P_{load} = 1/[\ 32*20*\pi^2]\ * E_o^2 * \lambda^2$$

which is a fundamental result, saying power received is proportional to λ^2 and to E_o^2 . We suspect that the E_o^2 proportionality will hold for any antenna, not just a short dipole.

5. Effective area concept. For any antenna, we can *imagine* an "effective antenna of area A" that scoops up 100% of the Poynting flux it receives and converts that 100% to P_{load} . We would then say

$$P_{load} = A * S$$

6. Poynting flux in mks units and Z_0. In cgs units, for a plane wave in free space we know that the time averaged S is given by

$$\mathbf{S} = (c/8\pi)\ \mathbf{E} \times \mathbf{H}^* \qquad \Rightarrow\ S = (c/8\pi)\ E_o^2$$

Converting this to mks as on page 619 Jackson requires

$$c \rightarrow 1/\sqrt{\mu_o\varepsilon_o} \qquad E_o \rightarrow \sqrt{4\pi\varepsilon_o}\ E_o$$

so we would expect to get

$$S = (1/8\pi\sqrt{\mu_o\varepsilon_o}\ )\ 4\pi\varepsilon_o\ E_o^2 = (1/\sqrt{\mu_o/\varepsilon_o}\ )\ E_o^2/2 = [\ E_o^2/2\ ]\ /\ Z_o \qquad \Rightarrow \qquad S = [\ E_o^2/2\ ]\ /\ Z_o$$

where Z_o = 376.7 = 120π ohms = $1/(\varepsilon_o c)$.

7. **Area of a Hertzian Dipole antenna.** Combining the previous two items, we find that

$$P_{load} = A * S = A * [\ E_o^2/2\ ]\ /\ Z_o$$

being the definition of this effective area A. But for the matched Hertzian Dipole we know that

$P_{load} = 1/[\ 32*20*\pi^2]\ \ * E_o^2 * \lambda^2$

from which we may find the effective area

$A * [\ E_o^2/2\ ] / Z_o = 1/[\ 32*20*\pi^2]\ \ * E_o^2 * \lambda^2$

$A = 1/[\ 32*20*\pi^2] * \lambda^2 * 2\ Z_o = 1/[\ 32*20*\pi^2] * \lambda^2 * 240\ \pi = 12/[\ 32*\pi] * \lambda^2 = (3/8\pi)\ \lambda^2$

$\Rightarrow$ $A(\text{short dipole}) = (3/8\pi)\ \lambda^2$

8. **Gain of the Hertzian dipole antenna.** Now the Hertzian dipole is not an isotropic radiator, but has a $\sin^2\theta$ dependence for $dP/d\Omega$. Let's compare a dipole radiator to an isotropic one:

$dP/d\Omega = D \sin^2\theta$ $\Rightarrow$ $P = D*8\pi/3$

$dP/d\Omega = I$ $\Rightarrow$ $P = I * 4\pi$

If we want both these radiators to radiate the same total power, we must have

$D*8\pi/3 = I * 4\pi$ $\Rightarrow$ $D * 2/3 = I$

Now if we consider the dipole in its peak radiating direction, we have $dP/d\Omega = D$, so the dipole antenna therefore has a GAIN over the same-total-power isotropic antenna of $G = 3/2$.

9. The hypothetical isotropic antenna. Imagine a transmitter sending a signal to our Hertzian antenna from a point in the peak of the beam of our receiving antenna. We have $P_{load} = A * S$ in our receiver load. Now, suppose we replace our *receiving* antenna with a hypothetical isotropic one. From reciprocity, we know that this antenna for the same incoming S is going to have *less* P_{load} by exactly the power gain factor of the Hertz antenna. So we would write

$P_{load} = A * S$ // Hertz antenna
$P'_{load} = A' * S$ // isotropic antenna
$P_{load} / P'_{load} = G = \text{gain}$

Therefore,

$A/A' = P_{load} / P'_{load} = G$

So you imagine that you start with an isotropic receiving antenna of area A_0 and the antenna's gain makes the effective area larger by $A = G * A_0$.

10. Effective area A0 of the hypothetical isotropic antenna. So, at this point we computed the A for a Hertzian short dipole to be A =$(3/8\pi)\lambda^2$, and we know that the gain for this antenna is G = 3/2. If we replaced this antenna with a hypothetical isotropic receiving antenna, that hypothetical antenna would have a smaller effective area A_o and would have a smaller power delivered to the receiving antenna load. We would have

$$A_o = (3/8\pi)\lambda^2/G = (3/8\pi)\lambda^2 * 2/3 = \lambda^2/4\pi = \pi \lambda\!\!\!^{-2} \qquad \lambda\!\!\!^{-} = \lambda/2\pi$$

11. Interaction between two antennas. Imagine now a transmitting antenna driven by P_{xmit} all of which is delivered into the transmitting radiation resistance. The flux from this antenna at distance r is going to be G x the flux from an isotropic radiator of the same power, so

$$S = G_t * P_{xmit} / (4\pi r^2)$$

The power delivered to the load in a receiving antenna will be

$$P_{load} = A_r * S = G_r A_o * \{ G_t * P_{xmit} / (4\pi r^2) \}$$

$$= G_r G_t P_{xmit} (\lambda^2/4\pi) / (4\pi r^2) = G_r G_t [\lambda/4\pi r]^2 P_{xmit} = [G_r \lambda^2/4\pi] [G_t \lambda^2/4\pi] P_{xmit}/r^2\lambda^2$$

$$= [A_r A_t/\lambda^2] P_{xmit} / r^2$$

This is the Friis Transmission Formula, symmetric in both directions of course.

12. Conclusions about Area

Therefore we arrive at these conclusions:

(1) The effective area A_0 of a hypothetical (and non-existent) isotropic antenna is a $\pi \lambda\!\!\!^{-2}$ which is the area of a circle of radius $\lambda\!\!\!^{-}$. The meaning of this area is that, if you actually had such a receiving antenna, and put it into the flux S of some transmitted plane wave, and if you matched this antenna to its load, the power delivered to the load would be S* A_0.

(2) Any real antenna has an effective area A = G A_0 = G $\pi \lambda\!\!\!^{-2}$, where G is the directivity or gain factor in presumably the best direction, which is assumed to be the way it would be used. Again, the power delivered to a matched load would be S* A.