

Electric Dipole moments as probes of physics beyond the Standard Model

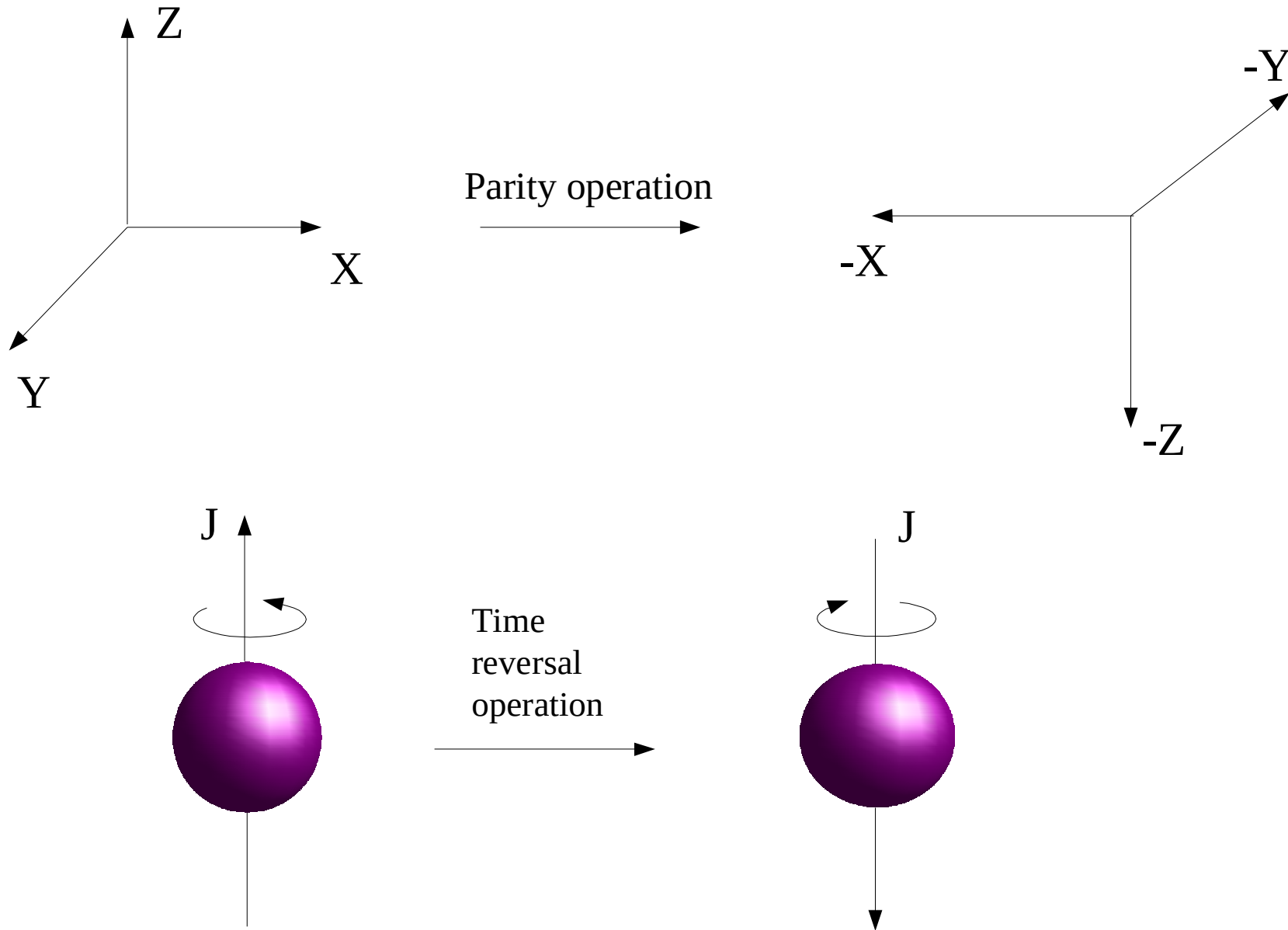
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Plan of the Talk

- Parity (P) and Time-reversal (T) operations
- Electric Dipole moments (EDMs) as consequences of P&T violations
- Sources of EDMs in atoms
- Experiments to measure EDMs – basic principle
- Closed-shell atoms – dominant interactions
- Calculation of EDMs – Requirement of atomic theory
- Present Limits and Implications for Particle Physics
- Conclusions

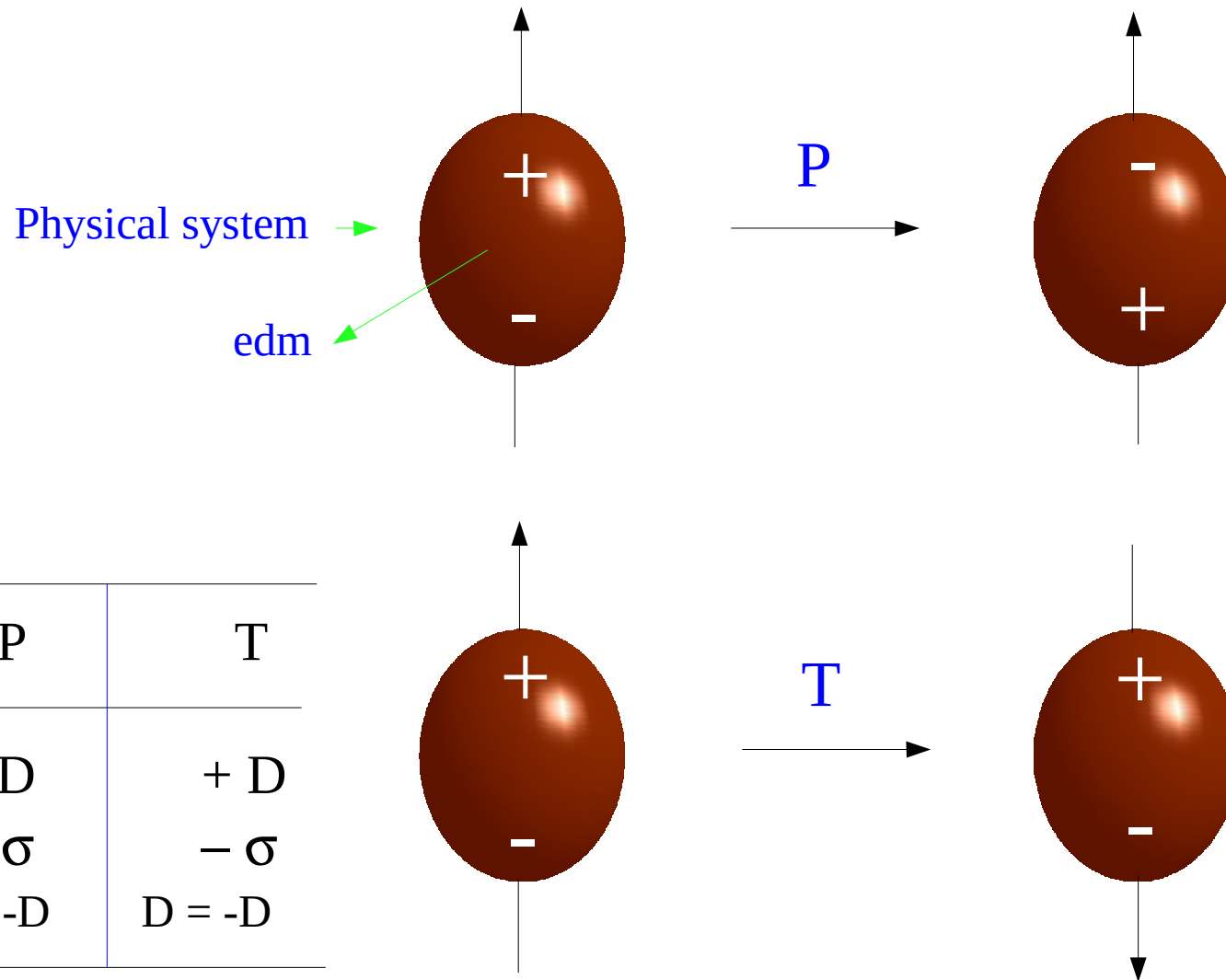
Parity and Time-reversal transformations



P & T violation for a non-zero EDM

Non-zero EDMs are direct evidence for Parity and Time-reversal violations

Angular momentum



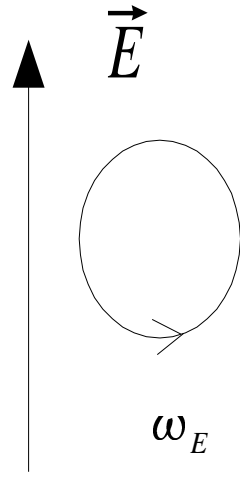
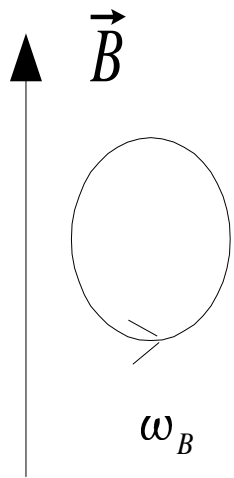
Quantity	P	T
D	- D	+ D
σ	+ σ	- σ
D = D	D = -D	D = -D

Sources of EDMs in atoms

Elementary particle	Nucleon	Nuclear		Atomic
d_e				D_a (open)
			Scalar-pseudo scalar	
e-q	e-N	e-Nuc		D_a (open)
			Tensor-pseudo tensor	D_a (closed)
d_q	d_N	d_{Nuc}		D_a (closed)
q-q	d_N , N-N	d_{Nuc}		D_a

- D_a is measured by atomic experiments
- D_a / C , ($C = C_T$, C_S , or Q) is calculated using atomic theory, C is the coupling constant of the corresponding interaction,
- Compare theory and experiment and extract the value of ' C ' .

Principle of measurement of EDM



$$\omega(\vec{B} \parallel \vec{E}) = (2\mu/\hbar)|\vec{B}| + (2D/\hbar)|\vec{E}|$$

$$\omega(\vec{B} \parallel -\vec{E}) = (2\mu/\hbar)|\vec{B}| - (2D/\hbar)|\vec{E}|$$

$$\Delta\omega = (4D|\vec{E}|)\hbar$$

If $D \sim 10^{-24}$ e-cm and $|\vec{E}| = 10^3$ V/cm

$$\Delta\nu \sim 10^{-3} \text{ Hz}$$

Closed-shell atoms

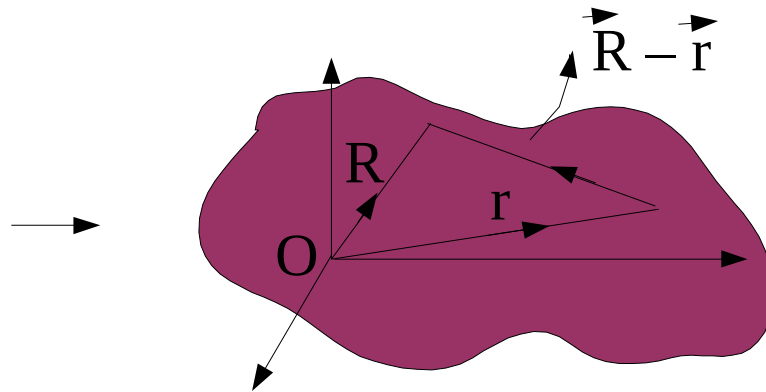
Closed-shell atoms with non-zero total angular momentum are sensitive to

- Nuclear Schiff moment
- Tensor-pseudo tensor e-N interactions

Examples : ^{199}Hg , ^{171}Yb , ^{129}Xe

- No contribution from the J of the electrons

Nuclear Schiff moment



The total charge density of the nucleus is

$$\rho(r) = \rho_0(r) + \delta\rho(r)$$

└ P, T violating interaction

The total nuclear potential at a point $\vec{R}$ from the origin is,

$$\phi(\vec{R}) = \int \frac{\rho(r)}{|\vec{R} - \vec{r}|} d^3r - \frac{1}{Z} \vec{d} \cdot \vec{E} \quad \text{where} \quad \vec{d} = e \int \vec{r} \delta\rho(\vec{r}) d^3r \quad \text{Is the nuclear electric dipole moment.}$$

The nuclear potential that is first order in the T and P violating interaction is

$$\delta\phi(\vec{R}) = e \int \frac{\delta\rho(\vec{r})d^3r}{|\vec{R} - \vec{r}|} - \frac{1}{Z}\vec{d} \cdot \vec{E}$$

The above equation can be expressed as

$$\delta\phi(\vec{R}) = e \int \frac{\delta\rho(r)}{|\vec{R} - \vec{r}|}d^3r + \frac{1}{Z}\vec{d} \cdot \nabla_R \int \frac{\rho_0(r)}{|\vec{R} - \vec{r}|}d^3r$$

where R and r are the electron and nuclear coordinates respectively. After simplification,

$$\delta\phi(\vec{R}) = 4\pi\vec{Q} \cdot \vec{\nabla}\delta(\vec{R})$$

where $\vec{Q} = Q\vec{I}$, $\vec{I}$ = nuclear spin and $\vec{d} = e \int \vec{r}\delta\rho(\vec{r})d^3r$

The interaction of the potential containing the Schiff moment with an electron in an atom is given by

$$H_Q = -4\pi e\vec{Q} \cdot \vec{\nabla}\delta(\vec{R})$$

where Q depends on $\rho_0(r)$, $\delta\rho(r)$ and quantities related to them.

Tensor-pseudo tensor e-N interaction

The P and T violating T-PT electron-nucleon interaction is

$$H_{e-N} = \sum_N \frac{G_F}{\sqrt{2}} C_T [i\bar{\Psi}_N \sigma_{\mu\nu} \Psi_N] [\bar{\Psi}_e \gamma^5 \sigma_{\mu\nu} \Psi_e]$$

$$H_{e-N} = iG_F (2\sqrt{2}) C_T \beta \vec{\alpha} \cdot \vec{I} \rho_N(\vec{r})$$

Where C_T Is the T-PT coupling constant,

G_F is the Fermi constant = $2.22 \cdot 10^{-14}$ a.u,

$\sigma_{\mu\nu}, \gamma^5$ are built from the Dirac matrices,

' I ' is the nuclear spin.

The matrix elements of the H_{e-N} operator $\sim Z^2$, hence heavy elements are preferred.

=> Relativistic many-body theory required.

Many-body theory to calculate D_a / C

Required are

- Knowledge of the Hamiltonian of the system
- Accurate relativistic electronwavefunctions

H_a = Dirac Hamiltonian for a many-electron atom = unperturbed Hamiltonian

$$= \sum_i (\mathbf{C}_i \cdot \mathbf{p}_i + m c^2 - Z e^2 / r_i) + \sum_{i < j} e^2 / r_{ij}$$

|
Coulomb interaction

In the presence of a P, T violating interaction,

$$H = H_a + C H_{PTV}$$

The Schroedinger equation for an exact atomic state is

$$H |\psi\rangle = E |\psi\rangle \text{ Where } |\psi\rangle = \underbrace{|\psi^{(0)}\rangle}_{\text{Unperturbed wavefunction}} + C \underbrace{|\psi^{(1)}\rangle}_{\text{First-order perturbed wavefunction}}$$

$|\psi^{(0)}\rangle$'s are obtained by solving the unperturbed Schroedinger equation,

$$H_a |\psi^{(0)}\rangle = E^{(0)} |\psi^{(0)}\rangle$$

The perturbed Schroedinger equation hence becomes,

$$(H_a - E^{(0)}) |\psi^{(1)}\rangle = -H_{PTV} |\psi^{(0)}\rangle$$

The atomic EDM is given by

$$D_a = \langle \psi | D | \psi \rangle / \langle \psi | \psi \rangle$$

= Expectation value of the Electric Dipole Operator.

$$\text{Hence } D_a / C = \frac{\langle \psi^{(0)} | D | \psi^{(1)} \rangle + \langle \psi^{(1)} | D | \psi^{(0)} \rangle}{(\langle \psi^{(0)} | \psi^{(0)} \rangle)}$$

$|\psi^{(0)}\rangle$ is calculated by atomic many-body methods.

- Many -body perturbation theory
- Configuration Interaction
- Coupled-cluster theory

Present Limits And Implications For Particle Physics

The limits for EDM of Hg induced by the Nuclear Schiff moment are

$$D_a / Q = -2.8 * 10^{-27} \text{ e-cm} / (\text{e fm}^3) \text{ (PRA 66, 012111 (2002))}$$

$$\text{And for } |D(\text{Hg})| < 2.1 * 10^{-28} \text{ e-cm}$$

$$\text{Gives the upper limit for Q as } -7.5 \cdot 10^{-2} \text{ e-fm}^3$$

To derive the CP violating coupling constants at the quark level from this limit on Q, nuclear structure calculations are necessary. The Schiff moment of Hg is primarily sensitive to the S-PS interaction between the proton and the neutron. This interaction is parametrized in terms of ξ , which is related to Q by

$$Q = -1.8 \cdot 10^{-7} \xi \text{ e. fm}^3$$

This gives the constraint on ξ . From ξ , the upper limit on the quark-chromo EDMs is

- Limit for the T-PT electron-nucleus coupling constant,

$$C_T = 1 * 10^{-8}$$

We need nuclear structure and particle theory to deduce C_T for electron-nucleon and electron-quark interactions.

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$$D_{Yb} = 4.75 C_T \sigma_N * 10^{-12} e a_0$$

(Angom Dilip et. al. Jphys – B, 34, 3089(2001))

Electron EDM can also be deduced from closed shell atoms by considering the hyperfine interaction as a perturbation.

The EDM of an electron, as predicted by the Standard model of Particle physics in comparison with other models :

Implications for Particle Physics

Model	d_e e-cm
Standard Model	$< 10^{-38}$
SUSY	$10^{-26} - 10^{-28}$
Multi-Higgs	
Left-right asymmetric	

- The current best limit for the EDM of an electron is obtained from the experiment on Tl atom,

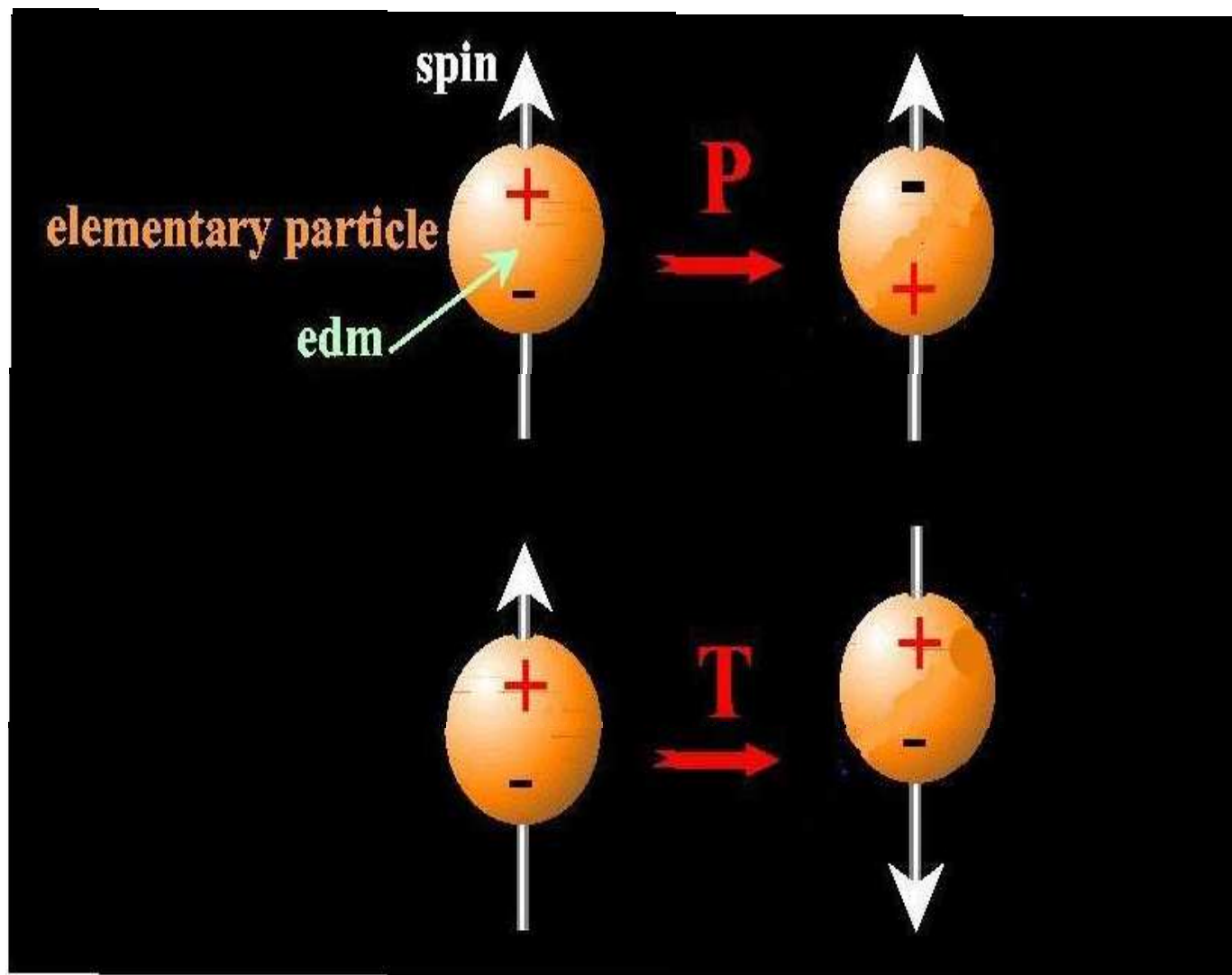
$|D_{\text{Tl}}| < 9.4 * 10^{-25}$ e-cm and comparing theory and experiment gives
 $d_e(\text{tl}) < 1.6 * 10^{-27}$ e-cm (PRL, 88 (071805) (2002)) is consistent with
the predictions of the non-standard models.

Conclusions

- Presence of EDMs is a direct evidence of T – violation.
- The knowledge of the T – PT coupling constants and the Schiff moment, Q gives deep insights into the interactions responsible for them.

P & T violation for a non-zero EDM

Non-zero EDMs are direct evidence for Parity and Time-reversal violations



Independent particle model – starting point

- Electrons are assumed to move independently of each other in an average field due to the nucleus and the other electrons.
- Residual interaction is treated as the perturbation
- The exact two-body interaction is approximated by an average one-body interaction. The many-electron anti-symmetric wavefunction in IPM is a Slater determinant

$$\phi = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_1(\mathbf{r}_1) & \psi_1(\mathbf{r}_2) & \dots & \psi_1(\mathbf{r}_N) \\ \psi_2(\mathbf{r}_1) & \psi_2(\mathbf{r}_2) & \dots & \psi_2(\mathbf{r}_N) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_N(\mathbf{r}_1) & \psi_N(\mathbf{r}_2) & \dots & \psi_N(\mathbf{r}_N) \end{vmatrix}$$

- The single particle orbitals are determined by using Variational principle which leads to Hartree Fock equations.

$$V_{\text{es}} = \sum_{i < j} e^2 / r_{ij} - \sum_i U_i(\mathbf{r}_i),$$

$U_i(\mathbf{r}_i)$ = Hartree-Fock potential ; V_{es} is treated as perturbation.

Many-body perturbation theory

- Different orders of perturbation are introduced in a systematic way
- MBPT takes care of all the excitations upto a given order of perturbation
- Residual Coulomb interaction is treated as a perturbation

The wavefunction in MBPT in terms of the perturbation parameter is

$$|\Psi_{\text{MBPT}}\rangle = |\phi\rangle + \lambda |\phi^{(1)}\rangle + \lambda^2 |\phi^{(2)}\rangle + \dots$$

Configuration interaction method

- The many-body wave function is expanded as a linear combination of determinantal wavefunctions, where the coefficients of various determinants are found using Variational principle. These determinants include

HF reference state

$$|\phi\rangle$$

Singly excited determinant

$$|\phi_a^r\rangle = C_S |\phi_S\rangle$$

Doubly excited determinant

$$|\phi_{ab}^{rs}\rangle = C_D |\phi_D\rangle$$

identified by the total angular momentum J and its projection M_J . Therefore, the CI wavefunction is given by

$$|\psi_{CI}\rangle = C_0|\phi\rangle + \sum_s C_s |\phi_s\rangle + \dots$$

- The problem of finding these coefficients reduces to the diagonalisation of the total Hamiltonian in the space of configurations. Hence the CI equation can be cast in the form

$$H C = E C$$

and solved for the energies and the coefficients.

Coupled-cluster theory

- Decomposing the wavefunction of the wavefunction of a many-particle system in terms of amplitudes for exciting clusters of a finite number of particles.

Let $|\psi\rangle$ be the exact wave function and $|\phi\rangle$ be the reference state. In CCT,

$$|\psi\rangle = e^T |\phi\rangle \quad \text{Where } T = \text{excitation operator}$$

$$|\phi\rangle = \text{Reference state}$$