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Macroscopically-Discrete Quantum Cosmology

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Abstract

To Milne's Lorentz-group-based spacetime and Gelfand-Naimark unitary representations of this group we associate a Fock space of 'cosmological preons'—quantum-theoretic universe constituents. Milne's 'cosmological principle' relies on Lorentz invariance of 'age'--*global* time. We divide Milne's spacetime into 'slices' of fixed macroscopic width in age, with 'cosmological rays' defined on (hyperbolic) *slice boundaries*—Fock space attaching *only* to these *exceptional* universe ages. Each (fixed-age) preon locates within a 6-dimensional manifold, one of whose 3 'extra' dimensions associates in Dirac sense to a self-adjoint operator that represents preon (continuous) *local* time. The operator canonically conjugate to preon local time is its (total) energy. Self-adjoint-operator expectations at any spacetime-slice boundary prescribe throughout the following slice a non-fluctuating 'mundane reality'— electromagnetic and gravitational potentials 'tethered' to current densities of locally-conserved electric charge and energy-momentum. The ray at the upper boundary of a slice is determined from the lower-boundary ray by *branched* slice-traversing Feynman paths that carry (divergence-free) potential-depending action. A macroscopically-stable positive-energy single-preon wave function identifies either with a Standard-Model elementary particle or with a graviton. Within intermediate-density universe regions such as the solar system, where reproducible measurement is meaningful, special relativity--Poincaré invariance—is accurate for spacetime scales far below that of Hubble and far above that of Planck.

Introduction

Nonexistence of unitary *finite-dimensional* Lorentz-group representations has heretofore obstructed, for dynamically-*changing* numbers of particles, a Dirac-type quantum theory that, at fixed (global) time, represents entity location, momentum, spin and energy by self-adjoint operators on a rigged Hilbert space. [(1)] The Standard Model, founded not on particles but on continuous-spacetime quantum fields associated to finite-dimensional Lorentz-group representations, and representing action by ill-defined *local-field-product* operators with perturbative renormalization procedures to manage consequent divergences, has failed to accommodate gravity. Problematic, furthermore, is the Standard-Model description of bound states ('condensed matter'), perturbation theory being unsuited to macroscopically-stationary composite wave functions.

The discrete quantum cosmology (DQC) proposed in Reference (2) and elaborated by the present paper introduces at exceptional (global) 'ages' a 'cosmological-preon' Fock space via *unitary* (infinite-dimensional) single-preon representations of the *complex* Lorentz-group. Cosmological preons are quantum-theoretic universe constituents. We divide global spacetime into 'slices' of fixed macroscopic width in age, with 'cosmological rays' defined on *slice boundaries*—our Fock space attaching *only* to these exceptional universe ages. The ray at the upper boundary of a slice is determined from that at the lower boundary by action-carrying Feynman paths that traverse the slice. Self-adjoint-operator expectations at a spacetime-slice boundary prescribe throughout the following slice a non-fluctuating 'mundane reality'— electromagnetic and gravitational potentials 'tethered' to current densities of locally-conserved electric charge and energy-momentum.

Any cosmological preon (henceforth throughout this paper simply called 'preon') is 'lightlike'—having velocity-magnitude c and a polarization transverse to velocity direction—and carries momentum, angular momentum and energy. However, only very-special preon wave functions exhibit, through expectations (at exceptional ages) of self-adjoint operators, the relation between energy, momentum, spin and spacetime location that characterizes 'ordinary matter'. A DQC Fock-space ray, representing the entire universe at some exceptional age, comprises sums of products of preon wave functions whose discrete quantum numbers allow *certain* macroscopically-stable positive-energy single-preon states to be interpreted as lepton, quark, weak boson, photon or graviton. The spacing between exceptional ages provides (precise) cosmological meaning for the adjective 'macroscopic'. Reference (2) specifies 'creation-annihilation' Feynman paths that traverse each spacetime slice and that carry action which represents gravity as well as electromagnetism and weak-strong interaction.

Although the DQC Fock space comprises Pauli-symmetrized superpositions of products of invariantly-normed single-preon functions, the present paper at the risk of obscuring total relativity chooses to emphasize the unfamiliar labels on which depends a wave function belonging to an *individual* preon. The special functions that represent elementary particles will be exposed.

Any single-preon basis carries six labels—the two 'extra' preon dimensions associating to a velocity of magnitude c whose direction is generally independent of momentum direction. To illustrate let us compare one DQC basis, with 4 continuous and 2 discrete labels, to the 'asymptotic Hilbert-space' labels of S-matrix theory. Our preon-label set comprises energy, momentum magnitude, momentum direction (2 angles) and a

pair of invariant (discrete) helicities. One of the latter is the (familiar) angular momentum in the direction of momentum while the other—here to be called 'velocity helicity'--is angular momentum in the direction of velocity. The energy of a preon is (*c* times) the component of its momentum in the direction of its velocity.

Identifiable as 'extra' in this label set are the preon's energy and its velocity-helicity. Although the continuous label called 'preon-momentum magnitude' sounds familiar, we warn readers that this label enjoys an association to Lorentz-group Casimirs which parallels the rotation-group-Casimir association of a massive particle's discrete (integer or half-integer) *spin magnitude*. Only for large values does 'preon-momentum magnitude' have the classical meaning suggested by its name.

Commutability of a complete set of 6 self-adjoint operators corresponding to the foregoing 6 preon attributes derives from commutability of right Lorentz transformations with left transformations. Our usage of the adjectives 'right' and 'left' will be explained. DQC Hilbert space unitarily represents a *12-parameter* group—the product of right and left Lorentz groups. DQC action is *right*-Lorentz invariant, its right transformations being those employed by Milne to *define* a global spacetime [(3)] and which we call 'Milne transformations' to avoid confusion with the Einstein-Poincaré meaning for a Lorentz transformation. The 6 self-adjoint operators that generate (non-commuting) Milne transformations constitute a second-rank antisymmetric tensor (a 6-vector) that represents preon momentum and angular momentum. DQC path action conserves momentum and angular momentum but not energy—which associates to one of the *left* Lorentz generators. Velocity helicity associates to another left generator, one which commutes with preon energy as well as with momentum and angular momentum.

Readers are warned that two *different* (commuting) continuous-spectrum self-adjoint operators represent, for different purposes, *preon energy*; the spectrum of one operator is positive while that of the other spans the real line. Expectations of the positive-energy operator determine 'mundane reality' and inertial and gravitational action. The indefinite-energy operator is the generator of local-time displacement—the 'Dirac conjugate' of the individual-preon (local-) time operator. DQC explicitly distinguishes local time from global time.

DQC *left* transformations lack any 6-parameter-symmetry-group precedent in natural philosophy but a 1-parameter left subgroup associates to *local-time* (*not* global 'age') translation. The generator of this left U(1) subgroup represents indefinite preon energy. (Spacetime slicing precludes meaning for continuous 'age translation'.) Another U(1) left subgroup, generated by preon velocity helicity, comprises rotations about the preon's velocity direction. In the DQC algebra of self-adjoint preon operators (which we avoid calling 'observables' because quantum cosmology admits no *a priori* meaning for 'measurement'), the two left Lorentz-group Casimirs are *equal* to the two right Casimirs (commuting with all 12 group generators).

Reference (2) addresses the DQC action of branched Feynman paths. The present paper is complementary--ignoring path action while addressing various 6-labeled bases for single-preon Hilbert space. Each basis corresponds to a complete set of 6 commuting self-adjoint operators (a 6-csco). Unitary Hilbert-space *regular* representation of the product of right and left Lorentz groups provides a DQC path-contactable basis that parallels the Feynman-path-contacting *coordinate* basis for Dirac's nonrelativistic

quantum theory. [1] An analog of Dirac's *momentum* basis associates to the unitary *irreducible* ('unirrep') SL(2,c) representation found by Gelfand and Naimark (G-N). [4]

The algebra of preon self adjoint operators collectively represents in Dirac sense preon spatial location, velocity, polarization, energy, momentum and angular momentum, as well as velocity-helicity and momentum-helicity. Of course not all these operators commute with each other. Each of the DQC bases discussed here associates to a different 6-csco. (To deal with a variety of issues the present paper will invoke seven alternative csco's.)

In the 'path basis' each preon's 'canonical coordinate' is a product of the 6 continuous coordinates of a manifold traversed by Feynman paths comprising straight positive-lightlike 'arcs' which may be created or annihilated at cubic vertices as a path progresses. The local time along any arc has *unit* derivative with respect to global age. Any path contributes, by Feynman's prescription, to the determination from some 'ray' of the succeeding ray. Each preon of a ray contacts exactly one starting arc of a Feynman path, and each finishing path-arc contacts exactly one preon of the subsequent ray. Any path-arc failing to reach its slice's upper bound is annihilated at a path branching where 1 or 2 new arcs are created.

We employ the term 'ray' because the norm and overall phase of a DQC wave function lack probabilistic or other significance. Heisenberg uncertainty-- leading to 'many worlds' in previous attempts to formulate a quantum cosmology--is absent from DQC.

Our quantum-theoretic version of Milne's 'cosmological principle' [3] recognizes time arrow and absoluteness of motion, while respecting Mach's principle in the sense discussed by Wilczek. [5] Einstein-Poincaré special relativity ignores time arrow and motion absoluteness, although partially accommodating Mach by invariance of maximum velocity. (General relativity disregards Planck and Dirac as well as Milne.) Application of DQC to *physics* requires scale-based *approximation*; DQC addresses an evolving universe that lacks *a priori* meaning for *reproducible* measurement while allowing scale-dependent approximate meaning. The general meaning of 'physics' and of special relativity in particular is confined to spacetime scales tiny compared to that of Hubble while huge compared to that of Planck. Reference (6) presents a Euclidean-group-based physics-scale gravity-less approximation to DQC that may be described as a (Higgsless) 'sliced-spacetime Standard Model' (ssSM).

The DQC (global) *age*, invariant under both right and left transformations, plays a discrete role paralleling that of continuous time in nonrelativistic quantum theory. DQC Feynman paths connect successive macroscopically-spaced *exceptional* ages at each of which is defined a cosmological Fock-space ray—in a sense recalling S-matrix theory. DQC spacetime divides into macroscopic-width 'slices' whose boundaries locate at the exceptional ages. Path branching—path-arc creation or annihilation--is forbidden at slice boundaries where a ray is defined—occurring at ages *interior* to a slice where rays are not defined. DQC dynamics prescribes quantum propagation in the discrete S-matrix sense of an "*in* state" leading to a subsequent "*out* state" without any wave function being defined between *in* and *out*.

Although not discussed in the present paper, the aggregation of DQC Hilbert space, path rules and initial condition "spontaneously" breaks *C*, *P* and *CP* symmetries. Because the DQC Hilbert space represents a group isomorphic to the *complex* Lorentz

group (as does analytic S-matrix theory), some cosmological counterpart to physical CPT symmetry promises to be revealed by future theoretical investigation.

The here-examined infinite-dimensional single-preon Hilbert space comprises normed functions of the continuous coordinates (path-basis labels) that 'classically locate' an individual preon within a 6-dimensional manifold which is at once a right *and* a left group manifold (*common* left-right Haar measure). The single-preon infinite-dimensional Hilbert space has as a factor a *finite*-dimensional (6^4 dimensions [7]) Hilbert subspace of *discrete* labels, invariant under both right and left continuous transformations, labels that are largely ignored by the present paper. Discrete labels carried by both path and ray distinguish different preon 'sectors' (e.g., electron, up-quark, photon, graviton) by specifying electric charge, color, generation, etc. [7]

We *shall* here attend to an invariant 2-valued parity-related "handedness" carried both by path arcs (between path-branching points) and by preons--in contrast to preon helicities that are undefined for a path arc. DQC Hilbert space correlates handedness to *sign* of helicity by assigning to each preon sector a unique velocity helicity that coincides with momentum helicity. Both these helicities are right-Lorentz (Milne) invariant.

G-N discussed two different bases for a Hilbert space that represents unitarily the group SL(2,c), [4] without attempting for either basis a natural-philosophical interpretation. The vectors of one basis—analog to the coordinate basis of nonrelativistic Dirac theory--are normed functions over the 6-dimensional (continuous, left-right) manifold. We call this the path basis because DQC Feynman paths traverse this 6-space. G-N's second basis--that we here call 'G-N unirrep'--parallels the Dirac-Fourier-Wigner momentum-spin basis of nonrelativistic quantum theory that unitarily and irreducibly represents the Euclidean group. [8] (The 6-parameter compound Euclidean group is a contraction of the 6-parameter semisimple Lorentz group.) Although G-N's transformation connecting their two bases differs from Fourier-Wigner, wave-function norm is preserved; the transformation is unitary.

We modify the G-N unirrep basis by (unitary) Fourier transformations of wave-function dependence on a pair of complex *directional* labels, so as to diagonalize simultaneously preon (indefinite) energy—the component of momentum in velocity direction--and components of momentum and angular momentum in some arbitrarily-specified direction. A continuous Casimir label, maintained undisturbed from the G-N unirrep basis, we call 'magnitude of momentum'. Two discrete labels are helicity interpretable—components of angular momentum in velocity and momentum directions. The modified basis, which facilitates meaning for 'preon parity reflection' while remaining a unitary irreducible SL(2,c) representation, we call the 'energy-momentum unirrep basis'.

The universe spacetime proposed by Milne in the nineteen thirties [3]—an open forward-lightcone interior whose boundary has 'big-bang' interpretation— endows 'Lorentz transformation' with a cosmological time-arrowed meaning different from the Einstein-Poincaré special-relativistic physics meaning (ignoring time arrow) that augments Lorentz invariance by spacetime-displacement invariance. (Arbitrary spacetime displacements are *not* allowed in Milne spacetime. A sufficiently large spacelike or negative-timelike displacement may move a point of Milne's spacetime *outside* his universe.) In contrast to an Einstein boost between different 'rest frames' that each assigns a different set of velocities to massive entities within some macroscopic region,

Milne boosts relate to each other different 'local frames' that each associates to a different spatial location. Milne boosts--*right* DQC transformations--are spatial displacements at fixed universe age in a curved (hyperbolic) 3-space.

General relativity's association of gravity to spacetime curvature may cause readers to suppose inability of flat Milne spacetime to represent gravity. But DQC gravitational action at a distance, plus creation and annihilation of *soft-gravitonic* arcs at path branching points, enables *discretized* spacetime curvature via 'gentle' branchings of stationary-action Feynman paths. [2] Any DQC path, as prescribed in Reference (2), is an 'event graph'—a set of spacetime-located cubic vertices connected by spacetime-straight arcs of positive-lightlike 4-velocity that carry positive energy as well as discrete attributes. By its disregard of Planck's constant, general relativity ignores gravitons and might be described as approximating, by a spacetime-curving Feynman-path trajectory (e.g., an electron trajectory), an arc-sector-maintaining stationary-action *sequence* of DQC straight 'hard' arcs that are separated at gentle events by 'soft' gravitonic-arc absorption or emission.

The adjectives 'hard' and 'soft' here refer to the energy scale set by Planck's constant h times the *inverse* of the age width of a spacetime slice—the time interval that *defines* cosmologically the adjective 'macroscopic'. Both the Standard Model and the Reference (6) ssSM revision thereof attend to the Planck constant, while setting G equal to 0 and achieving flat 3-space through disregard of the Hubble constant H--suppressing redshift by treating universe age as infinite. The ssSM physics approximation to DQC differs from the Standard Model through recognition of macroscopic spacetime slicing. Even though ignoring gravity and redshift, ssSM by accommodating soft photons *dynamically* represents the macroscopic *electromagnetic* observations that are taken for granted by the S-matrix standardly employed to interpret experiments. *Furthermore*, spacetime slicing frees ssSM from need for elementary Higgs scalar bosons.

In (un-approximated) DQC, the (inverse) *geometric mean* of slice macroscopic width and Planck-scale age step (along any path-arc) [2] establishes a *particle-mass* scale—a scale that in the Standard Model must be supplied by Higgs scalars. DQC particle scale survives into the ssSM approximation because the latter, although ignoring both Hubble and Planck scales, keeps fixed the *ratio* H^2/G as G and H individually approach zero. This ratio sets the universe mean energy density that is required by Mach's principle in conjunction with universality of maximum velocity. [2]

Treated as *a priori* by ssSM are 2 scale parameters: the foregoing particle scale, regardable as counterpart to DQC's Planck-scale path step, [2] and the macroscopic scale that the present paper introduces as an *a priori* DQC feature. The following section notes that number theory may eventually render *non-arbitrary* the integral ratio of spacetime-slice width to path step. Particle scale would thereby, like path step, become set by the trio of universal parameters G, h and c, whose values Planck realized do not require 'explanation'.

Milne Spacetime

The open interior of a forward lightcone—what we call 'Milne spacetime'--is the product of a lower-bounded one-dimensional 'age space' with an unbounded 3-dimensional 'boost space'. The spacetime displacement from the forward-lightcone vertex (whose spacetime location is meaningless) to any spacetime point is a positive-

timelike 4-vector $(t, \boldsymbol{x})$. Defining the "age" τ of a spacetime point to be its Minkowski distance from lightcone vertex—i.e., the Lorentz-invariant modulus $(t^2 - \boldsymbol{x}^2c^{-2})^{½}$ of its spacetime-location 4-vector—the set of points sharing some common age occupies a 3-dimensional (global) hyperboloid. Any point within such a hyperboloid may be reached from any other by a 3-vector *boost*. Once an origin within boost space is designated, an arbitrary spacetime point is specified by $(\tau, \boldsymbol{\beta})$, where $\boldsymbol{\beta}$ is the 3-vector boost-space displacement from the selected origin to the point. Writing $\boldsymbol{\beta} = \beta\boldsymbol{n}$, where $\boldsymbol{n}$ is a unit 3-vector and β is positive,

$$t = \tau \cosh\beta, \quad \boldsymbol{x} = c\tau\, \boldsymbol{n} \sinh\beta. \tag{1}$$

The spatial-location label $\boldsymbol{\beta}$ will in the following section and in Appendix A be identified within the path basis for DQC Hilbert space. Compatibility of age discretization with Milne's meaning for Lorentz invariance allows DQC's spacetime to be temporally discrete for its Feynman-path quantum dynamics even though spatially continuous. Discretization occurs at two very different fundamental scales: (1) Any DQC Feynman path traverses a 'slice' of Milne spacetime—a slice whose width in age is *macroscopic*. (2) Within each slice any (straight and lightlike) arc proceeds in *Planck-scale* age steps whose precise extension is established by Reference (2) from action quantization.

Although consistency requires slice width to be an integral multiple of arc step, the huge-integer ratio will not be addressed by the present paper—which ignores arc steps and number theory. Before attending to path arcs this paper chooses to address the single-preon Hilbert-space path basis. Nevertheless the termination of path arcs at exceptional (ray-age) hyperboloids might be taken as *defining* the path basis of preon Fock space.

To each point of boost space associates a "local" Lorentz frame in which $\boldsymbol{\beta} = 0$—i.e., a frame defined up to a rotation by the point's location 4-vector there being purely timelike. Age change and time change are equal in local frame. In local frame an infinitesimal spatial displacement $d\boldsymbol{x}$ at age τ relates to an infinitesimal boost-space displacement $d\boldsymbol{\beta}$ by $d\boldsymbol{x} = c\tau d\boldsymbol{\beta}$. (A phenomenological meaning for 'local frame' resides in the approximate isotropy of cosmic background radiation observed in that frame. This meaning parallels that of standard cosmology's 'co-moving coordinates'.) The rotational ambiguity of local-frame meaning is related below to arbitrariness of the origin of a 6-dimensional space. Origin location specifies not only a boost-space location but also an attached orthogonal and handed (1, 2, 3) set of 3 reference axes which may be parallel transported along a boost-space geodesic from the origin to any other boost-space location.

The (3-parameter) global orientation ambiguity is accommodated by total relativity—a DQC Fock-space restriction that requires rays to be *globally* rotationally invariant—unchanged when a common Milne rotation is applied to *all* preons. Ray *expectations* of Milne-*boost* generators are also globally invariant. Total relativity might be said to mean that both the total momentum and the total angular momentum of the universe are zero (6 conditions). The DQC universe is not only rotationally invariant but, associating right-boost generators with infinitesimal spatial displacements at fixed age, the universe is also 'classically-invariant' under (non-abelian) boost-space displacements.

Path Basis; Preon-Coordinate Matrix

The Dirac 'classical coordinate' of a preon is a product of 6 real continuous labels prescribing a *unimodular* 2×2 matrix that we shall call the "preon-coordinate matrix". Although (in contrast to 4-vectors representable by *hermitian* 2×2 matrices) preon-coordinate matrices are not linearly superposable, they may be *multiplied* either on the left or on the right by unimodular 2×2 matrices. *Right* multiplication corresponds to Milne transformation of preon coordinate—a Lorentz transformation that if applied to *all* preons is merely a change of basis origin and without significance. A right boost applied to an individual preon is a displacement in (hyperbolic) Milne 3-space. A right rotation of a preon has the familiar physics meaning designated by 3 Euler angles that refer to some *externally-fixed* set of 3 orthogonal and handed reference directions.

Left multiplication of a DQC preon's coordinate matrix represents an 'internal' preon modification--by this transformation's reference to a 'body-fixed' preon axis. The reader is invited to recall the body-fixed symmetry axis of a nonrelativistic symmetrical top; the velocity direction of a DQC preon (*distinct* from its momentum direction) may be regarded as the direction of its symmetry axis. Change of wave function by left transformation of path-basis coordinates, although incapable of altering either momentum or angular momentum, may shift a preon's energy, velocity and velocity helicity. (The mass of an elementary particle—a special preon state--may be altered by left transformation.) The meaning of 'preon' depends on the *inequivalence* of two different SL(2,c) representations--through left and right action on the preon-wave-function argument in the path basis. Preon description relies on the *commutability* of any (self-adjoint) Hilbert-space left-transformation generator with any right-transformation generator.

[The internal versus external meanings of left versus right transformations might be interchanged. The assignment chosen in this paper conforms to an arbitrary choice made by Gelfand and Naimark and our desire to maintain the notation of Reference (4). It is pedagogically unfortunate that Wigner, working with the group SU(2)—an SL(2,c) subgroup--made the opposite choice. [(8)] He represented rotation of a symmetric top with respect to some external coordinate system, or vice-versa, by *left* multiplication of the top-wave-function's matrix argument. Right multiplication of the Wigner-defined argument has capacity to vary the top's angular momentum *component* in the direction of its symmetry axis while leaving total angular momentum unaltered either in direction or magnitude. Wigner right multiplication may change top (rotational kinetic) energy while left multiplication lacks such capability, even though his left multiplication may alter angular-momentum direction with respect to some fixed set of external axes.]

Because the doubly-covered Lorentz group is isomorphic to the group of unimodular 2×2 matrices—one must be careful to distinguish a preon-coordinate matrix from a matrix that represents a coordinate *transformation* (either left or right). Although coordinate matrices and transformation matrices both depend on 6 real parameters, parameter meanings are of course different. Analog is found in the 3 Euler angles that coordinate a symmetrical top, as opposed to the 3 angles that label a rotation.

The *coordinate matrix* of Preon *i* or, alternatively, that of Arc *i* within a Feynman path, will here be denoted by the (boldface) G-N symbol $\boldsymbol{a}_i$. Apart from the double

covering of rotations, the content of $\boldsymbol{a}_i$ may be expressed through three Milne 4-vectors represented by hermitian 2×2 matrices defined by the $\boldsymbol{a}_i$ quadratic forms,

$$\boldsymbol{x}_i \equiv \tau\, \boldsymbol{a}_i^{\dagger} \boldsymbol{a}_i = \tau \exp\,(-\boldsymbol{\sigma} \bullet \boldsymbol{\beta}_i), \tag{2}$$

$$\boldsymbol{v}_i \equiv \boldsymbol{a}_i^{\dagger}\,(\sigma_0 - \sigma_3\,)\,\boldsymbol{a}_i, \tag{3}$$

$$\boldsymbol{e}_i \equiv \boldsymbol{a}_i^{\dagger}\,(-\,\sigma_1\,)\,\boldsymbol{a}_i, \tag{4}$$

where $\boldsymbol{\sigma} \equiv (\sigma_1, \sigma_2, \sigma_3)$ is the standard (handed) set of Pauli hermitian traceless self-inverse 2×2 matrices (determinant –1). The matrix σ_3 is real diagonal while σ_1 and σ_2 are antidiagonal, σ_1 real and σ_2 imaginary. The symbol • in (2) denotes the inner product of two 3-vectors. The symbol σ_0 denotes the unit 2×2 matrix. Although G-N did not employ Pauli matrices, we have found them convenient. Otherwise we largely maintain the notations of Reference (4). [In checking that (2), (3) and (4) transform as Milne 4-vectors, the reader needs remember that although Milne transformation of a coordinate matrix is multiplication from the *right*, Milne transformation of the hermitian conjugate of a coordinate matrix is a *left* multiplication.]

The positive-definite hermitian matrix $\boldsymbol{x}_i$, with determinant τ^2 and positive trace, represents [Formula (1) and Appendix A] a positive-timelike 4-vector whose components are $\tau\, cosh\, \beta_i$, $\tau\, \boldsymbol{n}_i\, sinh\, \beta_i$, where $\boldsymbol{\beta}_i = \beta_i \boldsymbol{n}_i$ with $\boldsymbol{n}_i$ a unit 3-vector and β_i a positive real number. We interpret the 4-vector represented by (2) as the spacetime displacement of the preon's location, or a location along some arc of a Feynman path, from the big-bang forward-lightcone vertex—the 'origin' of Milne spacetime. The 3-vector $\boldsymbol{\beta}_i$ is the above-discussed location in Milne boost space.

Components of the 4-vectors $\boldsymbol{v}_i$ and $\boldsymbol{e}_i$ may be pulled from the 2x2 matrices (3) and (4) in the usual way. The positive-lightlike 4-vector represented by the zero-determinant, positive trace hermitian matrix $\boldsymbol{v}_i$ is characterizable, in the path basis of a cosmological ray, as the 4-velocity of Preon *i*, with unit timelike component in this particle's local frame--where $\boldsymbol{\beta}_i = 0$. The symbol $\boldsymbol{v}_i$ also represents the 4-velocity of an *i*-labeled Feynman-path arc that passes through the 6-space point coordinated by $\boldsymbol{a}_i$.

The hermitian matrix $\boldsymbol{e}_i$ (determinant –1) represents the Preon-*i* (unit-normalized) *spacelike* transverse-polarization 4-vector that is (right) Lorentz orthogonal to $\boldsymbol{x}_i$ and $\boldsymbol{v}_i$. The same symbol may be used to denote the polarization of Arc *i*. Formulas (2, 3, 4) specify 4-vectors in that Lorentz frame—belonging to some arbitrarily designated '6-space origin'—in which the coordinate matrix is $\boldsymbol{a}_i$.

The *unit* 2×2 matrix σ_0 represents an *origin* of 6-space shared by *all* the preons represented in a cosmological ray. When a preon-coordinate $\boldsymbol{a}_i$ is a unit matrix, the preon (classically) locates at the origin of boost space with its velocity in the (arbitrarily-selected) 3 direction and its polarization in the 1 direction. The angle that specifies polarization in the general case, in a plane perpendicular to velocity direction, spans a 4π interval with the *sign* of $\boldsymbol{a}_i$ *reversing* under any 2π continuous displacement of polarization direction.

A path-basis single-preon wave function $\Psi(\boldsymbol{a}_i)$ has a finite norm

$$\int d\boldsymbol{a}_i \left| \Psi(\boldsymbol{a}_i) \right|^2 , \tag{5}$$

where the 6-dimensional invariant volume element $d\boldsymbol{a}_i$ (the Haar measure), is specified immediately below for the G-N parameterization of $\boldsymbol{a}_i$. The norm (5) is invariant under *both* of the above-discussed (right and left) coordinate transformations. Wave-function norm is preserved by the transformation to G-N unirrep basis as well as to various other bases we shall discuss.

G-N expressed the most general unimodular 2×2 coordinate-matrix $\boldsymbol{a}$ with $\boldsymbol{a}_{22} \neq 0$ through 3 *complex* labels s, y, z according to the product of 3 unimodular 2×2 matrices each representing an abelian 2-parameter subgroup of SL(2,c),

$$\boldsymbol{a}(s, y, z) = exp(-\sigma_3 s) \times exp\,(\sigma_+ y) \times exp(\sigma_- z)\,, \qquad (6)$$

where $\sigma_\pm \equiv \tfrac{1}{2}(\sigma_1 \pm i\sigma_2)$. [G-N employed, rather than y, a complex label equal to ye^{-s}.] Because the spatial direction of the 4-velocity $\boldsymbol{v}$ is completely determined by z (see Formula (3') below) we shall refer to z as the "velocity coordinate". Considerations in the following two sections reveal it appropriate to call 2 *Re s* either the preon's 'longitudinal coordinate' or its 'local time' while y is its 'transverse coordinate'. Calculation, substituting (6) into (4), shows the angle specifying polarization direction (in a plane transverse to velocity) to be 2 *Im s*. The polarization 4-vector $\boldsymbol{e}$ is independent of *Re s*.

The 6-dimensional volume element,

$$d\boldsymbol{a} = ds\, dy\, dz, \qquad (7)$$

is invariant under $\boldsymbol{a} \rightarrow \boldsymbol{a}\,\boldsymbol{\Gamma}^{-1}$, with $\boldsymbol{\Gamma}$ a 2×2 unimodular matrix representing an external (right) Milne transformation. It is also invariant under the internal (left) transformation $\boldsymbol{a} \rightarrow \boldsymbol{\gamma a}$, where $\boldsymbol{\gamma}$ is a unimodular 2×2 matrix.

By the symbol $d\xi$, with ξ complex, is meant $d\,Re\,\xi \times d\,Im\,\xi$. Formula (6) implies periodicity of $\Psi(s, y, z)$ in *Im s* with period 2π, and the norm (5), with a corresponding interval for *Im s*, is preserved by Fourier-series representation of Ψ dependence on *Im s*. The norm is also preserved by Fourier-integral representation of Ψ dependence on *Re s* over the real line. Such a 2-dimensional unitary Fourier transformation (*not* to be confused with 6-dimensional transformations to G-N unirrep and energy-unirrep bases) will immediately be discussed. The resulting basis we call "intermediate".

Intermediate Basis--Diagonalizing Preon Energy, Velocity and Velocity Helicity

The intermediate basis is labeled by the two (path-basis) complex-number symbols y, z and two real (positive or negative) Milne-invariant (right Lorentz-invariant) labels--an integer n and a continuous ω. In Dirac sense the label n is 'conjugate' to *Im s* while the label ω is conjugate to *Re s*. Fourier-series representation of wave-function dependence on *Im s* defines the discrete index n that, before its correlation with 'extra-Lorentz' discrete Hilbert-space labels, may take any integral value (positive, negative or zero). Fourier-integral representation of wave-function dependence on *Re s* analogously defines the continuous label ω that spans the real line. We make the unitary transformation,

$$\Psi(s, y, z) = \boldsymbol{\Sigma}_n \int d\omega\; exp(-in\; Im\; s - i\omega\; Re\; s)\, \psi_{n,\omega}(y, z), \qquad (8)$$

the integer label n prescribing in $\hbar$ units, as shown below, twice the preon velocity helicity. The label ω, after consideration of DQC Feynman-path arcs, will be seen to prescribe twice preon (total) energy in $\hbar/\tau$ units. The norm (5), up to factors of 2 and π, equals

$$\Sigma_n \int d\omega\, dy\, dz \left| \psi_{n,\omega}(y, z) \right|^2. \tag{9}$$

Multiplying the coordinate matrix $\boldsymbol{a}$ from the left by the unimodular matrix $\boldsymbol{\lambda}_0 \equiv exp(-\sigma_3 s_0)$, so that s changes to $s + s_0$ with y and z remaining fixed, effectuates a rotation of polarization by an angle $2\ Im\ s_0$ around velocity direction along with a shift of preon boost-space location along velocity direction by $2\ Re\ s_0$. The intermediate-basis wave function $\psi_{n,\omega}(w, z)$ is seen from (8) to be changed, under $\boldsymbol{\lambda}_0$ left multiplication, merely by the phase factor $exp(-in\ Im\ s_0 - i\omega\ Re\ s_0)$. For pure rotation (i.e., $Re\ s_0 = 0$) by an angle $\varphi = 2\ Im\ s_0$ about velocity direction, this phase factor is $exp(-in\ \varphi/2)$--hence our characterization of n's meaning.

Two self adjoint operators on the G-N Hilbert space generate the foregoing transformations. In the path basis these operators are

$$J_{3L}{}^{\wedge} = \tfrac{1}{2}\, i\, \partial/\partial\ Im\ s, \tag{10}$$

$$K_{3L}{}^{\wedge} = \tfrac{1}{2}\, i\, \partial/\partial\ Re\ s. \tag{11}$$

$J_{3L}{}^{\wedge}$ generates (at fixed $Re\ s$, y and z, which the following section shows to mean fixed preon spatial location) rotation of preon polarization about its velocity direction while $K_{3L}{}^{\wedge}$ generates (at fixed velocity and polarization as well as fixed transverse location) spatial displacement in this direction. According to (8) the eigenvalues of $J_{3L}{}^{\wedge}$ are $\frac{1}{2}\, n$ while those of $K_{3L}{}^{\wedge}$ are $\frac{1}{2}\, \omega$. It will be seen later that $J_{3L}{}^{\wedge}$ and $K_{3L}{}^{\wedge}$ are among the set of 6 commuting self-adjoint operators whose real eigenvalues label the energy-momentum unirrep basis. That is, n and ω are labels *both* of the energy-momentum unirrep basis *and* the intermediate basis.

Odd-even n distinguishes fermion from boson inasmuch as increasing the value of $Im\ s$ by π with $Re\ s$, y, z unchanged reverses the sign of the preon-coordinate matrix $\boldsymbol{a}$. Writing $\Psi(\boldsymbol{a}) = \Sigma_n\ \Psi_n(\boldsymbol{a})$, Formula (8) implies

$$\Psi_n(-\boldsymbol{a}) = (-1)^n\ \Psi_n(\boldsymbol{a})\ . \tag{12}$$

DQC Hilbert space correlates a 2-valued handedness label with the *sign* of n, in restricting *any* preon sector $\mathfrak{s}$ to a *single* value of $n = n_{\mathfrak{s}}$. A fermionic preon has $n = \pm 1$, a vector-bosonic preon has $n = \pm 2$ and a gravitonic preon $n = \pm 4$. Path-carried sector-specifying discrete labels and Milne-invariant path action perpetuate this reduction of DQC Hilbert space.

The continuous label ω, related to inertial and gravitational action through the positive-energy operator defined in Appendix C, [(2)] may appropriately be called 'preon frequency' or, in units specified below, 'preon energy'. Both preon velocity-helicity and preon energy, as well as preon momentum-helicity and preon 'momentum magnitude',

will be shown to be Milne (right-Lorentz) invariant. Energy and momentum-magnitude labels enjoy the meaning familiar for these terms in the preon's *local* frame.

The following section shows *Re s* to increase, along any arc of a Feynman path, with *unit* (local-frame) *age* derivative of the *local* time, while the remaining 5 path-basis labels remain fixed. Appendix C associates the label ω to the energy of any 'beginning arc'—one which contacts that ray at the lower boundary of the spacetime slice occupied by this arc. The ray's 'partial-expectation' of the positive-energy operator defined in Appendix C establishes the energy of any path arc originating at this ray. In the paper's penultimate section the full expectation of a certain self-adjoint *operator function* of $K_{3L}{}^{\wedge}$ prescribes the non-fluctuating near-future energy-momentum density that, by determining gravitational action (for ray propagation to the succeeding ray), constitutes an aspect of 'mundane reality'. The (indefinite) energy operator $\hbar/\tau\ K_{3L}{}^{\wedge}$ is diagonal (with eigenvalues $\hbar\omega/2\tau$) both in the intermediate basis and in the energy-momentum unirrep basis, although not in the path or G-N unirrep bases.

Path Arcs

Each (positive-lightlike) arc of a DQC Feynman path moves (in Planck-scale age steps [(2)]) through the 7-dimensional continuous space coordinated by τ and the 3 complex variables s, y, z. Because the Pauli-matrix identity

$$\sigma_0 = \tfrac{1}{2}\,(\sigma_0 + \sigma_3) + \tfrac{1}{2}\,(\sigma_0 - \sigma_3\,), \tag{13}$$

expresses the unit matrix as the sum of two zero-determinant unit-trace hermitian matrices, Formulas (2) and (6), by straightforward calculation, imply any spacetime location (with respect to Milne-lightcone vertex) to be the sum of two "oppositely-directed" positive-lightlike 4-vectors:

$$\boldsymbol{x} = \tau\,[|y|^2\, e^{-2Re\,s}\, \boldsymbol{h}(z + y^{-1}) + e^{2Re\,s}\, \boldsymbol{h}(z)]. \tag{14}$$

The symbol $\boldsymbol{h}(\varsigma)$ denotes a zero-determinant hermitian positive-trace 2×2 matrix,

$$\boldsymbol{h}(\varsigma) \equiv (\boldsymbol{\sigma}_0 + \varsigma^{*}\boldsymbol{\sigma}_+)\,\boldsymbol{\sigma}_-\boldsymbol{\sigma}_+\,(\boldsymbol{\sigma}_0 + \varsigma\,\boldsymbol{\sigma}_-), \tag{15}$$

a bilinear function of a direction-specifying complex variable. The second term within the square bracket of (14) relates to the following formula, deducible from (3), for the positive-lightlike 4-velocity (unit time-component in local frame) of a preon or arc:

$$\boldsymbol{v} = 2\, e^{2Re\,s}\, \boldsymbol{h}(z). \tag{3'}$$

Notice absence from (14) of any dependence on *Im s*.

Formula (14) reveals two distinct categories of positive-lightlike fixed-y, fixed-z trajectory through 7-dimensional (τ, $\boldsymbol{a}$) space. In one category the first term of (12) is age independent while the second's magnitude increases with age. In the other category the second term is fixed while the first's magnitude increases. We call the former category ***F*** and the latter ***B***. Along an ***F*** trajectory $e^{2Re\,s}$ is proportional to τ, while along a ***B*** trajectory $e^{2Re\,s}$ is *inversely* proportional to τ. DQC achieves unit age derivative of local

time along any Feynman path by requiring every arc to lie along an $\boldsymbol{F}$ trajectory--identifying the derivative with respect to age of local time as $\tau\, d(2Re\ s)/d\tau$.

An $\boldsymbol{F}$ trajectory is identified by the two complex dimensionless labels z, y and one real label $\tau_{\boldsymbol{F}} \equiv \tau\, e^{-2Re\ s}$ of time dimension. Formula ($14_{\boldsymbol{F}}$) represents the most general path spacetime trajectory by a hermitian-matrix that is a second-degree polynomial in age:

$$\boldsymbol{x}_{z,w}{}^{\boldsymbol{F}}(\tau) \equiv\ \tau_{\boldsymbol{F}}|y|^2\, \boldsymbol{h}(z + y^{-1}) + \tau^2\tau_{\boldsymbol{F}}{}^{-1}\, \boldsymbol{h}(z). \qquad (14_{\boldsymbol{F}})$$

The age-independent first term of ($14_{\boldsymbol{F}}$) --an "initial displacement" from the lightcone vertex that depends on both z and y—locates the $\boldsymbol{F}$ trajectory's zero-age *origin* on the (3-dimensional) Milne forward-lightcone boundary. *Any* arc, beginning and ending along *some* such trajectory, lies *inside* the lightcone. The second term of ($14_{\boldsymbol{F}}$), positive lightlike like the first term but in the opposite spatial direction and independent of y, leads from the trajectory's big-bang origin to the age-τ spacetime point. The second term's direction is determined entirely by z.

G-N Unirrep Basis

The unitary *irreducible* SL(2,c) representation found by Gelfand and Naimark provides a Hilbert-space basis, alternative to the path basis, that we shall call the 'G-N unirrep basis'--comprising functions of 2 real labels and 2 complex labels. A norm-preserving transform connects path and G-N unirrep bases.

The two real labels prescribe eigenvalues of two self-adjoint operators that represent Casimir-quadratics in the generators of *either* right *or* left Lorentz transformations—polynomials invariant under both right and left 6-parameter groups (commuting with all 12 generators). If we denote the trios of right-rotation and left-rotation generators by the 3-vector boldface operator symbols $\boldsymbol{J}_R{}^{\wedge}$, $\boldsymbol{J}_L{}^{\wedge}$ and the trios of boost generators by the symbols $\boldsymbol{K}_R{}^{\wedge}$, $\boldsymbol{K}_L{}^{\wedge}$ the Casimir, $\boldsymbol{K}_R{}^{\wedge} \bullet \boldsymbol{J}_R{}^{\wedge} = \boldsymbol{K}_L{}^{\wedge} \bullet \boldsymbol{J}_L{}^{\wedge}$, has eigenvalues $(\rho/2)\,(m/2)$ while the Casimir, $\boldsymbol{K}_R{}^{\wedge}\bullet\boldsymbol{K}_R{}^{\wedge} - \boldsymbol{J}_R{}^{\wedge}\bullet\boldsymbol{J}_R{}^{\wedge} = \boldsymbol{K}_L{}^{\wedge}\bullet\boldsymbol{K}_L{}^{\wedge} - \boldsymbol{J}_L{}^{\wedge}\bullet\boldsymbol{J}_L{}^{\wedge}$, has eigenvalues $(\rho/2)^2 - (m/2)^2 + 1$. The label m takes positive, negative or zero integer values that are shown below to correspond to *twice* preon helicity (angular momentum in momentum direction). Bosonic preons have even values of m while fermionic preons have odd. The second Casimir label, denoted by the symbol ρ, takes continuous real non-negative values. In $\hbar/c\tau$ units, $\rho/2$ will be called the preon's (local-frame) 'magnitude of momentum'.

The remaining labels on which a preon's G-N unirrep-basis wave function depends are 2 complex variables, z' and z_1, the former right-invariant and the latter left invariant. In Reference (4) the right-invariant variable z' is denoted by the symbol z; we have here added a prime superscript to avoid confusion with the path-basis label z. The left-invariant complex label z_1 prescribes in a cosmological ray the *direction* of preon momentum in a sense similar to that above in which the path-basis complex variable z prescribes velocity direction. Loosely speaking, the complex variable z' may be understood as representing the *difference* between momentum and velocity directions. A precise meaning for z' resides in the difference between ρ and ω and the difference between m and n. In a sense similar to that of Dirac, half these differences will be seen 'conjugate', respectively, to $ln\ |z'|$ and $arg\ z'$.

The set (z', m, ρ, z_1) of 2 real-Casimir and 2 complex directional labels we shall denote by the shorthand index $\boldsymbol{b}$, the G-N unirrep-basis wave function $\Phi(\boldsymbol{b})$ being a transform of the path-basis wave function $\Psi(\boldsymbol{a})$. The (right-left-invariant) G-N unirrep-basis norm is

$$\int d\boldsymbol{b} \; \big| \, \Phi \, (\boldsymbol{b}) \big|^{\,2} \, , \tag{16}$$

where the symbol $\int d\boldsymbol{b}$ means (with m assigned some 'preon-type' value)

$$\int d\boldsymbol{b} \equiv (1/2\pi)^4 \; \int d\rho \; (m^2 + \rho^2) \int dz' \int dz_1 \; , \tag{17}$$

the integration over $d\rho$ running from zero to + infinity. For Preon Sector $\mathfrak{s}$ of DQC Hilbert space the value of m is taken equal to $n_{\mathfrak{s}}$--*one* of the 6 options ±1, ±2, ±4.

The norm-preserving transform is achieved by an integration over the 4-dimensional manifold regularly representing those 2×2 unimodular matrices—a 4-parameter SL(2,c) subgroup—that have the form

$$\boldsymbol{k}(\lambda, \, y) \equiv exp(-\sigma_3 \, ln\lambda) \times \, exp(\, \sigma_+ y), \tag{18}$$

with λ and y almost-arbitrary complex numbers ($\lambda \neq 0$). The 4-dimensional volume element

$$d_\ell \boldsymbol{k} = |\lambda|^{-2} d\lambda \; dy \tag{19}$$

is 'left-invariant' in the sense that an integral $\int d_\ell \boldsymbol{k} \, f \, (\boldsymbol{k})$ over some function $f \, (\boldsymbol{k})$ of the matrix $\boldsymbol{k}$ equals $\int d_\ell \boldsymbol{k} \, f \, (\boldsymbol{k}_0 \, \boldsymbol{k})$, with $\boldsymbol{k}_0$ any *fixed* matrix within the subgroup. The transform from path to G-N unirrep basis is

$$\Phi(\boldsymbol{b}) = \int d_\ell \boldsymbol{k} \; \alpha_{m\rho}(\boldsymbol{k}) \; \Psi(\boldsymbol{z}'^{-1} \; \boldsymbol{k} \; \boldsymbol{z}_1), \tag{20}$$

where a bold-faced $\boldsymbol{z}$ symbol in (20) is to be understood as the 2×2 unimodular matrix $exp \; (\sigma_- z)$. The function $\alpha_{m\rho}(\boldsymbol{k})$—analog of the Fourier transform's exponential function and a cornerstone of G-N's analysis—is given by

$$\begin{aligned} \alpha_{m\rho}(\boldsymbol{k}) &\equiv | \, k_{22}|^{-m+i\rho-2} \; k_{22}{}^{m} \\ &= | \, k_{22}|^{-2} exp \; (im \; arg \; k_{22} + i\rho \; ln \; | \, k_{22}|). \end{aligned} \tag{21}$$

Note that $\alpha_{m\rho}(\boldsymbol{k})$ is independent of the parameter y in (18), depending only on $k_{22} = \lambda$. Although (20) is written in a form facilitating left transformation, an alternative form facilitates right transformation. Our choice above is arbitrary—motivated by desire to maintain the notation of Reference (4). Later the alternative form will also be invoked.

The G-N reverse transform, expressed in a notation where the path-basis wave-function is understood not as a function of a unimodular matrix $\boldsymbol{a}$ but as a function of the three complex labels s, y, z defined by (6), is

$$\Psi_m(s, y, z) = (1/2\pi)^4 \int d\rho\, (m^2 + \rho^2) \int dz_1 |\lambda'|^{-m- i\rho+2}\, \lambda'^{*m}\, \Phi(m, \rho, \check{z}, z_1), \tag{22}$$

the symbols λ' and $\check{z}$ denoting quotients that depend on z_1 as well as on s, y, z, :

$$\check{z} \equiv e^s \lambda' (z_1 - z), \quad \lambda' \equiv e^s / [1 - (z_1 - z)\, y]. \tag{23}$$

The right-invariant G-N unirrep-basis complex label z' is constrained in (22) to the value $\check{z}$, vanishing when momentum and velocity directions are parallel. The point relation (23)--two real point relations--between path-basis and G-N unirrep-basis labels, 'entangled' by y involvement, enjoys a 'disentangled' counterpart in our use below of the intermediate-basis labels n, ω as also labels on the *energy-momentum* unirrep basis. Disentanglement will make the latter basis appropriate for specification of preon sectors.

External (Right) and Internal (Left) Transformations

Let us denote by the symbol $\boldsymbol{\Gamma}^{-1}$ a 2×2 unimodular matrix applied on the *right* of a coordinate matrix so as to effectuate an "external" (Milne) Lorentz transformation $\Psi(\boldsymbol{a}_1, \boldsymbol{a}_2 \ldots) \rightarrow \Psi^{\boldsymbol{\Gamma}}(\boldsymbol{a}_1, \boldsymbol{a}_2 \ldots) \equiv \Psi(\boldsymbol{a}_1\boldsymbol{\Gamma}^{-1}, \boldsymbol{a}_2\boldsymbol{\Gamma}^{-1}, \ldots)$ of a path-basis cosmological wave function --a sum of products of single-preon functions. A corresponding transformation is induced in *any* Hilbert-space basis, although generally a wave-function *factor* accompanies transformation of wave-function argument. (Absence of such factor characterizes the group-manifold path basis). Here ignored are discrete coordinates such as handedness, electric charge and color that are invariant under both right and left coordinate transformation. Although Milne's cosmological principle associates in DQC to *global* right transformation, the present section addresses the effect on an *individual* preon function of either (external) right or (internal) left transformation.

In terms of the 3 complex path-basis coordinates s, y, z the right transformation is

$$\Psi (s, y, z) \rightarrow \Psi(s^{\boldsymbol{\Gamma}}, y^{\boldsymbol{\Gamma}}, z^{\boldsymbol{\Gamma}}), \tag{24}$$

where by straightforward calculation of $\boldsymbol{a}\boldsymbol{\Gamma}^{-1}$ with $\boldsymbol{a}$ given by (6),

$$z^{\boldsymbol{\Gamma}} = (\Gamma_{22} z - \Gamma_{21})\, (-\Gamma_{12} z + \Gamma_{11})^{-1}\,, \tag{24a}$$

$$y^{\boldsymbol{\Gamma}} = (-\Gamma_{12} z + \Gamma_{11})[(-\Gamma_{12} z + \Gamma_{11}) y - \Gamma_{12}], \tag{24b}$$

$$s^{\boldsymbol{\Gamma}} = s + ln(-\Gamma_{12} z + \Gamma_{11})\,. \tag{24c}$$

Notice how right transformation of z is independent of the variables s, y, while all right coordinate transformations involve z and right transformation of s (y) is independent of y (s). [Taken alone, (24a) provides a representation of SL(2,c) by action of this group on the points of a complex plane.]

Notice further how, with respect to the complex coordinate s, the (24c) right transformation is merely a z-dependent displacement, implying right invariance of the intermediate-basis labels n and ω. Right Lorentz transformation of the intermediate basis (with labels n, ω, y, z) alters the wave-function arguments y and z according to (24b) and

(24a) while leaving n and ω undisturbed although multiplying the wave function by a phase, dependent on n, ω and z, that is deducible from (8) and (24c).

Milne (right) transformation induces, by calculation according to (20), in the G-N unirrep basis the transformation

$$\Phi\,(\boldsymbol{b}) \rightarrow \Phi^{\Gamma}\,(\boldsymbol{b}) = \alpha_{m\rho}{}^{r}[(-\Gamma_{12}\, z_1 + \Gamma_{11})^{-1}]\; \Phi(m, \rho, z', z_1{}^{\Gamma}), \tag{25}$$

where $z_1{}^{\Gamma}$ is the polynomial quotient (24a) with z_1 in place of z, and

$$\alpha_{m\rho}{}^{r}(k_{22}) \equiv |k_{22}|^4\; \alpha_{m\rho}(k_{22}). \tag{26}$$

Both the momentum and the angular momentum of a preon may be changed (relative to a fixed origin of Milne 6-space) by right transformation, but helicity and 'magnitude of momentum' are invariant as also (we have emphasized above) is energy and velocity helicity. Right-Lorentz transformation leaves undisturbed the complex argument z' of the G-N unirrep-basis wave-function.

The individual-preon transformation from the left in the G-N unirrep basis is as simple as (25). Left transformation alters wave-function dependence on the "internal" parameter z' while dependence on momentum direction z_1 is unchanged. One finds

$$\Phi\,(\boldsymbol{b}) \rightarrow {}^{\gamma}\Phi\,(\boldsymbol{b}) = \alpha_{m\rho}(-\,\boldsymbol{\gamma}_{12}\, z' + \boldsymbol{\gamma}_{11})\; \Phi(m, \rho, {}^{\gamma}z', z_1), \tag{27}$$

where

$${}^{\gamma}z' = (\boldsymbol{\gamma}_{22}z' - \boldsymbol{\gamma}_{21})(-\,\boldsymbol{\gamma}_{12}z' + \boldsymbol{\gamma}_{11})^{-1}. \tag{28}$$

Neither the momentum nor the angular momentum (nor of course the helicity) changes, as the 6 coordinates (path-basis labels) of some preon are left-shifted (*other* preons remaining unshifted). Left transformation of Preon i may change not only its spatial location and local time but its energy, its velocity and its velocity helicity.

The earlier-considered special left transformation $\boldsymbol{\gamma} = \boldsymbol{\lambda}_0$, where $\boldsymbol{\lambda}_0$ is the unimodular matrix $exp(-\sigma_3\, ln\lambda_0)$, preserves preon velocity while prescribing a rotation around velocity direction and a longitudinal shift of preon location along this direction ($\boldsymbol{\gamma}_{12} = \boldsymbol{\gamma}_{21} = 0$ and $\boldsymbol{\gamma}_{22} = \boldsymbol{\gamma}_{11}{}^{-1} = \lambda_0$). One finds, from (27) and (28),

$${}^{\gamma}\Phi\,(m, \rho, z', z_1) = \alpha_{m\rho}{}^{-1}(\lambda_0)\; \Phi\,(m, \rho, \lambda_0{}^2\, z', z_1). \tag{29}$$

Although momentum and angular momentum remain undisturbed, there is λ_0 "rescaling" of the variable z'. We have seen earlier that preon energy and velocity helicity are unchanged by the foregoing transformation. A more general $\boldsymbol{\gamma}_{21} = 0$ left transformation, with arbitrary $\boldsymbol{\gamma}_{12}$ and $\boldsymbol{\gamma}_{22}$, continues to preserve velocity while shifting preon spatial location transversely as well as longitudinally. Here, in the path basis,

$${}^{\gamma}z = z, \tag{30a}$$

$${}^{\gamma}y = y + \boldsymbol{\gamma}_{12}\, \boldsymbol{\gamma}_{22} e^{2s}, \tag{30b}$$

$$^{\gamma}s = s + ln\ \gamma_{22}\,. \qquad (30c)$$

In this case preon energy and velocity helicity are affected if (as finite norm renders unavoidable) the wave function varies with changing y. Extension of the path-basis formulas (30) to the most general left transformation, where the velocity coordinate z also may change, is straightforward. [Formula (30b) is relevant to the elsewhere-discussed transverse Planck-scale 4-stranded ('cable') structure of a path arc.]

Momentum, Angular-Momentum 6-Vector

The self-adjoint-operator set of 6 right generators for Preon i, $\boldsymbol{K}^i{}_R{}^\wedge$ and $\boldsymbol{J}^i{}_R{}^\wedge$, under Milne transformation behaves as a second-rank antisymmetric tensor--a '6-vector'--that we denote by the symbol $J^i{}_{\mu\upsilon}{}^\wedge$. The two preon Casimirs are invariants bilinear in $J^i{}_{\mu\upsilon}{}^\wedge$. Preon momentum (in certain units) is represented by $\boldsymbol{K}^i{}_R{}^\wedge$ and preon angular momentum by $\boldsymbol{J}^i{}_R{}^\wedge$.

Because DQC path action is Milne-transformation invariant, the tensor $J^i{}_{\mu\upsilon}{}^\wedge$ represents 6 'conserved' preon attributes. The sum $J_{\mu\upsilon}{}^\wedge = \Sigma_i\, J^i{}_{\mu\upsilon}{}^\wedge$ of *all* preon 6-vectors is the total momentum and angular momentum of the universe. DQC recognizes Mach's principle as implying universe rotational invariance--a quantum condition of zero total angular momentum. It is well known that, even though the 3 angular-momentum generators do not commute with each other, a quantum state may be a simultaneous eigenvector of angular-momentum components if the eigenvalue is zero for all three.

Because Hilbert-space vectors have finite norm, the quantum-theoretic notion of zero total momentum for the universe is problematic, but not the classical notion. In DQC certain self-adjoint-operator expectations over a cosmological ray play a central role, defining 'mundane reality' and specifying arc energy. We postulate zero expectation for all 3 total-universe-momentum operators—a condition sustained by DQC dynamics.

Energy-Momentum Unirrep Basis; Parity Reflection

Returning attention to an *individual* preon, the set of 4 self-adjoint Hilbert-space operators, $J_{3L}{}^\wedge$, $K_{3L}{}^\wedge$, $J_{3R}{}^\wedge$, $K_{3R}{}^\wedge$ (out of a 12-generator algebra) not only commute with each other but also with the pair of Casimir self adjoint operators—*bilinears* of group-algebra elements that commute with *all* group-algebra elements. The pair of G-N unirrep-basis labels m and ρ together specify Casimir eigenvalues. These two labels attach to *any* unirrep, including the energy-momentum unirrep basis, whose 6 labels we now address. The associated 6-csco supplements the above generator quartet by the Casimir pair.

The symbols m, ρ, n and ω, for four energy-momentum unirrep-basis labels, have already been introduced—specifying, respectively, the eigenvalues of the Casimir pair and of $J_{3L}{}^\wedge$ and $K_{3L}{}^\wedge$. A meaning for n and ω with respect to preon velocity has been achieved in the path basis through left-group action on a velocity eigenvector. Appendix B shows how these two labels are unitarily equivalent to the complex label z' on the G-N unirrep basis. The two remaining labels on the energy-unirrep basis are the eigenvalues of $J_{3R}{}^\wedge$ and $K_{3R}{}^\wedge$, which Appendix B shows are equivalent to the complex label z_1.

The self-adjoint operator $J_{3R}{}^\wedge$ generates rotations about some (arbitrarily-assigned) external 3-direction while $K_{3R}{}^\wedge$ generates spatial displacements in this direction. The eigenvalues of $J_{3R}{}^\wedge$ span the same range as those of $J_{3L}{}^\wedge$ and will be denoted by using the

right integer symbol m_3 to accompany the *left* integer symbol n. (In both cases a factor ½ connects integer symbol to generator eigenvalue.) Similarly the *right* continuous symbol p_3 (spanning the real line and, with a factor ½, designating the eigenvalue of $K_{3R}^{\wedge}$) will accompany the *left* continuous symbol ω. The complete set of energy-unirrep-basis labels is then n, ω, m, ρ, m_3, p_3.

[Our attachment of the subscript 3 to right-generator eigenvalues emphasizes the *arbitrariness* of the designated *external* direction. The *left* labels n and ω refer to the preon's velocity direction. It is because Pauli arbitrarily used the subscript 3 to designate a *diagonal*, rather than antidiagonal, 2×2 hermitian traceless self-inverse matrix that this subscript in Formulas (3) and (6) became associated to the velocity direction. By ignoring Pauli, G-N avoided notational awkwardness.]

One immediate application of the energy-unirrep basis is to define 'preon parity reflection' as sign reversal for n, m and p_3 with ω, ρ and m_3 unchanged. Helicities and momentum direction reverse, while energy, momentum magnitude and angular momentum are unchanged. Implied is velocity-direction reversal. Another immediate application is to the definition of DQC Hilbert-space *sectors*.

Path-Arc-Defined Hilbert-Space Sectors

Any preon shares with a path-contacting arc certain *spacetime-independent* ('extra-Lorentz') discrete labels, mostly ignored by the present paper, that in *any* of the 4 DQC bases define preon *sectors* for photons, gravitons, leptons, quarks and weak bosons of 3 generations, 3 colors, 2 weak isospins, 2 chiralities and 2 values of boson handedness. The latter path label divides a photon or graviton sector into a pair of subsectors. (Although gluons are unrepresented in the DQC Hilbert space, certain path arcs that never contact a ray carry gluonic labels.) The DQC Fock space is constrained so that (path-defined) Sector $ş$ has, in the energy-momentum unirrep basis, a *unique* (twice) helicity $n = m = n_ş$ –i.e., equal components of angular momentum in velocity *and* momentum directions. Sector $ş$ then needs only 4 energy-momentum unirrep-basis labels ω, ρ, m_3 and p_3—one discrete (m_3) and three continuous (ω, ρ, p_3) 'spacetime' labels -- each of the continuous trio spanning some portion of the real line.

The labels m_3 and p_3 determine, respectively, the components of preon angular momentum and momentum in the 3-direction and not only depend on the choice of this direction but are altered by right (Milne) transformations. We have above discussed how a preon's momentum-angular-momentum 6-vector behaves under right transformation. The two other labels, ω and ρ, are *invariant* under the 6-parameter group of (Milne-interpretable) *right* Lorentz transformations--rotations and spatial displacements—as well as under a (sector-changing) parity reversal that changes the signs of $n_ş$ and p_3. We have called (*half* the values of) ω and ρ, respectively, the preon 'energy' and 'momentum magnitude' in τ-dependent units—both these terms to be understood in the preon's local frame.

Positive-Energy Elementary-Particle Preon Wave Functions

Certain special preon wave functions with sharply-defined *positive* energy are interpretable as Standard-Model elementary particles or gravitons. There is qualitative *difference* between a preon wave function that represents a photon or graviton and a

preon wave function that represents any *other* type of elementary particle—which we characterize as 'massive'.

A photon or graviton is a state reachable by right transformation of a single-preon state that in the energy-momentum unirrep basis has $\omega = \rho = p_3$ and $n = m = m_3$ ($= n_ṣ$). An arbitrary (fixed) positive energy specifies a Hilbert subspace in one of four discrete preon sectors--$n_ṣ = \pm 2, \pm 4$--each such sector-subspace representing the right Lorentz group (of Milne transformations). The subspace is isomorphic to normed functions over the complex z_0 plane with the measure dz_0 and an origin corresponding to the foregoing special state. The most general massless state is a superposition of states separately labeled by $n_ṣ$, ω, and z_0, where z_0 designates a direction related to the specified (3) direction by the zenith angle θ_0 and the azimuthal angle φ_0 according to $z_0 = i \tan(\theta_0/2) \exp(i\varphi_0)$.

For a massive elementary particle *at rest* (in local frame) the 'spin-directed' preon wave function with $m_3 = |n_ṣ|$ is parity symmetric—with *zero* expectation for momentum, velocity and helicity and with (non-fluctuating) $\omega > 0$. For a Standard-Model massive particle the expectation of (fluctuating) ρ is at Planck-mass scale--hugely larger than the particle-mass scale of (non-fluctuating) ω. Right rotations generate the other m_3 values without changing the foregoing collection of zero expectations. The fluctuating velocity and momentum directions are *not* parallel to each other although rigidly correlated by the value of particle mass--tending to be almost orthogonal for $\omega << \rho$. (DQC elementary-particle rest mass does not require Higgs scalars.)

Einstein boosts--transformations in Milne local frame from at-rest to moving massive-particle wave functions--have been specified in the ssSM physics approximation to DQC. [6] The status of Einstein-Poincaré boost as an *approximate* symmetry of the universe at macroscopic and particle scales requires further study. Reference (6) finds essential here the dilation invariance of non-gravitational, non-inertial action.

Mundane Reality

The cosmological ray at the lower boundary of some spacetime slice prescribes within that slice, through *expectations* of certain self-adjoint operators, nonfluctuating ('classical') current densities of conserved electric charge and energy-momentum together with associated electromagnetic and gravitational (Lorentz-gauge) 'tethered' potentials that specify the *non-radiation* electromagnetic and gravitational action for the Feynman paths traversing this slice. We refer to these current densities, which enjoy one to one correspondence with tethered potentials (hence the adjective 'tethered'), as 'mundane reality'. (Heisenberg uncertainty is *absent* from DQC—accompanying absence of 'many-worlds' interpretation.) Path-action-contributing vector and tensor (classical) *radiation* potentials will be addressed by another paper.

DQC prescription of the electromagnetic tethered vector potential (in Lorentz gauge) employs real polynomials of the 6-csco whose continuous eigenvalues label the path basis. As in our treatment above of these eigenvalues, it proves notationally economical to represent the continuous-product Preon-*i* 6-csco by a unimodular 2×2 complex matrix—a matrix $a_i\hat{}$ of 8 commuting self-adjoint operators, two of which are determined by the remaining six. (The 12, 21 and 22 matrix elements may be regarded as the 6 independent operators, with the 11 matrix element an operator pair determined by the *complete* operator sextet.)

Consider a pair of self-adjoint hermitian 2×2 matrix operators $\boldsymbol{x}^{\tau}{}_i\hat{}$ and $\boldsymbol{v}_i\hat{}$ which have the real eigenvalues represented by the hermitian 2×2 matrices (2) and (3). That is, the 8 self-adjoint commuting operators represented by the symbols $\boldsymbol{x}^{\tau}{}_i\hat{}$ and $\boldsymbol{v}_i\hat{}$ are real bilinears of the 8 self-adjoint commuting operators represented by the symbol $\boldsymbol{a}_i\hat{}$. The operator bilinear coefficients are the same as those of the eigenvalue bilinears. Similarity transformation of $\boldsymbol{x}^{\tau}{}_i\hat{}$ and $\boldsymbol{v}_i\hat{}$ by our unitary representation of the right Lorentz group allows these operators to be recognized as a pair of commuting Milne 4-vectors whose timelike components are positive. It follows further, if the symbol • denotes the Lorentz inner product of two 4-vectors, that $\boldsymbol{x}^{\tau}{}_i\hat{} \bullet \boldsymbol{x}^{\tau}{}_i\hat{} = \tau^2 I\hat{}$, where $I\hat{}$ is the *unit*-operator (commuting with *all* operators), and $\boldsymbol{v}_i\hat{} \bullet \boldsymbol{v}_i\hat{} = 0$. That is, the 4-vector $\boldsymbol{x}^{\tau}{}_i\hat{}$ is positive timelike, while $\boldsymbol{v}_i\hat{}$ is positive lightlike.

We define a (right-Lorentz) 4-vector Preon-i self-adjoint operator $\mathcal{A}^{\tau}{}_i(\boldsymbol{x})\hat{}$ that depends on the preon's electric charge Q_i and on a spacetime-location 4-vector $\boldsymbol{x}$, with $\boldsymbol{x} \bullet \boldsymbol{x} > \tau^2$, according to the following operator quotient of a 4-vector and a scalar,

$$\mathcal{A}^{\tau}{}_i(\boldsymbol{x})\hat{} \equiv Q_i \, \boldsymbol{v}_i\hat{} \,/\, \boldsymbol{v}_i\hat{} \bullet (\boldsymbol{x}\, I\hat{} - \boldsymbol{x}^{\tau}{}_i\hat{}). \tag{31}$$

For $\boldsymbol{x}$ within the spacetime slice following the exceptional age τ, we call this operator the 'electromagnetic vector potential tethered to Preon i'. The expectation $\mathcal{A}^{\tau}{}_i(\boldsymbol{x})$ of (31) over the cosmological ray at age τ is the contribution from Preon i to the (full, Lorentz-gauge) tethered vector potential $\mathcal{A}^{\tau}(\boldsymbol{x})$—the sum over i of $\mathcal{A}^{\tau}{}_i(\boldsymbol{x})$. Because the preon-velocity 4-vectors $\boldsymbol{v}_i\hat{}$ are lightlike, the 4-divergence vanishes for the spacetime-dependent vector-potential-operator (31), as well as for the τ-ray expectation thereof and for the full (sum over i) classical tethered vector potential.

The classical tethered electromagnetic potential contributes (often importantly) to the action of Feynman paths traversing the near future of the τ-ray. Further, the Dalembertian of (non-fluctuating) $\mathcal{A}^{\tau}(\boldsymbol{x})$ gives the (divergence-less) 4-current density of conserved electric charge within the slice--one component of DQC 'mundane reality'.

The Preon-i-generated tethered gravitational-tensor potential $\mathcal{G}_{\mu\upsilon}{}^{\tau}{}_i(\boldsymbol{x})$ may analogously be specified through a symmetric-tensor self-adjoint operator that parallels (31) but is proportional to the energy rather than to the electric charge of Preon i. We begin by defining the divergence-less zero-Lorentz-trace symmetric-tensor operator

$$V_{\mu\upsilon}{}^{\tau}{}_i(\boldsymbol{x})\hat{} \equiv \; v_{\mu i}\hat{}\, v_{\upsilon i}\hat{} / \, \boldsymbol{v}_i\hat{} \bullet (\boldsymbol{x}\, I\hat{} - \boldsymbol{x}^{\tau}{}_i\hat{}). \tag{32}$$

Thence we define the gravitational-tensor operator

$$\mathcal{G}_{\mu\upsilon}{}^{\tau}{}_i(\boldsymbol{x})\hat{} \equiv \tfrac{1}{4} G \, [E^{\tau}{}_i\hat{}\, V_{\mu\upsilon}{}^{\tau}{}_i(\boldsymbol{x})\hat{} + V_{\mu\upsilon}{}^{\tau}{}_i(\boldsymbol{x})\hat{}\, E^{\tau}{}_i\hat{}], \tag{33}$$

where $E^{\tau}{}_i\hat{}$ is the (right-Lorentz-scalar) Preon-i energy operator (Appendix C), which does not commute with $V_{\mu\upsilon}{}^{\tau}{}_i(\boldsymbol{x})\hat{}$, and G is the gravitational constant. (A factor ½ in Formula (33) arises from Newton's nonrelativistic definition of the gravitational constant.) Summing (33) over i (over all preons) and taking the τ–ray expectation gives the full classical tethered gravitational potential $\mathcal{G}_{\mu\upsilon}{}^{\tau}(\boldsymbol{x})$ within the near future of this ray. The Dalembertian of $\mathcal{G}_{\mu\upsilon}{}^{\tau}(\boldsymbol{x})$ divided by the factor ½G is the current density of conserved energy-momentum within the slice.

Notice that the photons and gravitons of a cosmological ray contribute to the energy-momentum tensor although not contributing to the electric-charge current density. The former (tensor) current density and the latter (vector) current density, both conserved, collectively comprise mundane reality.

Conclusion

Explored has been a Hilbert space for preons--elementary quantum-theoretic entities in Dirac sense that populate Milne spacetime. Our proposal, based on the complex Lorentz group rather than the Poincaré group, is suitable for quantum cosmology although not immediately for physics. Our Hilbert space is defined only at macroscopically-spaced exceptional universe ages. The results here support the gravitational-action proposals of Reference (2) and suggest a sliced-spacetime Higgs-less revision of the particle-physics Standard Model—the revised model being related to the present paper's cosmological theory by group contraction, as reported in Reference (6).

Appendix A. A Pair of Real 3-Vector Path-Basis Labels

The most general (6 real-parameter) coordinate matrix of Preon *i* or Arc *i* may be expressed, rather than by Formula (6) of the main text, through a 3-vector boost $\boldsymbol{\beta}_i$ together with a 3-vector rotation $\boldsymbol{\chi}_i$ ($0 \leq |\boldsymbol{\chi}_i| \leq 4\pi$) such that the coordinate matrix is

$$\boldsymbol{a}_i = exp(i\boldsymbol{\sigma} \bullet \boldsymbol{\chi}_i /2) \times exp(-\boldsymbol{\sigma} \bullet \boldsymbol{\beta}_i /2). \qquad (A.1)$$

The right factor in (A.1) is a *hermitian* unimodular matrix while the left factor is a *unitary* unimodular matrix. (*Any* square matrix may be written as the product of a hermitian matrix and a unitary matrix.) The choice of matrix order in (A.1) correlates with the definition (2) in the main text of the preon's spacetime-location 4-vector. The 3-vector parameter $\boldsymbol{\beta}_i$ in (A.1) is the location in Milne boost space of Preon *i* or Arc *i*. When the 6-space origin is chosen to coincide in boost space with this location--i.e., when $\boldsymbol{\beta}_i = 0$, the 3-vector $\boldsymbol{\chi}_i$ appearing in (A.1), or an equivalent set of 3 angles,

$$0 \leq \theta_i \leq \pi\,, \;\; 0 \leq \varphi_i \leq 2\pi, \;\; 0 \leq \varphi_i' \leq 4\pi, \qquad (A.2)$$

specifies Preon-velocity direction by the polar coordinates θ_i, φ_i and transverse polarization direction by the angle $-\frac{1}{2}(\varphi_i' + \varphi_i)$. The relation between $\boldsymbol{\chi}_i$ and θ_i, φ_i, φ_i' is

$$exp(i\boldsymbol{\sigma} \bullet \boldsymbol{\chi}_i /2) = exp(i\sigma_3\varphi_i'/2)\, exp(i\sigma_1\theta_i/2)\, exp(i\sigma_3\varphi_i/2)\,. \qquad (A.3)$$

The invariant 6-dimensional volume element is $d\boldsymbol{a}_i = d\boldsymbol{\chi}_i\, d\boldsymbol{\beta}_i$ with

$$d\boldsymbol{\chi}_i = sin\ \theta_i\, d\theta_i\, d\varphi_i\, d\varphi_i', \qquad d\boldsymbol{\beta}_i = sinh^2\beta_i\, d\beta_i\, d\boldsymbol{n}_i, \qquad (A.4)$$

the symbol $d\boldsymbol{n}$ standing for (2-dimensional) solid-angle element.

The parameterization (A.1) is convenient when relating 3 *different* arcs to each other at a Feynman-path vertex. The different arcs at a vertex share the same $\boldsymbol{\beta}$ but have independent $\boldsymbol{\chi}$. The G-N complex coordinates are convenient for arcs *between* (rather than *at*) vertices as well as for wave functions contacted by arcs.

Appendix B. Eigenvectors of the Energy-Momentum Csco.

Formula (25), specifying the outcome of right Lorentz transformation in the G-N unirrep basis, reveals in this basis the eigenfunctions of the two commuting right Lorentz-group generators J_{3R}^ and K_{3R}^. If $\Gamma_{12} = \Gamma_{21} = 0$ so that $\Gamma_{11} = \Gamma_{22}^{-1}$—the right (Lorentz) transformation here being a boost in (some arbitrarily chosen) 3 direction combined with a rotation about this direction--we find

$$\Phi^{\Gamma}(\boldsymbol{b}) = |\Gamma_{22}|^2 \exp(im \arg \Gamma_{22} + i\rho \ln|\Gamma_{22}|)\, \Phi(m, \rho, z', \Gamma_{22}^2 z_1). \qquad \text{(B.1)}$$

Formula (B.1) determines both the eigenvalues and the eigenfunctions of the operators that generate right rotations and boosts with respect to the 3 direction. Writing $\Gamma_{22} = e^{\varepsilon}$ and $\Phi^{\Gamma}(\boldsymbol{b}) = \Phi^{\varepsilon}{}_{m,\rho,z'}(z_1)$, Formula (B.1) becomes

$$\Phi^{\varepsilon}{}_{m,\rho,z'}(z_1) = e^{2\,Re\,\varepsilon + im\,Im\,\varepsilon + i\rho\,Re\,\varepsilon}\, \Phi_{m,\rho,z'}(e^{2\varepsilon} z_1). \qquad \text{(B.2)}$$

A function of z_1 proportional to $|z_1|^{-1} exp\,(i\ell_1 arg\, z_1 + i\,\rho_1 ln|\,z_1|)$ is transformed by (B.2) into itself multiplied by $exp[i(m+2\ell_1)Im\,\varepsilon + i(\rho+2\rho_1)Re\,\varepsilon]$. Such a function (not normalizable but admissable in the sense of 'rigged Hilbert space')--an eigenvector of both J_{3R}^ and K_{3R}^—is a state where components in the 3 direction of both angular momentum and of momentum are sharply defined. The eigenvalues of J_{3R}^ are seen to be $m/2+\ell_1 \equiv m_3/2$ while those of K_{3R}^ are $\rho/2+\rho_1 \equiv p_3/2$.

A function of z_1 that vanishes except in the neighborhood of $z_1 = 0$ may, according to (B.2), loosely be described as a momentum-*direction* eigenvector--with preon momentum direction in the externally-prescribed 3-direction. In this case Formula (B.2), with $Re\,\varepsilon = 0$, supports our calling $m/2$ 'helicity'—component of angular momentum in the direction of momentum. But for this wave function *no* individual component of momentum is sharply defined.

The curvature of Milne 3-space renders non-commuting the trio of self-adjoint 'right'-boost generators that represent the 3 components of a preon's momentum. It is nevertheless reasonable to call $\rho/2$ the 'magnitude of preon 3-momentum' in a sense similar to the language which in nonrelativistic quantum theory calls the label j the 'magnitude of angular momentum'. The relation between self-adjoint operators,

$$\boldsymbol{K}_R\text{^}\bullet\boldsymbol{J}_R\text{^} = (\rho\text{^}/2)(m\text{^}/2), \qquad \text{(B.3)}$$

is here helpful to keep in mind, together with the helicity interpretability of $m/2$.

Although the most general internal single-preon transformation of a ray from the *left*, $\Psi(\boldsymbol{a}_1, \boldsymbol{a}_2 \ldots \boldsymbol{a}_i \ldots) \rightarrow \Psi(\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots \boldsymbol{\gamma}_i \boldsymbol{a}_i \ldots)$, is less transparent than (24) when expressed in s_i, y_i, z_i coordinates, the transformation induced in the G-N unirrep basis is as simple as (25). The difference from (25) is that left transformation alters wave-function dependence on the "internal" parameter z'_i while dependence on preon helicity and momentum magnitude and direction (m_i, ρ_i, z_{1i}) is unchanged apart from an overall factor. Dropping the subscript i one finds, matching Formula (25),

$$\Phi(\boldsymbol{b}) \rightarrow {}^{\gamma}\Phi(\boldsymbol{b}) = \alpha_{m\rho}(-\boldsymbol{\gamma}_{12}\, z' + \boldsymbol{\gamma}_{11})\, \Phi(m, \rho, {}^{\gamma}z', z_1), \qquad \text{(B.4)}$$

where

$$^{\gamma}z' = (\gamma_{22}z' - \gamma_{21})(-\gamma_{12}z' + \gamma_{11})^{-1}. \qquad \text{(B.5)}$$

Neither the magnitude nor the direction of momentum changes nor does helicity, as the coordinates (path-basis labels) of some preon shift from one set of values to another (the coordinates of *other* preons remaining unshifted). Change in Preon i is manifested by change in the z'_i dependence of its G-N unirrep-basis wave function. Accompanying the earlier-considered special internal shift $\boldsymbol{\gamma} = \boldsymbol{\lambda}_0$, where $\boldsymbol{\lambda}_0$ is the unimodular matrix $exp(-\sigma_3 ln\lambda_0)$ that prescribes an orientation rotation around velocity direction together with a longitudinal shift of preon location along velocity direction ($\gamma_{12} = \gamma_{21} = 0$ and $\gamma_{22} = \gamma_{11}^{-1} = \lambda_0$), one finds from (B.4),

$$^{\gamma}\Phi\,(m, \rho, z', z_1) = \alpha_{m\rho}^{-1}(\lambda_0)\;\Phi\,(m, \rho, \lambda_0^2 z', z_1). \qquad \text{(B.6)}$$

Here, although momentum and helicity (together with wave-function norm) remain undisturbed, there is λ_0 "rescaling" of the variable z'.

Because a function $\Phi(m, \rho, z', z_1)$ with z' dependence of the same form as that discussed for z_1 , immediately following (B.2) above, is an eigenvector of $J_{3L}\hat{}$ and $K_{3L}\hat{}$, it is possible simultaneously to diagonalize the six self-adjoint operators whose eigenvalues are determined by the labels n, ω, m, ρ, m_3, p_3--the main-text labels of the energy-momentum unirrep basis.

Appendix C. Starting-Arc Energy at a Spacetime-Slice Lower Boundary

In either the intermediate basis or the energy-momentum unirrep basis the Preon i positive-energy operator at some exceptional age τ is

$$E^{\tau}{}_i\hat{} = \delta_+(\omega_i\hat{})\;\hbar\omega_i\hat{}\,/2\tau, \qquad \text{(C.1)}$$

where $\delta_+(\omega\hat{})$ is the unit operator for positive $\omega\hat{}$ and zero otherwise. Note the distinction between the positive energy operator and the frequency operator $\omega_i\hat{}$ whose spectrum covers the real line. With respect to those special (main text) single-preon wave functions which represent macroscopically-stationary elementary particles, this distinction is immaterial. However the completeness of a csco that includes $\omega\hat{}$ would be lost were $\omega\hat{}$ replaced by the positive-energy operator.

An intermediate-basis '*partial expectation*' of the positive-energy operator $E^{\tau}{}_i\hat{}$ we define by the quotient

$$E^{\tau}{}_i\,(Im\; s_i, y_i, z_i) = (\hbar/2\tau)\textstyle\int_+ d\omega\;\left|\,\psi_\omega\,(Im\; s_i, y_i, z_i)\right|^2\omega\;/\;\int_+ d\omega\;\left|\,\psi_\omega\,(Im\; s_i, y_i, z_i)\right|^2, \qquad \text{(C.2)}$$

where $\psi_\omega\,(Im\; s_i, y_i, z_i)$ is the Fourier transform of $\psi\,(s_i, y_i, z_i)$ with respect to $Re\; s_i$. The symbol $\int_+ d\omega$ denotes an integral over the *positive* real line. Formula (C.2) assigns a positive energy to an arc i starting at the lower boundary of a spacetime slice, an arc which there contacts Preon i. This arc is characterized by fixed values of $Im\; s_i, y_i, z_i$ but not of $Re\; s_i$. Implicit in (C.2) is a dependence of Arc-i energy on all $\boldsymbol{a}_j$ with $j \neq i$.

Along any arc its (positive) energy decreases in inverse proportion to age as age increases (Milne redshift), but measurable redshift--over age intervals much longer than the macroscopic width of a spacetime slice--depends on Feynman-path dynamics. At each (cubic) path vertex the sum of the (1 or 2) ingoing ω values equals the sum of the (2 or 1) outgoing values, so total path energy decreases in inverse proportion to the age increase between path start and finish. But Feynman's formula for the ray at the upper slice boundary in terms of the ray at the lower boundary does not imply a transfer of finishing path energy to the upper-boundary ray.

Because the positive-energy-operator partial-expectation over a ray, and other path-action-influencing self-adjoint-operator expectations, depend *nonlinearly* on this ray, DQC ray propagation over more than one slice fails to be linear. Linearity lacks macroscopic significance in DQC where at each exceptional age there is exactly one universe ray and within each spacetime slice exactly one mundane reality. Nevertheless the special character of *photon* and *graviton* wave functions allow *approximately* linear propagation of a ray's *radiation* component over macroscopic spacetime regions that are tiny on Hubble scale.

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