April 25, 2008
DRAFT

Cosmological Hilbert Space

Geoffrey F. Chew

Theoretical Physics Group
Physics Division
Lawrence Berkeley National Laboratory
Berkeley, California 94720, U.S.A.

Abstract

A Fock space of 'cosmological preons'—quantum-theoretic universe constituents—associates to a 'Milne spacetime' based on the Lorentz group and to Gelfand-Naimark unitary group representations. Lorentz invariance of Milne-universe 'age' accommodates 'total relativity'. Two 'extra' dimensions of the 6-dimensional space occupied by preons at certain exceptional ages of the universe define self-adjoint single-preon operators that include a canonically-conjugate pair representing preon energy and *local time*. Global spacetime divides into 'slices' of fixed macroscopic width in age, with 'cosmological rays' defined on slice *boundaries*. Self-adjoint-operator expectations at such a boundary prescribe throughout the subsequent slice a non-fluctuating 'mundane reality'—current densities of conserved electric charge and energy-momentum together with electromagnetic and gravitational potentials. The ray at the lower boundary of a slice is propagated to the upper boundary by cosmological *branched* Feynman paths across the slice that carry (divergence-free) potential-depending action. A macroscopically-stable positive-energy single-preon wave function identifies either with a Standard-Model elementary particle or with a graviton. (Unstable negative-energy preon wave functions remain to be interpreted.) Special relativity--Poincaré invariance--although inexact, is accurate for low-density regions of the universe at spacetime scales which are far below that of Hubble and far above that of Planck.

Introduction

Nonexistence of unitary finite-dimensional Lorentz-group representations has long been supposed to preclude, for dynamically-changing numbers of particles, a Dirac-type quantum theory that represents individual-particle properties such as location, momentum and spin by self-adjoint operators on a rigged Hilbert space. [(1)] The Standard Model, which employs as foundation not particles but quantum fields associated to finite-dimensional Lorentz-group representations and which represents action by ill-defined *local-field-product* operators with perturbative renormalization procedures to manage consequent divergences, has failed to accommodate gravity. Problematic, furthermore, is the Standard-Model description of bound states ('condensed matter'), perturbation theory being unsuited to macroscopically-stationary composite wave functions.

The discrete quantum cosmology (DQC) of Reference (2) and the present paper introduces a 'cosmological-preon' Fock space via *unitary* (infinite-dimensional) representations of the *complex* Lorentz-group. Any cosmological preon (henceforth throughout this paper simply called 'preon') is 'lightlike'—in the sense of having velocity c and a polarization transverse to velocity direction—and carries momentum, angular momentum and energy. However, only very-special preon wave functions exhibit, through expectations of self-adjoint operators, the relation between energy, momentum, spin and spacetime location that characterizes 'ordinary matter'. A DQC Fock-space ray, representing the entire universe, comprises sums of products of preon wave functions whose discrete quantum numbers allow *certain* macroscopically-stable positive-energy preon states to be interpreted as lepton, quark, weak boson, photon or graviton. (A cosmological meaning is given below for the adjective 'macroscopic'.) Reference (2) specifies 'creation-annihilation' Feynman paths whose action includes gravity as well as electromagnetism and weak-strong interaction.

The DQC Fock space is built from Pauli-symmetrized superpositions of products of invariantly-normed single-preon functions. Displayed in this paper is a single-preon basis that is '6-labeled' by preon energy, 'momentum magnitude', direction of momentum (2 angles) and a *pair* of invariant helicities. One of the latter—here called 'velocity helicity'--is angular momentum with respect to velocity direction while the other is the (usual) angular momentum in the direction of momentum. Preon energy is the component of its momentum in the direction of its velocity. The term 'momentum magnitude' has a continuous significance, through Lorentz-group Casimirs, that parallels the significance in nonrelativistic quantum theory of a particle's discrete 'angular-momentum magnitude'.

Commutability of the complete set of 6 corresponding self-adjoint operators derives from commutability of right Lorentz transformations with left transformations. Our usage of the adjectives 'right' and 'left' will be explained. DQC Hilbert space unitarily represents a *12-parameter* group—the product of right and left Lorentz groups. Action is *right*-Lorentz invariant, DQC right transformations being those employed by Milne to *define* a spacetime [(3)] and which we call 'Milne transformations' to avoid confusion with the Einstein-Poincaré meaning for a Lorentz transformation. The 6 self-adjoint-operator generators of Milne transformations, which do *not* commute with each other, represent preon momentum and angular momentum. DQC path action conserves momentum and angular momentum but not energy—which associates to one of the *left*

Lorentz generators. Velocity helicity associates to another left generator, one which commutes with preon energy as well as with momentum and angular momentum.

To the best of our knowledge DQC *left* transformations have no 6-parameter-group precedent in natural philosophy but a 1-parameter left subgroup associates to *local-time* translation (*not* translation of global time--which we call 'age'). The generator of this left subgroup represents preon (total) energy. Another U(1) left subgroup, generated by velocity helicity, comprises rotations about the velocity direction. In the DQC algebra of self-adjoint operators (which, because DQC accords no *a priori* meaning to 'measurement', we avoid calling 'observables') the two left Lorentz-group Casimirs are *equal* to the two right Casimirs (commuting with all 12 group generators).

Reference (2) addresses the DQC action of branched Feynman paths. The present paper is complementary--ignoring path action while addressing various 6-labeled bases for single-preon Hilbert space. Each basis corresponds to a complete set of 6 commuting self-adjoint operators (a 6-csco). Unitary Hilbert-space *regular* representation of the product of right and left Lorentz groups provides a DQC path-contactable basis that parallels the Feynman-path-contacting *coordinate* basis for Dirac's nonrelativistic quantum theory. [(1)] An analog of Dirac's *momentum* basis associates to the unitary *irreducible* SL(2,c) Gelfand-Naimark (G-N) representation ('unirrep'). [(4)]

The algebra of preon self adjoint operators represents in Dirac sense preon spatial location, velocity, polarization, energy, momentum and angular momentum, as well as velocity-helicity and momentum-helicity. Of course not all these operators commute with each other. Each of the DQC bases discussed here associates to a different 6-csco.

In the 'path basis' each preon is 'classically specified' by (a product of) the 6 continuous coordinates of a manifold traversed by Feynman paths comprising straight lightlike 'arcs' which may be created or annihilated as a path progresses. (The local time along any arc has *unit* derivative with respect to global age.) A path propagates some 'ray'--a fixed-age cosmological wave function whose norm lacks significance through probability or otherwise--to the subsequent ray. Each preon of a 'starting' ray contacts exactly one starting arc of a Feynman path. Before reaching the age of the subsequent ray, any path arc may be annihilated at a cubic vertex—an 'event'—where new arcs are created.

One of two 'extra' velocity-associated manifold dimensions, by defining a self-adjoint operator associated to individual-preon local time, allows reality in the 'near-future' of a ray to be prescribed by self-adjoint-operator *expectations* over that ray. The meaning of 'near future' attaches to a DQC 'macroscopic slicing' of Milne spacetime that will be discussed below. Cosmological rays are defined only on slice boundaries.

DQC Fock space comprises sums of Pauli-symmetrized products of normed single-preon functions. At the risk of obscuring total relativity, the present paper chooses to emphasize the unfamiliar labels on which depends a function belonging to an *individual* preon. Special such functions that represent elementary particles (quarks, leptons and weak bosons, together with photons and gravitons) will be exposed.

What we call 'total relativity' recognizes time arrow and absoluteness of motion while respecting Mach's principle in the sense discussed by Wilczek. [(5)] Einstein-Poincaré special relativity ignores time arrow and motion absoluteness; special relativity further disregards Mach. Application of DQC to *physics* requires scale-based approximation; DQC addresses an expanding universe that lacks meaning for

reproducible measurement. The general meaning of 'physics' and of special relativity in particular is confined to spacetime scales tiny compared to that of Hubble while huge compared to that of Planck. Reference (6) presents a Euclidean-group-based physics-scale gravity-less approximation to DQC that may be described as a (Higgsless) 'sliced-spacetime Standard Model' (ssSM).

The DQC (global) *age*, invariant under both right and left transformations, plays a discrete role paralleling that of continuous time in nonrelativistic quantum theory. DQC Feynman paths connect successive macroscopically-spaced *exceptional* ages at each of which is defined a cosmological Fock-space ray—in a sense recalling S-matrix theory. DQC spacetime divides into 'slices' of macroscopic width whose boundaries locate at the exceptional ages. Path branching—path-arc creation or annihilation--is forbidden at slice boundaries where a ray is defined—occurring at ages *interior* to a slice where rays are not defined. DQC dynamics prescribes quantum propagation in the discrete S-matrix sense of an "*in* state" leading to a subsequent "*out* state" without any wave function being defined between *in* and *out*.

Although not discussed in the present paper, the aggregation of DQC Hilbert space, path rules and initial condition "spontaneously" breaks *C*, *P* and *CP* symmetries. Because the DQC Hilbert space represents a group isomorphic to the *complex* Lorentz group (as does analytic S-matrix theory), it is plausible that some cosmological counterpart to 'CPT symmetry' will eventually become recognized.

The here-examined infinite-dimensional single-preon Hilbert space comprises normed functions of the continuous coordinates (path-basis labels) that 'locate' an individual preon within a 6-dimensional manifold which is at once a right *and* a left group manifold (*common* Haar measure). The single-preon Hilbert space has as a factor a *finite*-dimensional Hilbert subspace of *discrete* labels, invariant under both right and left continuous transformations, labels that are largely ignored by the present paper. Discrete labels carried by both path and ray distinguish different preon 'sectors' (e.g., electron, up-quark, photon, graviton) by specifying electric charge, color, generation, etc. [7]

We *shall* here attend to an invariant 2-valued parity-related "handedness" carried both by path arcs (between path-branching points) and by preons--in contrast to preon helicities that are meaningless for a path arc. DQC Hilbert space correlates handedness to sign of helicity in assigning to each preon sector a unique velocity helicity that coincides with momentum helicity.

G-N discussed two different bases for a Hilbert space that represents unitarily the group SL(2,c), [4] without attempting for either a natural-philosophical interpretation. The vectors of one basis—analog to the coordinate basis of nonrelativistic Dirac theory--are normed functions over the 6-dimensional (continuous, left-right) manifold. We call this the path basis because DQC Feynman paths traverse this 6-space. G-N's second basis--that we here call 'G-N unirrep'--parallels the Dirac-Fourier-Wigner momentum-spin basis of nonrelativistic quantum theory that unitarily and irreducibly represents the Euclidean group. [8] (The 6-parameter compound Euclidean group is a contraction of the 6-parameter semisimple Lorentz group.) Although the transformation connecting the two G-N bases is not entirely of Fourier-Wigner form, wave-function norm is preserved; the transformation is unitary.

We modify the G-N unirrep basis by (unitary) Fourier transformations of wave-function dependence on a pair of complex *directional* labels, so as to diagonalize

simultaneously preon energy—the component of its momentum in its velocity direction--and components of momentum and angular momentum in some arbitrarily-specified direction. A continuous Casimir label, carried over undisturbed from the G-N unirrep basis, we call 'magnitude of momentum'. Two discrete labels are helicity interpretable—components of angular momentum in velocity and momentum directions. The altered basis, which facilitates meaning for 'preon parity reflection', we call the 'energy-unirrep' basis.

The universe spacetime identified by Milne in the nineteen thirties [3]—an open forward-lightcone interior whose boundary allows 'big-bang' interpretation— endows 'Lorentz transformation' with a cosmological time-arrowed meaning different from the Einstein-Poincaré special-relativistic physics meaning (ignoring time arrow) that augments Lorentz invariance by spacetime-displacement invariance. (A sufficiently large spacelike or negative-timelike displacement may move a point within Milne spacetime outside that spacetime—i.e., outside the universe.) In contrast to an Einstein boost between different 'rest frames' that each assigns a different set of velocities to massive entities within some 'laboratory' spacetime-localized region, Milne boosts relate to each other different 'local frames' that each associates to a different spatial location. Milne boosts--*right* DQC transformations--are spatial displacements at fixed universe age in a curved (hyperbolic) 3-space.

Although Milne-spacetime flatness--manifested in DQC by straight lightlike arcs within Feynman paths--might seem incompatible with general relativity's association of gravity to spacetime curvature, DQC gravitational action at a distance plus creation and annihilation of *soft-gravitonic* arcs enables *discretized curvature* via 'gentle' branchings of stationary-action classical paths. [2] Any DQC path, as prescribed in Reference (2), is an 'event graph'—a set of spacetime-located cubic vertices connected by arcs of positive-lightlike 4-velocity that carry energy as well as discrete attributes. By disregarding Planck's constant, general relativity ignores gravitons and approximates by a spacetime-curving trajectory (e.g., an electron trajectory) an arc-sector-maintaining stationary-action *sequence* of DQC straight 'hard' arcs that are separated at gentle events by 'soft' gravitonic-arc absorption or emission.

The adjectives 'hard' and 'soft' refer to the energy scale set by Planck's constant times the *inverse* of the age width of a spacetime slice—the time interval that *defines* cosmologically the adjective 'macroscopic'. Both the Standard Model and the Reference (6) ssSM revision thereof attend to the Planck constant, while making a $G \rightarrow 0$ approximation and achieving flat 3-space through disregard of the Hubble constant--suppressing redshift by regarding universe age as infinite. The ssSM physics approximation to DQC—differs from the Standard Model by its recognition of macroscopic spacetime slicing. The revision accommodates soft photons and thereby maintains capacity (even while ignoring gravity and redshift) to provide an *electromagnetic* theory of physical measurement.

Milne Spacetime

The open interior of a forward lightcone—what we call 'Milne spacetime'--is the product of a lower-bounded one-dimensional 'age space' with an unbounded 3-dimensional 'boost space'. The spacetime displacement from the forward-lightcone vertex (whose spacetime location is meaningless) to any spacetime point is a positive-

timelike 4-vector $(t, \boldsymbol{x})$. Defining the "age" τ of a spacetime point to be its Minkowski distance from lightcone vertex—i.e., the Lorentz-invariant modulus $(t^2 - \boldsymbol{x}^2c^{-2})^{½}$ of its spacetime-location 4-vector—the set of points sharing some common age occupies a 3-dimensional (global) hyperboloid. Any point within such a hyperboloid may be reached from any other by a 3-vector *boost*. Once an origin within boost space is designated, an arbitrary spacetime point is specified by $(\tau, \boldsymbol{\beta})$, where $\boldsymbol{\beta}$ is the 3-vector boost-space displacement from the selected origin to the point. Writing $\boldsymbol{\beta} = \beta\boldsymbol{n}$, where $\boldsymbol{n}$ is a unit 3-vector and β is positive,

$$t = \tau \cosh \beta, \quad \boldsymbol{x} = c\tau\, \boldsymbol{n} \sinh \beta. \tag{1}$$

The spatial-location label $\boldsymbol{\beta}$ will in the following section and in Appendix A be identified within the path basis for DQC Hilbert space. Compatibility of age discretization with Milne's meaning for Lorentz invariance allows DQC's spacetime to be temporally discrete for its Feynman-path quantum dynamics even though spatially continuous. Discretization occurs at two *different* fundamental scales: (1) Any DQC Feynman path traverses an age-discretized 'macroscopic slice' of Milne spacetime—a slice bounded above in age as well as below. (2) Within each slice any (straight and lightlike) arc proceeds in Planck-scale age steps, whose precise value is established in Reference (2) from action quantization.

Although consistency requires slice width to be an integral multiple of arc step, the huge-integer ratio will not be addressed by the present paper—which ignores arc steps. Before attending at all to path arcs this paper chooses to address the single-preon Hilbert-space path basis. Nevertheless the termination of path arcs at exceptional (ray-age) hyperboloids might be taken as *defining* the path basis of preon Fock space.

To each point of boost space associates a "local" Lorentz frame in which $\boldsymbol{\beta} = 0$—i.e., a frame defined up to a rotation by the point's location 4-vector being purely timelike in that frame. Age change and time change are equal in local frame. In local frame an infinitesimal spatial displacement $d\boldsymbol{x}$ at age τ relates to an infinitesimal boost-space displacement $d\boldsymbol{\beta}$ by $d\boldsymbol{x} = c\tau d\boldsymbol{\beta}$. (A phenomenological meaning for 'local frame' resides in the approximate isotropy of cosmic background radiation observed in that frame. This meaning parallels that of standard cosmology's 'co-moving coordinates'.) The rotational ambiguity of local-frame meaning is reduced below through an origin of a 6-dimensional space that (arbitrarily) designates not only the origin's boost-space location but also an attached orthogonal and handed (1, 2, 3) set of 3 reference axes which may be parallel transported along a boost-space geodesic from the origin to any other boost-space location.

The (3-parameter) global orientation ambiguity is accommodated by total relativity—a DQC Fock-space restriction that requires rays to be *globally* rotationally invariant—unchanged when a common Milne rotation is applied to *all* preons. Ray *expectations* of Milne-*boost* generators are also globally invariant. Total relativity might be said to mean that both the total momentum and the total angular momentum of the universe are zero (6 conditions). The DQC universe is not only rotationally invariant but, associating right-boost generators with infinitesimal spatial displacements at fixed age, the universe is also 'classically-invariant' under (non-abelian) boost-space displacements.