

1 Useful Formulae, All in One Place

Fourier transformation in 3 dimensions:

$$\phi(\vec{x}) = \int \frac{d^3\vec{p}}{(2\pi)^3} e^{+i\vec{p}\cdot\vec{x}} \phi(\vec{p}), \quad (1)$$

$$\phi(\vec{p}) = \int d^3\vec{x} e^{-i\vec{p}\cdot\vec{x}} \phi(\vec{x}). \quad (2)$$

Performing integrals over space:

$$\int d^3\vec{x} e^{i(\vec{p}+\vec{q})\cdot\vec{x}} = (2\pi)^3 \delta^3(\vec{p}+\vec{q}), \quad (3)$$

which forces $\vec{q} = -\vec{p}$. Similarly,

$$\int \frac{d^3\vec{p}}{(2\pi)^3} e^{-i(\vec{x}+\vec{y})\cdot\vec{p}} = \delta^3(\vec{x} + \vec{y}), \quad (4)$$

forcing $\vec{x} = -\vec{y}$.

Fourier transformation in 4 dimensions: recall that

$$x \cdot p \equiv \eta_{\mu\nu} x^\mu p^\nu = x^0 p^0 - \vec{x} \cdot \vec{p}, \quad (5)$$

so the signs in the exponents are switched:

$$\phi(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} \phi(p), \quad (6)$$

$$\phi(p) = \int d^4x e^{ip\cdot x} \phi(x). \quad (7)$$

Lorentz invariant phase space for a particle of mass m :

$$\int \frac{d^4p}{(2\pi)^4} 2\pi \delta(p^2 - m^2) \theta(p^0) = \int \frac{d^3\vec{p}}{(2\pi)^3 2E_{\vec{p}}}, \quad E_{\vec{p}} \equiv \sqrt{\vec{p}^2 + m^2}. \quad (8)$$

Here $E_{\vec{p}}$ is also written $\omega_{\vec{p}}$ or p^0 .

Free field theory:

$$S = \int d^4x \mathcal{L}[\phi(x), \partial_\mu \phi(x)], \quad (9)$$

$$\mathcal{L}_{\text{Klein-Gordon}} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2, \quad (10)$$

$$\pi^\mu \equiv \frac{\delta \mathcal{L}[\phi, \partial_\mu \phi]}{\delta \partial_\mu \phi} \rightarrow_{\text{K-G}} \partial^\mu \phi, \quad (11)$$

$$\text{Euler-Lagrange : } \frac{\delta \mathcal{L}[\phi, \partial_\mu \phi]}{\delta \phi} = \partial_\mu \pi^\mu \rightarrow_{\text{K-G}} \partial_\mu \partial^\mu \phi = -m^2 \phi. \quad (12)$$

The field can be expanded in creation and annihilation operators:

$$\phi(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 2E_{\vec{p}}} \left[a_{\vec{p}} e^{-ip \cdot x} + a_{\vec{p}}^\dagger e^{+ip \cdot x} \right], \quad (13)$$

$$\left[a_{\vec{p}}, a_{\vec{q}}^\dagger \right] = 2E_{\vec{p}} (2\pi)^3 \delta^3(\vec{p} - \vec{q}). \quad (14)$$

Note that x and the phases here are 4-vector and 4-vector product, with $p^0 \equiv E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$.

Two-point correlation functions (the book uses D rather than G):

$$G^>(x-y) \equiv \langle 0 | \phi(x) \phi(y) | 0 \rangle, \quad (15)$$

$$G^<(x-y) \equiv \langle 0 | \phi(y) \phi(x) | 0 \rangle, \quad (16)$$

$$\begin{aligned} G_R(x-y) &\equiv \langle 0 | [\phi(x), \phi(y)] | 0 \rangle \Theta(x^0 - y^0) \\ &= (G^>(x-y) - G^<(x-y)) \Theta(x^0 - y^0), \end{aligned} \quad (17)$$

$$\begin{aligned} G_T(x-y) &\equiv \langle 0 | \phi(x) \phi(y) | 0 \rangle \Theta(x^0 - y^0) + \langle 0 | \phi(y) \phi(x) | 0 \rangle \Theta(y^0 - x^0) \\ &= G^>(x-y) \Theta(x^0 - y^0) + G^<(x-y) \Theta(y^0 - x^0) \\ &= \langle 0 | \mathbf{T}(\phi(x) \phi(y)) | 0 \rangle. \end{aligned} \quad (18)$$

Free theory values:

$$G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} G(p) \quad (19)$$

with

$$G^>(p) = 2\pi \delta(p^2 - m^2) \Theta(p^0) \quad (20)$$

$$G^<(p) = 2\pi \delta(p^2 - m^2) \Theta(-p^0) \quad (21)$$

$$G_R(p) = \frac{i}{p^2 - m^2 + i\epsilon \operatorname{sign}(p^0)} \quad (22)$$

$$G_T(p) = \frac{i}{p^2 - m^2 + i\epsilon}. \quad (23)$$

G_T is also written G_F , D_F , Δ_F etc and is called the time ordered correlator or the Feynman correlator.