**Lai Chapter 6 Meta Notes: Newtonian Viscous Fluids** PhL 6.28.12

This is Lai's offering on the subject of fluids. Here is a little hierarchy

fluid mechanics
  fluid statics
  fluid dynamics (before 1900 this whole field was called hydrodynamics, and this name lingers)
    aerodynamics (air and other gases in motion)
    hydrodynamics (water and other liquids in motion)

Wiki gives a list of 172 "popular" books on the subject of fluid mechanics!!! Amazing. I have attached that list at the end of this meta notes.

Chapter is 57 pages, raw notes are 34 pages, these meta notes are 9 pages.


**6.1 Fluids. (353) ..... 2**
**6.2. Incompressible subclass of fluids (354). ..... 2**
**6.3 Hydrostatics (355). ..... 2**
**6.4 Newtonian Fluids (357). ..... 2**
**6.5 Interpretation of λ and μ (358). ..... 2**
**6.6 Incompressible Newtonian Flow (359). ..... 3**
**6.7 Navier-Stokes for incompressible fluids (360). ..... 3**
**6.8 Navier-Stokes for incompressible fluids in cylindricals and sphericals (364). ..... 4**
**6.9 Boundary Conditions (365). ..... 4**
**6.10 Streamline vs Pathline, Steady vs Unsteady, Laminar vs Turbulent (365). ..... 4**
**6.11 Plane Couette Flow (368). ..... 4**
**6.12 Plane Poiseuille Flow (368). ..... 5**
**6.13 Hagen-Poiseuille Flow (371). ..... 5**
**6.14 Plane Couette Flow with two layers, gravity on, no tilt (372). ..... 5**
**6.15 Couette Flow (374). ..... 5**
**6.16 Trying to drive a transverse wave into a NS fluid (375). ..... 5**
**6.17 Dissipation functions for Newtonian fluids (376). ..... 6**
**6.19 The Vorticity Vector (379). ..... 6**
**6.20 Irrotational Flow (381). ..... 6**
**6.21 Irrotational Flow for inviscid incompressible (381). ..... 7**
**6.22 Irrotational Flow and Navier-Stokes (384). ..... 7**
**6.23 The Vorticity Transport Equation (385). ..... 7**
**6.24 Concept of a Boundary Layer (388). ..... 7**

**6.25 7 Equations for a Compressible Newtonian fluid (389).** ..... **8**
**6.26. Energy equation in terms of enthalpy (390).** ..... **8**
**6.27. Waves in a compressible inviscid fluid (392).** ..... **8**
**6.28. Irrotational inviscid + (barotropic compressible) flow (395).** ..... **8**
**6.29. One dimensional compressible flow (398).** ..... **9**
**6.30. Compressible fluid through a nozzle at bottom of a big tank (399)** ..... **9**
**6.31. Steady laminar flow in an elastic tube (blood vessel) (401).** ..... **9**
**Wiki book list on fluid dynamics:** ..... **10**

### 6.1 Fluids. (353)

If there is no on-going deformation ($D_{ij} = 0$) near a fluid cube, then $T_{ij}(x) = -p(x)\,\delta_{ij}$. There are only normal forces, no shear forces.

### 6.2. Incompressible subclass of fluids (354).

Incompressible means from continuity that div $\mathbf{v} = 0$ and $\rho$ = constant. Later we see that this means $\Delta = \mathrm{tr}(D) = 0$ as well.

### 6.3 Hydrostatics (355).

General Cauchy EOM says $\partial_j T_{ij} + \rho B_i = \rho a_i$, so when $T_{ij} = -p\delta_{ij}$ get $-\partial_i p + \rho B_i = \rho a_i$. Consideration of the static situation leads to $p = \rho g h$ + constant (incompressible). If compressible, you need some $p = p(\rho)$ type connection (such as Boyle or adiabatic) later called barotropic. Cylinder floating on bottom-tether in a pool leads to Archimedes. Glass accelerating to the right has tipped surface. Notion of "static" situations like this where motion is "rigid" and $D_{ij} = 0$.

### 6.4 Newtonian Fluids (357).

Newtonian fluid assumes first isotropic, second $T_{ij} = -p\delta_{ij} + T'_{ij}$ where $T'_{ij} = Q_{ijab}D_{ab}$, and then isotropic tensor theory tells you that $T'_{ij} = \lambda\Delta\delta_{ij} + 2\mu D_{ij}$, so the Newtonian model is this

$$T_{ij} = -p\delta_{ij} + \lambda\Delta\delta_{ij} + 2\mu D_{ij} \qquad \Delta = \mathrm{tr}(D) \qquad // = 0 \text{ when incompressible}$$

$\Delta$ = the **rate of dilatation**

Recall $D_{ij} = (\nabla v)^{\mathbf{S}}{}_{ij}$ (**rate of deformation tensor**) and $W_{ij} = (\nabla v)^{\mathbf{A}}{}_{ij}$ (**spin tensor**). I show why this second object does not appear in $T_{ij}$. Note that $\lambda$ and $\mu$ are unrelated to elastic solid parameters of the same names.

### 6.5 Interpretation of λ and μ (358).

In a river where 1 is the flow direction and 2 across, find $T_{12} = \mu\, \partial_2 v_1$ shear force. There *is* a rate of deformation here $D_{12} = \partial_2 v_1/2$, so there *is* a shear force. The meaning of μ is that large μ means large shear force for a given $D_{12}$ and we interpret this as **viscosity**. As for the meaning of λ, we learn that

$$\text{(average of normal } T'_{ii} \text{ elements)} = (\lambda+2\mu/3)\,\Delta = k\,\Delta \qquad k = \textbf{bulk viscosity}$$

So consider $\langle T'_{ii}\rangle = k\Delta$ and then $\Delta = \langle T'_{ii}\rangle/k$. If you have some set force $\langle T'_{ii}\rangle$ crushing in or pulling out, then large k means the rate of dilatation is small, it is "gooey" in a bulk sense. So the name bulk viscosity is reasonable. We don't have a separate interpretation for λ outside of this combination. The case k = 0 does occur, meaning the material is extremely squashable so any $\langle T'_{ii}\rangle$ gives a huge Δ rate. This case is called the **Stokes Assumption**.

**6.6 Incompressible Newtonian Flow (359).**

If your Newtonian fluid is incompressible, then Δ = 0 and then $T_{ij} = -p\delta_{ij} + 2\mu D_{ij}$ is the simplified constitutive equation (this is Hooke's law in the elastic theory). Since div **v** = 0 as well, it is easy to show that (**divT**) = $-\nabla p + \mu\nabla^2\mathbf{v}$ where $(\mathbf{divT})_i = \partial_j T_{ij}$. So these two terms are going to appear in the equation of motion for this kind of fluid.

**6.7 Navier-Stokes for incompressible fluids (360).**

Put the above in the Cauchy EOM and out comes Navier-Stokes for an incompressible Newtonian fluid

$$\underbrace{\rho\,[\,\partial_t\mathbf{v} + (\nabla\mathbf{v})\mathbf{v}\,]}_{\text{Navier-Stokes}} = \rho\mathbf{B} - \nabla p + \mu\nabla^2\mathbf{v} \qquad \text{and} \qquad \underbrace{\text{div}\,\mathbf{v} = 0}_{\text{incompressible}}$$

More generally, the last term should be written μ (**divT**) where as usual $(\text{divT})_i = \partial_j T_{ij}$. In the general case we have $T_{ij} = -p\delta_{ij} + \lambda\Delta\delta_{ij} + 2\mu D_{ij}$ and then $\partial_j T_{ij} = -\partial_i p + \lambda\partial_i\Delta + 2\mu\partial_j D_{ij}$. But in this last term we have $2\partial_j D_{ij} = \partial_j(\partial_i v_j + \partial_j v_i) = \nabla^2 v_i + \partial_i\,(\text{div}\,\mathbf{v})$ and then NS says

$$\rho\,[\,\partial_t\mathbf{v} + (\nabla\mathbf{v})\mathbf{v}\,] = \rho\mathbf{B} - \nabla p + \mu(\mathbf{divT})$$
$$\rho\,[\,\partial_t\mathbf{v} + (\nabla\mathbf{v})\mathbf{v}\,] = \rho\mathbf{B} - \nabla p + \mu\nabla^2\mathbf{v} + \mu\,\nabla\,(\text{div}\,\mathbf{v}) + \lambda\,\nabla\,[\,\text{tr}(D)]$$

But we also know that $\text{tr}(D) = D_{ii} = \partial_i v_i = \text{div}\,\mathbf{v}$, so we can combine the last two terms

$$\rho\,[\,\partial_t\mathbf{v} + (\nabla\mathbf{v})\mathbf{v}\,] = \rho\mathbf{B} - \nabla p + \mu\nabla^2\mathbf{v} + (\mu+\lambda)\,\nabla\,(\text{div}\,\mathbf{v})$$

and further if we write $k = \lambda + 2\mu/3$ then $(\mu+\lambda) = \mu + (k - 2\mu/3) = \mu/3 + k$, and then

$$\rho\,[\,\partial_t\mathbf{v} + (\nabla\mathbf{v})\mathbf{v}\,] = \rho\mathbf{B} - \nabla p + \mu\nabla^2\mathbf{v} + (\mu/3 + k)\,\nabla\,(\text{div}\,\mathbf{v})$$

so the above is NS for a compressible fluid, and this is the way you see things in wiki

http://en.wikipedia.org/wiki/Navier%E2%80%93Stokes_equations#Derivation_and_description

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\mu\nabla^2\mathbf{v}+\left(\tfrac{1}{3}\mu+\mu^{v}\right)\nabla(\nabla\cdot\mathbf{v})+\mathbf{f}$$

So we now have to use the term "inviscid" to mean that μ=0 *and* k = 0, which means λ = 0, and then even when div **v** ≠0, we still recover our old NS form

$$\rho\,[\,\partial_t\mathbf{v}+(\nabla\mathbf{v})\mathbf{v}\,]=\rho\mathbf{B}-\nabla p+\mu\nabla^2\mathbf{v}$$

Even the simpler compressible NS equation shown just above is non-linear due to the (**∇v**)**v** term. It seems to have "heat conduction equation" aspects to it since $\partial_t$ and $\nabla^2$ appear, but the variable is not scalar temperature, it is vector **v**. You sort of have diffusion of **v**. Since no $\partial_t^2$, it seems distant from the wave equation, but somehow waves will appear below. The longitudinal waves require ρ ≠ constant I think, and transverse ones die out quickly.

First example is for uniform flow where have only $v_1(x_2,x_3)$ and $v_1$ constant along flow axis. $D_{ij}= 0$ and $T'_{ij} = 0$ and nothing but $T_{ij}= -p\delta_{ij}$. Second example has a river going downhill and defines the so called piezometric head h = p/(ρg) + z , but this had no real meaning for me. Need to see more.

Third example seems to justify the intuitive ρgh idea you would apply at any point in any body of water that is not deforming.

**6.8 Navier-Stokes for incompressible fluids in cylindricals and sphericals (364).**

Lai writes NS and div **v** = 0 in Cylindricals and Sphericals. I am totally happy with this.

**6.9 Boundary Conditions (365).** We learn about the **non-slip condition** that seems valid for almost all kinds of fluids. All three components of **v** = 0 at a hard boundary. This is the BC set for a problem, seems Dirichlet like.

**6.10 Streamline vs Pathline, Steady vs Unsteady, Laminar vs Turbulent (365).**

We learn here the meaning of a **streamline** as opposed to a **pathline**, the first being a strobe photo thing, the second you put in a little boat. Flow can be **steady** or **unsteady**, which must means steady or changing in time. The final distinction is between **laminar** flow and **turbulent** flow. Laminar means adjacent layers don't mix at all. We may later learn that things are laminar when the Reynolds number is less than 2100, but for now we don't even know what this number even means.

**6.11 Plane Couette Flow (368).**

Plane Couette Flow. Think of an infinitely deep river with vertical walls. One wall is at rest, the other moves at $v_0$, wall spacing is d. This gives the simplest of all possibly cases of varying v -- linear variation p 368 picture. No gravity. My notes talk about this as a vertical situation instead where the top of the river (infinitely wide) has a plate dragging it along. There is shear force between the layers, you get $D_{12}$ = ($v_0$/2d) and $T_{12}$ = $2\mu D_{12} = \mu\,(v_0/d)$. The two walls exert shear force on the river touching them.

This very prototype of cases shows Lai's general method. We make an ansatz about a few things (in this case that p = constant along the flow, for example) and then we show that we have a self-consistent situation which solves both NS and continuity. This then generates the prototype, and luckily we have nice names to glue to the prototypes.

**6.12 Plane Poiseuille Flow (368).**

Now both walls are at rest, separation 2b, we no longer get a linear variation for v, it is in fact a simple quadratic curve shown p 369. So think of this as an infinitely deep river no gravity and both side walls are at rest (both dragging on the water). One ansatz here is a constant driving force $\partial_1 p$ to make the flow go! So in this river, p drops as you go downstream, but at any point the flow Q is the same and he computes it. He then tips the river at an angle and deduces some facts concerning pressure. // **pwa-zoy'-ee**

**6.13 Hagen-Poiseuille Flow (371).**

Whereas plane is the river situation with duct-like bed, this HP thing is flow in a rigid tube and again you expect some quadratic v profile and that is what you get. Q is again computed. I do a calculation here regarding water in my PVC pipe sprinkler system and get a fairly reasonable answer. We are using incompressible Newtonian N-S all the time in these examples.

Flow rate Q is proportional to $\partial_z p$ and $d^2$ for a pipe. If you have the same end pressures and double the length of a pipe, $\partial_z p$ drops in half and Q drops in half, so hose flow drops as hose gets long! The dependence on diameter is quadratic as you would expect (area). I did not study the nozzle effect, maybe in the problems. I had to get actual $\mu$ for water.

**6.14 Plane Couette Flow with two layers, gravity on, no tilt (372).**

Lai solves this, using $T_{12}$ continuous at the boundary and p as well and v as well (no slip between layers!). A bit of fiddling was needed here.

**6.15 Couette Flow (374).**

This is a rather strange apparatus and you don't just get a linear velocity ramp as in the plane case. You have two cylinders rotating at different rates with the fluid between them! Lai computes everything.

Note on naming flows:

| Lai | wiki |
|---|---|
| plane Couette | Couette |
| Couette | Taylor-Couette. |

Couette was 1890 or so, Taylor 1923 actually did those cylinders. There was a 1999 paper in fact accounting for effects of the ends of finite cylinders

6. ^ M.C. Wendl (1999) *General Solution for the Couette Flow Profile*, Physical Review E **60**, 6192-6194.

Not surprisingly, Bessel functions appear.

**6.16 Trying to drive a transverse wave into a NS fluid (375).**

But there is no wave equation, it is a diffusion equation as shown p 375 6.16.2. If you shear-wiggle a plane boundary, the effect just diffuses into the fluid and quickly dies away. An interesting situation to consider I think.

**6.17 Dissipation functions for Newtonian fluids (376).**

Lai shows that for an incompressible fluid we find that the stress power (per unit volume) is $P_s = 2\mu\, tr(D^2) \equiv \Phi_{inc}$. This is the rate at which power is converted to heat due to the viscosity. For a compressible fluid, there is an extra term here, $\Phi = \Phi_{inc} + \lambda\, (trD)^2$, and then $P_s = \Phi - p\, tr(D)$ for a second extra term. Somehow this means that work goes not just into heat but also into pressure storage (last term). An example is given and I conclude that for plane Couette action a cubic meter of water would only dissipate 1 mW. This is where I concluded that μ for water is "low".

**6.18. Energy Equation for Newtonian fluids (378).**

We did this earlier in the general continuum case and $T_{ij}$ appears in the energy conservation equation (6.18.1). Also appearing are energy density u and flow rate of heat div **q** and also possible heat sources. If you assume only Fourier heat conduction flow, get 6.18.2. Then if you can say that $u = u(\Theta) = c\Theta$, you end up with a PDE in temperature Θ which is 6.18.3.

Note that earlier $P_s = T_{ij}\partial_j v_i = 2\mu\, tr(D^2)$ and this same term appears in the Θ equation and that is why you see $\Phi_{inc}$ sitting in there. If you then neglect the viscous energy loss, you end up with a simple diffusion equation for heat flow in a fluid where then the constant is a combination of the Fourier κ and ρ and c. So here is a derivation of the heat equation in one circumstance, an equation Stakgold studied in amazingly great detail.

In an example, Lai then finds the Θ distribution in plane Couette flow where your two planes are held at certain fixed temperatures. This example is interesting for several reasons: (1) it is a combination of the NS problem for velocity, and the energy equation for Θ; (2) if you ignore μ losses, the plot is linear; (3) this is justified if a lot of heat is flowing through the fluid from the plates.

**6.19 The Vorticity Vector (379).**

We first learn that if you go into the principal axes system, then the three normal vectors in that system rotate together (George Shearing) according to spin tensor W. In this system, the sugar cube does not shear, though it can scale its edges and rotate as just stated. Next, we learn that if you call **ω** the vector associated with the spin tensor W (going way back on this), then in fact you find that $2\boldsymbol{\omega} = \text{curl}\, \mathbf{v}$. For irrotational flow, $\boldsymbol{\omega} = 0$ and $W = 0$ and a cube in the principle axis system neither rotates nor shears, but only stretches along its axes! The axes **n** are fixed. The vorticity vector is defined as $\boldsymbol{\zeta} = 2\boldsymbol{\omega} = \text{curl}\, \mathbf{v}$.

Lai computes curl **v** in our various prototype situations.

**6.20 Irrotational Flow (381).**

If you want curl **v** = 0 and div **v** = 0 (incompressible), you can say $\mathbf{v} = -\nabla\varphi$ and you automatically satisfy the curl thing and the div thing becomes Laplace. So here is the origin of "potential theory" in fluid dynamics.

**6.21 Irrotational Flow for inviscid incompressible (381).**

For Lai, the term **inviscid** means both $\mu = 0$ and incompressible ($\Delta=0$), and so for Lai, inviscid $\Leftrightarrow T_{ij} = -p\delta_{ij}$. Our NS equation was only stated by Lai for incompressible, so we just set $\mu = 0$ in the NS and the simpler resulting equations are called the **Euler Equations of Motion** 6.21.2. Now, if you also have $\mathbf{B} = -\nabla\Omega$ , this Euler equation becomes 6.21.6. Finally, if in addition to the above the flow is *irrotational*, Euler then becomes **Bernoulli's equations** of motion:

$-\partial_t\varphi + v^2/2 + p/\rho + \Omega = f(t)$ $\quad\quad$ $\mathbf{v} = -\nabla\varphi$

$v^2/2 + p/\rho + \Omega = \text{constant}$ $\quad\quad$ // for steady flow $\quad\quad$ $\Omega = gy$ for gravity.

So again, Bernoulli is for an inviscid ($\Rightarrow$ incompressible) Newtonian fluid which is doing irrotational flow. The last equation relates p,ρ,v at different points in a steady flow fluid. Lai notes that when $\mu = 0$, you can violate the no-slip condition. Two examples are given, the first academic, the second is the hole in can problem. Here you end up with the famous Torricelli's Formula $v^2 = 2gh$, as if the water were dropped from h. I find a table that shows that μ for water is quite low compared to kerosene or oil, etc, so treating it as inviscid is probably OK in many circumstances.

**6.22 Irrotational Flow and Navier-Stokes (384).**

When $\mu \neq 0$, for irrotational flow $\nabla^2\mathbf{v} = 0$ as is trivially shown raw notes. So you still get the Euler equations when $\mu \neq 0$! *But*: the problem is that in this case, the non-slip condition is in force and you then get both Dirichlet and Neumann BC's on φ for every problem, so there is really no solution except in contrived cases.

The example here shows the interesting fact that in Couette flow, there is a setting for the two rotation rates that causes curl $\mathbf{v} = 0$ in the fluid, so you can have irrotational in this special case. This has nothing to do with the observation above.

**6.23 The Vorticity Transport Equation (385).**

If you just define $\boldsymbol{\zeta} = \text{curl } \mathbf{v}$ , Lai and I separately show that the NS and div $\mathbf{v} = 0$ can be written in terms of $\boldsymbol{\zeta}$ as

$D_t\boldsymbol{\zeta} = (\nabla\mathbf{v})\boldsymbol{\zeta} + (\mu/\rho)\nabla^2\boldsymbol{\zeta}$ $\quad\quad$ p 386

in the case of incompressible. **B** can be present if in its potential form **Ω**, and as you see, **Ω** does not even show in this vorticity equation. In the example of 2D rotational flow, Lai shows that $\zeta \equiv \zeta_3$ solves the diffusion equation, so you can think of "vorticity" as something that diffuses away from a source, like heat. The source is usually a boundary where the no-slip condition creates vorticity and then this created substance diffuses away from the boundary. If it stays close to the boundary only, you get a boundary layer.

**6.24 Concept of a Boundary Layer (388).**

See previous section. You tend to get the layer if the flow is fast on a wing, so things are fairly laminar except right at the surface of the wing where v = 0. Lift is not covered by Lai!

**6.25 7 Equations for a Compressible Newtonian fluid (389).**

For general analysis, there are 7 equations and 7 variables, **v**,p,ρ,u,Θ . The 7 equations are (1,2,3) NS, which counts as three; (4) continuity; (5) energy conservation; (6) equation of state relating p and ρ; (7) equation relating stored energy u to temperature Θ. So you have a tough problem to solve in the fully general case where everything moves.

**6.26. Energy equation in terms of enthalpy (390).**

Enthalpy is the thermodynamic "energy" thing (potential) which is conserved in a constant S and constant p process (H = U + pV, dH = TdS - Vdp). In the fluid context, this thing is written h = u + p/ρ and if you add in KE $v^2/2$ you get $h_0$, the stagnation enthalpy. All this Lai stuff is per unit mass so think δm = 1 and so ρ = 1/δV. You can then write the energy equation from the above equation list as 6.26.3, in which energy u is replaced by energy $h_0$ and we find this $h_0$ is then driven by $T'_{ij}$. If you have a steady inviscid fluid situation (T'=0), this becomes $h_o$ = constant, and you can solve problems that way. If you also have ideal gas type equations, you get 6.26.9 for $h_0$ which is constant. But no example problems are solved using this result.

**6.27. Waves in a compressible inviscid fluid (392).**

Lai obtains the wave equation for longitudinal sound waves in an inviscid compressible fluid μ=0,k=0,λ=0. He starts with NS-for-inviscid (which recall works for compressible or incompressible inviscid fluids) and throws out quadratic terms in smallness, so waves are valid when they have small amplitude. The wave speed comes out being $c_0^2 = (dp/d\rho)_0$ .

In the usual gas case of adiabatic (aka isentropic) expansion, $p = \beta\rho^\gamma$ and then you get $c^2 = \gamma p/\rho$. [ Ie, this is the equation of state describing p(ρ).]

What about sound in water? On page 218 we did longitudinal waves in an elastic solid and we found there that $c_L^2 \sim k/\rho_0$ where k is the solid's bulk modulus. A similar equation holds for hard-to-compress fluids, but Lai never derives it. This site http://hyperphysics.phy-astr.gsu.edu/hbase/sound/souspe2.html claims that this is in fact the formula for water, and k is a large number so we get $c_L$ = 1500 m/sec. Speed of sound in air is about 350 m/sec, and in steel 4500 m/sec.

In first example, we learn that small amplitude rule means $\varepsilon << \rho_0 c_o^2 = \gamma p_o$. The second example does head on reflections and shows that $\rho c_L$ is the equivalent of the index of refraction (fluid impedance).

**6.28. Irrotational inviscid + (barotropic compressible) flow (395).**

Here we derive Bernoulli's equations for the *compressible* case (previous shot was incompressible only). We end up with p/ρ replaced by $\int dp/\rho$ as in 6.28.7,8. If you have p = p(ρ), that is called **barotropic**, and adiabatic gas law is an example where $p(\rho) = \beta\rho^\gamma$, the isentropic deal. In this case we get 6.28.10 which relates p, ρ and v (and γ). In the example we compare two points in a gas system. At the $p_0$ point we have zero velocity $v_0 = 0$ and $\rho_0$. At the other point we have p, v, ρ. We can then use 6.28.10 being equal at

these two locations to solve for $(p_0/p)$ as in 6.28.11, no big deal. What is interesting is that the ratio v/c appears for the first time, where c is that adiabatic sound speed $c^2 = \gamma p/\rho$ at the p,v,ρ location. This really does remind me of special relativity. This ratio is the Mach number v/c! In the second exercise, we get a simplified version of 6.28.11 (namely, 6.28.12) by assuming $M << 1$.

**6.29. One dimensional compressible flow (398).**

Imagine flow through a duct whose cross section A varies in some manner. We assume that our v field is uniform across this cross section (perhaps ignore boundary layers at the duct surfaces). By using the fancier Bernoulli equation and mass conservation ($\rho Av$ = constant), we find $(dA/A) = (dv/v)(M^2-1)$ known as the Hugoniot equation. Normally you would think $dA>0$ would give $dv < 0$ as is the case for compressible and which is just intuitive. This is in fact true below Mach One! Above the opposite is true! If you increase A, v increases as well, very strange!

**6.30. Compressible fluid through a nozzle at bottom of a big tank (399).**

The first section discusses a "convergent nozzle". We apply our new Bernoulli tool and solve for $v_2$, the speed at the tank section 2 region 6.30.4. We then compute dm/dt at that same location as 6.30.6. We next show that when $(p_2/p_1)$ = some critical value, $v_2 = c$, the speed of sound. Now if we apply Hugoniot to the convergent shape of the nozzle, we conclude that since $dA < 0$ in $(dA/A) = (dv_2/v_2)(M^2-1)$. If we could lower $p_2$, trying to make $dv_2 > 0$ from our $v_2$ formula, we get $v_2 > c$ or $M > 1$. This is a contradiction, so we have to conclude that $v_2 \leq c$ at any point like section 2 in such a nozzle. The flow cannot go supersonic and we cannot have $(p_2/p_1) < f(\gamma)$, there is a floor on the nozzle interior pressure.

In effect we have two conditions for the nozzle (the first in order to get any flow at all! )

$p_2 \geq p_R$ $\quad\Rightarrow\quad p_2 \geq \max(p_R, p_c)$ $\quad\quad p_R$ = ambient pressure outside tank

$p_2 \geq p_c$

Case 1: if max = $p_R$ the dm/dt is given by 6.30.9

Case 2: if max = $p_c$ the dm/dt is given by 6.30.10 which result is independent of $p_R$

Lai next considers a convergent nozzle followed by a divergent one as in the p 401 picture. This thing has a throat. We know that to the left of this throat, we are subsonic. If sonic exactly at the throat, "choked flow". Lai gives a qualitative description of what might happen as you vary $p_R$ in such a way that you hit this choke point and then "go beyond". You end up with a pressure discontinuity which is a shock front, but we have never really studied such things, so he is just commenting. Thinking flight 477 Nova and the shocks coming off the wings when stalling.

**6.31. Steady laminar flow in an elastic tube (blood vessel) (401).**

In a completely systematic manner, Lai obtains an expression for the flow Q of fluid through an elastic "artery" tube as a function of the pressures at the two ends. Along the way he finds r(p), the radius at some point along the tube, as in 6.31.7. If $E_Y$ for the tubing is large enough, you can approximate this as in 6.31.10 and then the flow rate Q simplifies to 6.31.12. This problem combines elastic solid theory and compressible flow fluid theory!

**Wiki book list on fluid dynamics:**

Fluid Mechanics Books List

This is the compilation of popular fluid mechanics books (172 books !!!, I have the one in red.


1. Acheson, D. J., **Elementary Fluid Dynamics**, Clarendon Press, Oxford (1990).
2. Ahmed, N., **Fluid Mechanics**, Engineering Press (1987).
3. Alexandrou, A. N., **Principles of Fluid Mechanics**, Prentice Hall, NJ (2001).
4. Aris, R., **Vectors, Tensors, and the Basic Equations of Fluid Mechanics**, Prentice Hall, Englewood Cliffs, NJ (1962).
5. Batchelor, G. K., **Theory of Homogeneous Turbulence,** Cambridge Univ. Press, Cambridge, UK (1953).
6. Batchelor, G. K., **An Introduction to Fluid Dynamics**, Cambridge Univ. Press, Cambridge, UK (1967).
7. Bennet, A., **Lagrangian Fluid Dynamics**, Cambridge Univ. Press, Cambridge (2006).
8. Bertin, J. J., **Engineering Fluid Mechanics**, Prentice Hall, NJ (1984).
9. Bird, R. B., W. E. Stewart, and E. N. Lightfoot, **Transport Phenomena,** 2nd ed., John Wiley, New York (2002).
10. Birkhoff, G., **Hydrodynamics**, Princeton Univ. Press, New Jersey (**1953**).
11. Bloomer, J. J., **Practical Fluid Mechanics for Engineering Applications**, Marcel Dekker, New York (1999).
12. Bober, W. and R. A. Kenyon, **Fluid Mechanics**, Wiley, New York (1980).
13. Bradshaw, P., **An Introduction to Turbulence and its Measurement**, Pergamon Press, Oxford (1971).
14. Bradshaw, P., T. Cebeci, and J. H. Whitelaw, **Engineering Calculation Methods for Turbulent Flow**, Academic Press, New York (1981).
15. Brodkey, R. S., **The Phenomena of Fluid Motions**, Brodkey Publishing (2004).
16. Brower Jr., W., **Primer in Fluid Mechanics**, CRC Press, New York (1999).
17. Cannon, J. R. and B. K. Shivamoggi, (ed.), **Mathematical and Physical Theory of Turbulence** , Chapman & Hall/CRC Press, New York (2006).
18. Cebici, T. and P. Bradshaw, **Momentum Transfer in Boundary Layers**, Hemisphere, Washington DC (1977).
19. Cebici, T. and A. M. O. Smith, **Analysis of Turbulent Boundary Layers**, Academic Press, New York (1974).
20. Cengel, Y. A. and J. M. Cimbala, **Fluid Mechanics**, McGraw-Hill, New York (2005).
21. Chandrashekhar, S., **Hydrodynamic and Hydromagnetic Stability**, Dover Publications, New York (1981).
22. Chen, C.-J. and S-Y. Jaw, **Fundamentals of Turbulence Modeling**, Taylor & Francis (1998).

23. Cheremisinoff, N. P., **Practical Fluid Mechanics for Engineers and Scientists**, CRC Press, New York (1990).

24. Chorin, A. J., **Vorticity and Turbulence**, Applied Mathematical Sciences, Vol 103, Springer-Verlag (1994).

25. Chorin, A. J. and J. E. Marsden, **A Mathematical Introduction to Fluid Mechanics**, 3rd ed., Springer-Verlag (1993).

26. Chorlton, F., **Textbook of Fluid Dynamics**, D. Van Nostrand (1967).

27. Churchill, S. W., **Viscous Flows - The Practical Use of Theory**, Butterworth Publishers, Stoneham, MA (1988).

28. Cimbala, J. M. and Y. A. Cengel, **Essentials of Fluid Mechanics**, McGraw-Hill, New York (2006).

29. Crow, C. T., J. A. Robertson, and D. F. Elger, **Engineering Fluid Mechanics**, 8th ed., John Wiley, New York (2004).

30. Currie, I. G., **Fundamental Mechanics of Fluids**, 3rd ed., CRC Press, New York (2002).

31. Daily, J. W. and D. R. F. Harleman, **Fluid Dynamics**, Addison-Wesley, Reading, MA (1966).

32. Darby, R., **Chemical Engineering Fluid Mechanics**, 2nd ed., Marcel Dekker, Inc., New York (2001).

33. Daugherty, R. L., J. B. Franzini, and E. J. Finnemore, **Fluid Mechanics with Engineering Applications**, 8th ed., McGraw-Hill, New York (1985).

34. Davidson, P. A., **Turbulence: An Introduction for Scientists and Engineers**, Oxford Univ. Press, (2004).

35. Douglas, J. F., J. M. Gasiorek, J. A. Swaffield, and L. B. Jack, **Fluid Mechanics**, 5th ed., Prentice Hall, NJ (2006).

36. Drazin, P. G., **Introduction to Hydrodynamic Stability**, Cambridge Univ. Press, London (2002).

37. Drazin, P. G. and W. H. Reid, **Hydrodynamic Stability**, Cambridge Univ. Press, London (1981).

38. Duncan, W. J., A. S. Thom, and A. D. Young, **An Elementary Treatise on the Mechanics of Fluids**, 2nd ed., Arnold, London (1970).

39. Emanuel, G., **Analytical Fluid Dynamics**, 2nd ed., CRC Press, New York (2000).

40. Evans, H., **Laminar Boundary Layers**, Addison-Wesley, Reading, MA (1968).

41. Faber, T. E., **Fluid Dynamics for Physicists**, Cambridge Univ. Press, London (1995).

42. Fay, J. A., **Introduction to Fluid Mechanics**, MIT Press (1994).

43. Finnemore, E. J. and J. B. Franzini, **Fluid Mechanics with Engineering Applications**, 10th ed., McGraw-Hill, New York (2001).

44. Fomin, V. M. and S. P. Kiselev, **Foundations of Fluid Mechanics with Applications**, Jaico Publishing House (2005).

45. Fox, R. W. and A. T. McDonald, **Introduction to Fluid Mechanics**, 6th ed., John Wiley, New York (2004).

46. Friedlander, S. and D. Serre (eds.), **Handbook of Mathematical Fluid Dynamics**, Elsevier B.V., Amsterdam (2004).

47. Frisch, U., **Turbulence**, Cambridge Univ. Press (1996).

48. Gallavotti, G., **Foundations of Fluid Dynamics**" , Springer (2005).

49. Garde, R. J., **Fluid Mechanics through Problems**, Wiley (1989).

50. Gerhart, P. M., R. J. Gross, and J. I. Hochstein, **Fundamentals of Fluid Mechanics**, Addison-Wesley, Reading, MA (1992).

51. Goldstein, S. (ed.), **Lectures on Fluid Mechanics**, John Wiley, New York (1960).

52. Goldstein, S. (ed.), **Modern Developments in Fluid Dynamics**, Dover Publications, New York (1965).

53. Graebel, W. P., **Engineering Fluid Mechanics**, Taylor & Francis (2001).

54. Graebel, W. P., **Advanced Fluid Mechanics**, Academic Press, New York (2007).

55. Granet, I., **Fluid Mechanics for Engineering Technology**, Prentice Hall, NJ (1981).

56. Granger, R. A., **Fluid Mechanics**, 2nd ed., Dover Publications, New York (1995).

57. Greenspan, H. P., **The Theory of Rotating Fluids**, Cambridge Univ. Press, Cambridge, UK (1968).

58. Guyon, E., J.-P. Hulin, L. Petit, and C. D. Mitescu, **Physical Hydrodynamics**, Oxford Univ. Press (2001).

59. Hansen, A. G., **Fluid Mechanics**, Wiley, New York (1967).

60. Happel, J. and H. Brenner, **Low Reynolds Number Hydrodynamics**, Prentice Hall, NJ (1965).

61. Hinze, J. O., **Turbulence**, 3rd ed., McGraw-Hill, New York (1987).

62. Holland, F. A. and R. Bragg, **Fluid Flow for Chemical Engineers**, 2nd ed., Edward Arnold, London (1995).

63. Hughes, W. F., **An Introduction to Viscous Flow**, Hemisphere, New York (1979).

64. Hughes, W. F. and J. A. Brighton, **Schaum's Outline of Fluid Dynamics**, 3rd ed., McGraw-Hill, New York (1999).

65. Janna, W. S., **Introduction to Fluid Mechanics**, 3rd ed., William S. Janna (1993).

66. John, J. E. A. and W. L. Haberman, **Introduction to Fluid Mechanics**, 3rd ed., Prentice- Hall, NJ (1988).

67. Johnson, R. W., **The Handbook of Fluid Dynamics**, CRC Press/Springer-Verlag (1998).

68. Joseph, D. D., **Stability of Fluid Motions, I and II**, Springer-Verlag, New York (1976).

69. Joseph, D. D. and Y. Renardy, **Fundamentals of Two-Fluid Dynamics. Part 1: Mathematical Theory and Applications**, Springer-Verlag, New York (1993).

70. Joseph, D. D. and Y. Renardy, **Fundamentals of Two-Fluid Dynamics. Part 2: Lubricated Transport, Drops and Miscible Liquids**, Springer-Verlag, New York (1993).

71. Kambe, T., **Elementary Fluid Mechanics**, World Scientific Publishing (2007).

72. Karamcheti, K., **Principles of Ideal-Fluid Aerodynamics**, Wiley, New York (1966).

73. Kaufmann, W., **Fluid Mechanics**, McGraw-Hill, New York (1979).

74. Khan, I. A., **Fluid Mechanics**, Oxford Univ. Press (1987).

75. King, R. P., **Introduction to Practical Fluid Flow**, Butterworth-Heinemann (2002).

76. Kiselev, S. P., E. V. Vorozhtsov, and V. M. Fomin, **Foundations of Fluid Mechanics with Applications: Problem Solving Using Mathematica**, Springer-Verlag (1999).

77. Kleinstreuer, C., **Engineering Fluid Dynamics: An Interdisciplinary Systems Approach**, Cambridge Univ. Press, Cambridge, UK (1997).

78. Krause, E., **Fluid Mechanics**, Springer, New York (2005).

79. Kreider, J., **Principles of Fluid Mechanics**, Allyn & Bacon (1985).

80. Kundu, P. K. and I. M. Cohen, **Fluid Mechanics,** 4th ed., Academic Press (2008).

81. Ladyzhenskaya, O. A., T**he Mathematical Theory of Viscous Incompressible Flow**, 2nd ed., Gordon & Breach (1969).

82. Lagerstrom, P. A., T**he Laminar Flow Theory**, Princeton Univ. Press (1996).

83. **Lamb**, H., H**ydrodynamics**, 6th ed., Dover Publications, New York (**1945**).

84. Landahl, M. T. and E. Mollo-Christensen, **Turbulence and Random Processes in Fluid Mechanics**, Cambridge Univ. Press, London (1986).

85. **Landau**, L. D. and E. M. **Lifshitz**, **Fluid Mechanics**, 3rd ed., Pergamon Press, London (**1966**).

86. Langlois, W. E., **Slow Viscous Flow**, Macmillan, New York (1964).

87. Launder, B. E. and D. B. Spalding, **Lectures in Mathematical Models of Turbulence**, Academic Press, New York (1972).

88. Leal, L. G., **Advanced Transport Phenomena: Fluid Mechanics and Convective Transport Processes**, Cambridge Univ. Press (2007).

89. Lesieur, M., **Turbulence in Fluids**, Kluwer Academic Publishers, New York (1990).

90. Leslie, D. C., **Developments in the Theory of Turbulence**, Clarendon Press, Oxford (1973).

91. Li, W. H. and S. H. Lam, **Principles of Fluid Mechanics**, Addison-Wesley, Reading, MA (1964).

92. Lin, C. C., **The Theory of Hydrodynamic Stability**, Cambridge Univ. Press, Cambridge (1955).

93. Lighthill, M. J., **An Informal Introduction to Theoretical Fluid Mechanic**s, Clarendon Press, Oxford, England (1986).

94. Massey, B. S. and J. Ward-Smith, **Mechanics of Fluids,** 7th ed., Nelson Thornes, UK (1998).

95. Mathieu, J. and J. Scott, **An Introduction to Turbulent Flow**, Cambridge Univ. Press, Cambridge, UK (2000).

96. McComb, W. D., **The Fluid Turbulence**, Oxford Univ. Press, UK (1992).

97. Meyer, R., **Introduction to Mathematical Fluid Dynamics**, Wiley, New York (1971).

98. Middleman, S., **An Introduction to Fluid Dynamics**, Wiley, New York (1997).

99. Milne-Thompson, L. M., **Theoretical Hydrodynamics**, 5th ed., Macmillan, New York (1968).

100. Monin, A. S. and A. M. Yaglom, **Statistical Fluid Mechanics**, Dover Publications, New York (2007).

101. Mott, R. L., **Applied Fluid Mechanics**, 6th ed., Prentice Hall, NJ (2006).

102. Munson, B. R., D. F. Young, and T. H. Okiishi, **Fundamentals of Fluid Mechanics**, 5th ed., Wiley, New York (2005).

103. Muralidhar, K. and G. Biswas, **Advanced Engineering Fluid Mechanics**, 2nd ed., Narosa Publ. House, New Delhi (2005).

104. Nakayama Y. and R. F. Boucher, **Introduction to Fluid Mechanics**, Arnold Publishers (1998).

105. Nunn, R. H., **Intermediate Fluid Mechanics**, Taylor & Francis (1989).

106. Ockendon, H. and J. R. Ockendon, **Viscous Flow**, Cambridge Univ. Press, London (1995).

107. Oleinik, O. and V. Samokhin, **Mathematical Models in Boundary Layer Theory**, Chapman & Hall (1999).

108. Olson, R. M. and S. J. Wright, **Essentials of Engineering Fluid Mechanics**, Harper-Collins College Div. (1990).

109. O'Neill, M. E. and F. Chorlton, **Ideal and Incompressible Fluid Dynamics**, Ellis Horwood, Chichester (1986).

110. O'Neill, M. E. and F. Chorlton, **Viscous and Compressible Fluid Dynamics**, Ellis Horwood, Chichester (1989).

111. Panton, R. L., **Incompressible Flow**, 3rd ed., John Wiley, New York (2005).

112. Papanastasiou, T. C., **Applied Fluid Mechanics**, Prentice Hall, NJ (1994).

113. Papanastasiou, T. C., G. C. Georgiou, and A. N. Alexandrou, **Viscous Fluid Flow**, CRC Press, New York (2000).

114. Paterson, A. R., **A First Course in Fluid Dynamics**, Cambridge Univ. Press (1983).

115. Petrila, T. and D. Trif, **Basics of Fluid Mechanics and Introduction to Computational Dynamics**, Springer (2005).

116. Pope, S. B., **Turbulent Flows**, Cambridge Univ. Press, Cambridge, UK (2000).

117. Potter, M. C. and J. F. Foss, **Fluid Mechanics**, 2nd ed., Great Lakes Press (1983).

118. Potter, M. C., D. C. Wiggert, M. Hondzo, and T. I-P. Shih, **Mechanics of Fluids**, 3rd ed., Brooks/Cole, Pacific Grove, CA (2001).

119. Pozrikidis, C., **Introduction to Theoretical and Computational Fluid Dynamics**, Oxford Univ. Press, Oxford (1997).

120. Pnueli, D. and C. Gutfinger, **Fluid Mechanics**, Cambridge Univ. Press (1997).

121. Prandtl, L., **Essentials of Fluid Dynamics**, Hafner Publications, New York (1952).

122. Prasuhn, A. L., **Fundamentals of Fluid Mechanics**, Prentice Hall, NJ (1980).

123. Robertson, J. M., **Hydrodynamics in Theory and Application**, Prentice Hall, NJ (1965).

124. Rogers, D. F., **Laminar Flow Analysis**, Cambridge Univ. Press (1992).

125. Rosenhead, L. (ed.), **Laminar Boundary Layers**, Oxford Univ. Press (1963).

126. Rouse, H. (ed.), **Advanced Mechanics of Fluids**, John Wiley, New York (1959).

127. Rouse, H., **Elementary Mechanics of Fluids**, Dover Publications, New York (1979).

128. Roy, D. N., **Applied Fluid Mechanics**, Ellis Horwood (1988).

129. Sabersky, R. H., A. J. Acosta, E. G. Hauptmann, and E. M. Gates, **Fluid Flow: A First Course in Fluid Mechanics**, 4th ed., Prentice Hall, NJ (1998).

130. Saffman, P.G., **Vortex Dynamics**, Cambridge Univ. Press, New York (1992).

131. Samimy, M., K. S. Breuer, L. G. Leal, and P. H. Steen, **A Gallery of Fluid Motion**, Cambridge Univ. Press, New York (2003).

132. Schaschke, C., **Fluid Mechanics: Worked Examples for Engineers**, Institution of Chemical Engineers (1998).

133. Schetz, J. A., **Foundations of Boundary Layer Theory**, Prentice Hall, NJ (1984).

134. Schetz, J. A. and A. E. Fuhs, **Fundamentals of Fluid Mechanics**, 3rd ed., Wiley-Interscience (1999).

135. Schlichting, H., **Boundary Layer Theory**, 7th ed., McGraw-Hill, New York (1979).

136. Serrin, J., **Mathematical Principles of Classical Fluid Mechanics**, in Encyclopedia of Physics, edited by S. Flgge, Springer-Verlag, Berlin-Heidelberg (1959).

137. Seshadri, C. V. and S. V. Patankar, **Elements of Fluid Mechanics**, Prentice Hall of India, New Delhi (1971).

138. Shames, I. H., **Mechanics of Fluids**, 4th ed., McGraw-Hill, New York (2002).

139. Shaughnessy, E. J., I. M. Katz, and J. P. Schaffer, **Introduction to Fluid Mechanics**, Oxford Univ. Press (2004).

140. Sherman, I. H., **Viscous Flow**, McGraw-Hill, New York (1990).

141. Shinbrot, M., **Lectures on Fluid Mechanics**, Gordon & Breach (1973).

142. Shivamoggi, B. K., **Theoretical Fluid Dynamics**, John Wiley, New York (1998).

143. Slattery, J. C., **Advanced Transport Phenomena**, Cambridge Univ. Press (1999).

144. Smits, A. J., **A Physical Introduction to Fluid Mechanics**, John Wiley, New York (1999).

145. Spurk, J. H. and N. Aksel, **Fluid Mechanics**, 2nd ed., Springer-Verlag, New York (2007).

146. Stanisic, M. M., **The Mathematical Theory of Turbulence**, Springer-Verlag, New York (1985).

147. Streeter, V. L., E. B. Wylie, and K. W. Bedford, **Fluid Mechanics**, 9th ed., McGraw-Hill, New York (1998).

148. Streeter, V. L. (ed.), **Handbook of Fluid Dynamics**, McGraw-Hill, New York (1961).

149. Sullivan, J. A., **Fundamentals of Fluid Mechanics**, Reston Pub. Co. (1978).

150. Swinney, H. L. and J. P. Gollub, **Hydrodynamic Instabilities and Transition to Turbulence**, Springer, New York (1981).

151. Telionis, D. P., **Unsteady Viscous Flows**, Springer-Verlag, New York (1981).

152. Tennekes, H. and J. L. Lumley, **A First Course in Turbulence**, MIT Press, Cambridge, MA (1972).

153. Tritton, D. J., **Physical Fluid Dynamics**, 2nd ed., Oxford Univ. Press (1988).

154. Tropea, C., A. L. Yarin, and J. F., Foss (eds.), **Springer Handbook of Experimental Fluid Mechanics**, Springer-Verlag (2007).

155. Truesdell, C. and K. R. Rajagopal, **An Introduction to the Mechanics of Fluids**, Birkhäuser, Boston (1999).

156. Vallentine, H. R., **Applied Hydrodynamics**, 5th ed., Dover Publications (1996).

157. Van Dyke, M., **Perturbation Methods in Fluid Mechanics**, Parabolic Press, Stanford, CA (1970).

158. Van Dyke, M., **An Album of Fluid Motion**, Parabolic Press, Stanford, CA (1982).

159. Vanyo, J. P., **Rotating Fluids in Engineering and Science**, Butterworth-Heinemann (1993).

160. Vennard, J. K. and R. L. Street, **Elementary Fluid Mechanics**, 6th ed., John Wiley, New York (1982).

161. von Mises, R. and K. O. Friedrchs, **Fluid Dynamic**, Springer-Verlag, New York (1971).

162. Walshaw, A. C. and D. A. Jobson, **Mechanics of Fluids**, Longman (1979).

163. Warsi, Z. U. A., **Fluid Dynamics - Theoretical and Computational Approaches**, 3rd ed., Taylor & Francis (2006).

164. Whitaker, S., **Introduction to Fluid Mechanics**, Krieger Publishing Co. (1992).

165. White, F. M., **Fluid Mechanics**, 5th ed., McGraw-Hill, New York (2003).

166. White, F. M., **Viscous Fluid Flow**, 3rd ed., McGraw-Hill, New York (2006).

167. Wieghardt, K. and W. Tillmann, **Boundary Layer Theory**, McGraw-Hill, New York (1968).

168. Wilcox, D. C., **Basic Fluid Mechanics**, DCW Industries, La Canada, CA (2000).

169. Yih, C.-S., **Fluid Mechanics: A Concise Introduction to Theory**, West River Press, Ann Arbor, MI (1977).

170. Young, D. F., B. R. Munson, T. H. Okiishi, and W. W. Huebsch, **A Brief Introduction to Fluid Mechanics**, Wiley, New York (2007).

171. Yuan, S. W., **Foundations of Fluid Mechanics**, Prentice Hall, NJ (1970).

172. Zeytounian, R. Kh., Theory and Applications of Viscous Fluid Flows, Springer-Verlag, New York (2004).

TOP