Lai Chapter 8 Meta Notes: Non-Newtonian Fluids PhL 10.21.12

Part A: Linear ViscoElastic Fluid (444)

8.1 Linear Maxwell Fluid (444)

Here is the new feature involved with the "linear Maxwell fluid" in the constitutive equation:

$$T_{ij} = -p\,\delta_{ij} + S_{ij} \qquad S_{ij} = 2\mu D_{ij} - \lambda \partial S_{ij}/\partial t \qquad D = (\partial_i v_j + \partial_j v_i)/2$$

The fluid is studied first using a simple model of spring and dashpot. The fluid has two parameters, μ is viscosity, and λ is a (viscosity/elasticity) ratio = relaxation time. The creep and relaxation basic experiments are described which let you measure. There is a lot going on here which I skip here.

8.2 Fluid with multiple components (450). [generalized linear Maxwell discrete]

A mixture of linear maxwell fluids each component having its own μ_n and λ_n and S_n but all components have the same v and this the same D. So

$$S_n + \lambda_n \partial_t S_n = 2\mu_n D \qquad S = \Sigma_{n=1}^{N} S_n$$

I proved (in line) a theorem regarding this equation that Lai just quoted.

8.3 Green's Function solution to our ODE's (451)

The constitutive equation is an ODE and you can solve it in a simple way to get

$$S = 2 \int_{-\infty}^{t} dt' \, \Sigma_n [(\mu_n/\lambda_n) e^{-(t-t')/\lambda_n}] \, D(t') = 2 \int_{-\infty}^{t} dt' \, \Sigma_n [\varphi_n(t-t')] \, D(t') \qquad (*)$$

for the discrete mix of components. Now we no longer have an ODE and our constitutive equation involves a time integral over a history, and φ_n is called a stress relaxation function.

8.4 Continuous spectrum version (450).

We now assume a continuous mix of "components" and the above becomes

$$S(t) = 2 \int_{-\infty}^{t} dt' \, [\int_{0}^{\infty} d\lambda \, (\mu/\lambda) e^{-(t-t')/\lambda}] \, D(t') = 2 \int_{-\infty}^{t} dt' \, [\varphi(t-t')] \, D(t') = (8.3.1)$$

for a "continuous relaxation spectrum", so now

$$\varphi(t) = \int_{0}^{\infty} [\mu(\lambda)/\lambda] \, e^{-t/\lambda} d\lambda = \int_{0}^{\infty} H(\lambda) \, e^{-t/\lambda} d\lambda \quad \text{instead of} \quad \varphi(t) = \Sigma_n [\varphi_n(t)]$$

where $H(\lambda)/\lambda$ is called the relaxation spectrum.

For small strain, $D = \partial_t E$ and you can write the above in many different ways as shown p 453. What now appears is $f(t) = d/dt(\varphi(t))$ and $f(t)$ is called the memory function, done with parts integration.

Along the way we get the famous "oscillating plate" experiment where you measure both the stress on the plate, and the response of the fluid (in terms of velocity and thus D). Actually, the plate has a displacement function $u_x(y,t) = u_0(y/h)e^{i\omega t}$ (plate is at y = h) from which you find D_{12} = a certain value. You then use the Maxwell ODE to solve for stress S_{12} and it is a complex number due to the phase shift of the fluid, and one writes $S_{12} = (u_0/h)\, G^* e^{i\omega t}$ and then it is G* which you measure in this experiment. But then Lai shows the theoretical expressions for $G^*(\omega) = G' + iG''$ as integrals of the above $H(\lambda)$ function. From experimental data on G for synovial fluid, he shows how you can reverse these integrals numerically to find $H(\lambda)$ where recall λ is time. This data is page 456.

Part B: Nonlinear Viscoelastic Fluid (456)

8.6 Current configuration as the reference configuration (456)

I do this much better in my tensor doc Appendix K with nice pictures etc.

8.7 The Relative Deformation Gradient (457)

This thing is defined in terms of the new "configuration" picture, he writes it in a few coordinate systems, nothing too exciting now that I understand it all.

8.8 The Relative "other tensors" (459)

All in Appendix K.

8.9 The Relative C tensor in various coordinates (460)

He writes C in various coordinate systems, but this is all stuff we have done earlier in the book back in that horrendous Sectoin 3.29.

8.10 "Deformation History" and the Rivlin-Ericksen Tensors (463)

These A_n tensors are computed for some simple flow situations, some in curvilinear coordinates.

8.11 Relating A_1 to D; a recursion formula for the A_N (468)

Note Added: I have done all this now in full detail in "RE tensors A_n.doc".

8.12 Some tensor relations involving F_t and (∇v) and the polar decomposition (471)

I sort of missed this section. Basically he shows that

$$[d_\tau F_t(\mathbf{x},\tau)]^{\tau=t} = d_t F_t = \nabla_{\mathbf{x}}[\mathbf{v}(\mathbf{x},t)] = (\nabla v) = D + W$$

and if you use the polar decomposition for F_t you find $D = d_t U_t$ and $W = d_t R_t$. I did not put this into tensor doc App K because I was more interested in d_t applied to a tensor that was objective, and F_t is not objective. Not surprising that the spin tensor W is associated with the R_t part of F_t. I suppose it is possible that $d_t F_t$ is objective, I would have to try it. Lai makes no comments at all on this.

8.13 Change of Frame Stuff (471)

This is what I show now in App K concerning which "relative" tensors are really tensors and which are not.

8.14 Change of Frame for the A tensors (474)

This tiny section just shows that the A tensors are objective relative to Q(t), nothing more or less.

8.15 Incompressible Simple Fluid (474)

This is the idea that

$$S(t) = 2 \int_{-\infty}^{t} d\tau \text{ [some kernel] } C_t(\tau)$$

which I now mentino in App K.

8.16 Two specific models which reduce to Maxwell for small deformation (475)

Here Lai produces two different constitutive models using the above simple form which in the limit of small strain become linear Maxwell fluids. The names Tanner and Simmons (I think 1967) appear.

8.17 Single-integral non-linear constitutive equations (478)

This is another single-integral $C_t(\tau)$ constitutive equation but we have both C_t and C_t^{-1} terms so he claims this is as general as you can get. The BKZ fluid of 1963 fits in here.

I forgot to mention above that every model makes a prediction for viscosity $\mu(k)$ where k is the simple shear parameter. For Newtonian we of course have μ = constant. The BKZ model has its own prediction. In this model it is possible to have μ be large at small k, and then drop off dramatically. That is what synovial fluid in fact does, as shown page 480.

8.18 Differential-type constitutive equations for incompressible fluid (481)

This has the little list of models I put in Appendix K, like second order fluid.

8.19 Objective Rate of Stress (483)

This covers the three $d_t J$ derivatives as in Appendix K.

8.20 Rate-type constitutive equations (487)

This is the fancier list of rate-type fluids involving these fancier derivatives.

Part C: Viscometric flow for incompressible simple fluid (491)

8.21 Viscometric Flow (491)

I should have put this into my list -- these have only A_1 and A_2 but there are also special requirements on the form that A_1 and A_2 can have! He shows that in fact many of our prototype flows are of this type. Simple shear, its generalization unidirectional flow, cylindrical pipe flow, Couette cylinders flow.

8.22 Form of T_{ij} for Viscometric Flow (493)

Lai shows that the flow can be characterized by just three functions page 495. He uses a reflection matrix Q to find some properties of these three functions $\tau(k)$, $\sigma_{1,2}(k)$ where $S_{12} = \tau(k)$. You might then try a polynomial for each function and thereby define some constant parameters.

8.23 Channel Flow (495)

This flow between parallel plates meets the viscometric requirement and it can be used measure $\tau(k)$ for a fluid.

8.24 Couette Flow (497)

The channel method is a bit impractical, so here things are redone with those concentric cylinders, which we know already is viscometric.

Appendix 8.1

This is where Lai does his (∇T) expansion in curvilinears, which is my Appendix J.