

Ch. 4 Reynolds Transport Theorem

Representing Fluids Systems as Control Volumes

Control Volume and System Representations

□ Applying fundamental physical laws to fluids

■ **Systems approach**

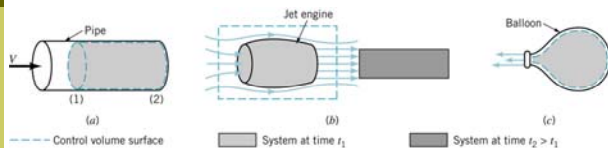
A system is a collection of matter of fixed identity (always the same atoms, fluids, etc). They may move, flow, interact, etc.

■ **Control volume approach**

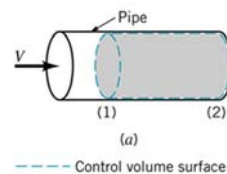
A control volume is volume in space (a geometric entity, independent of mass) through which a fluid may flow.

Control Volume and System Representations

EXAMPLES of **CONTROL VOLUMES**

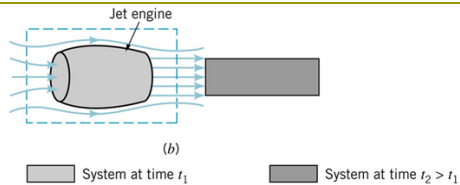


Control Volume and System Representations



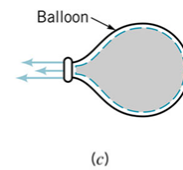
Case a: Fluid flows through a pipe with a fixed **control surface**. The inside surface of the pipe, the outlet end at section (2), and a section across the pipe at section (1). Fluid flows across part of the control surface.

Control Volume and System Representations



Case b: If the plane is moving, the control volume is fixed relative to the observer on the plane; however it is a moving control volume relative to an observer on the ground.

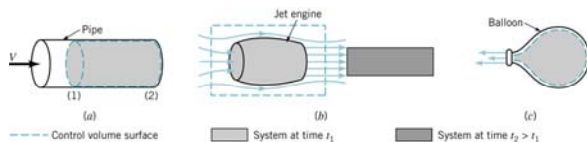
Control Volume and System Representations



Case c: A deforming control volume. If we do not hold on to the balloon, it becomes a moving control volume.

Control Volume and System Representations

EXAMPLES of **CONTROL VOLUMES**



MOST problems in fluid mechanics can be solved using a fixed, non-deforming control volume.

The governing laws of fluid motion are stated in fluid systems, not control volumes.

Reynolds Transport Theorem

- Laws governing fluid motion are stated in terms of a system approach.

For example the “mass of a system remains constant.” Note the word system, not control volume, in these statements.

- **Reynolds Transport Theorem** allows us to rephrase these laws in terms of control volumes.

Reynolds Transport Theorem

- Let B be any physical fluid parameter. (velocity, mass, density, temperature)
- b is the amount of that parameter per unit mass so that $B = m b$, where m is the mass of the portion of the fluid of interest.
- B is an **extensive property** (FUNCTION of the amount (quantity of mass))
- b is an **intensive property** (INDEPENDENT of the amount of mass)

Reynolds Transport Theorem



B	$b = B/m$
m	1
mV	V
$\frac{1}{2}mV^2$	$\frac{1}{2}V^2$

EXAMPLES.

IF $B = mV^2/2$, the kinetic energy of the mass, then

$b = V^2/2$, the kinetic energy per unit mass.

PARAMETER B is an extensive parameter (depends upon mass)

PARAMETER b is an intensive parameter (does not depend upon mass)

Reynolds Transport Theorem

The amount of an **extensive property** that a system possesses at a given instant, B_{sys} , can be determined by summing the amount associated with each fluid particle in the system.

For particles of size δV and mass $\rho \delta V$, this summation (in the limit of $\delta V \rightarrow 0$) takes the form of an integration over all the particles in the system and can be expressed as:

$$B_{sys} = \lim_{\delta V \rightarrow 0} \sum_i b_i (\rho_i \delta V_i) = \int_{sys} \rho b dV$$

Reynolds Transport Theorem

The limits of integration cover the entire system – a (usually moving volume). **UNDERSTAND** that we have used the fact that the amount of B (any **extensive property**) in a fluid particle of mass $\rho \delta V$ is given in terms of b by $\delta B = b \rho \delta V$.

Reynolds Transport Theorem

- Most laws governing fluid motion involve the time rate of change of an extensive property in a system.

$$\frac{dB_{\text{sys}}}{dt} = \frac{d\left(\int_{\text{sys}} \rho b dV\right)}{dt}$$

- The corollary for the laws for a control volume can be written as the time rate of change of an extensive property in a control volume.

$$\frac{dB_{\text{cv}}}{dt} = \frac{d\left(\int_{\text{cv}} \rho b dV\right)}{dt}$$

Reynolds Transport Theorem

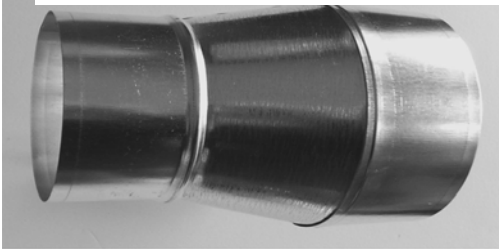
$$\frac{dB_{\text{sys}}}{dt} \text{ does not necessarily equal } \frac{dB_{\text{cv}}}{dt}$$

Even if they temporarily occupy the same volume in space.

Reynolds transport theorem provides a relationship between the time rate of change of an extensive property of a system and that of a control volume.

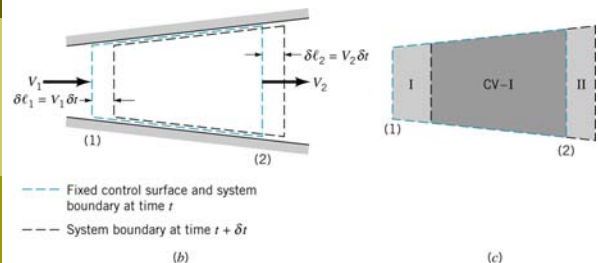
Control Surface and System Boundary

CENTER SECTION OF THE FIGURE



(a)

Control Surface and System Boundary



Reynolds Transport Theorem

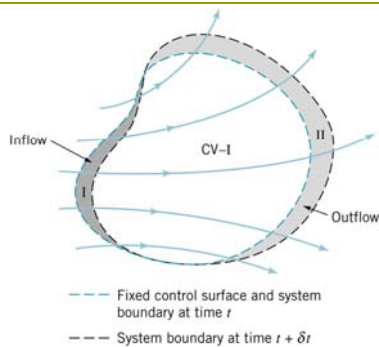
$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \dot{B}_{out} - \dot{B}_{in} \quad \text{EQ 4.14}$$

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_1 A_1 V_1 b_1 \quad \text{EQ 4.15}$$

Restrictive Assumptions

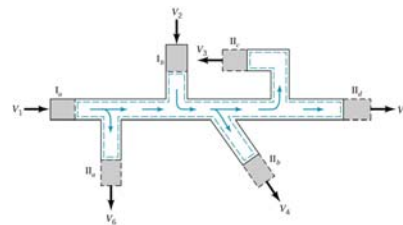
1. Fixed control volume with one inlet and one outlet.
2. Uniform properties (density, velocity, and the parameter ***b*** across both inlet and outlet.
3. Velocity normal to the exit sides.

Generalizing Reynolds Transport Theorem



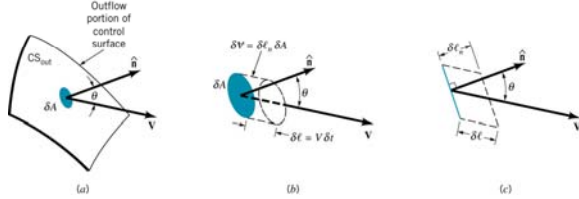
Generalizing Reynolds Transport Theorem

To generalize we must give the correct interpretation to $\dot{B}_{out}$ and $\dot{B}_{in}$. The control volume may contain more (or less) than one inlet or one outlet.



Generalizing Reynolds Transport Theorem

For $\dot{B}_{out}$, $\delta V = \delta \ell_n \delta A$, $\delta \ell_n = \delta \ell \cos \theta$
 $\delta V = V \cos \theta \delta t \delta A$



$$\delta B = b(\rho \delta V) = b\rho[(V \cos \theta \delta t) \delta A]$$

The rate at which B is carried out of the control volume across δA is denoted $\delta \dot{B}_{out}$

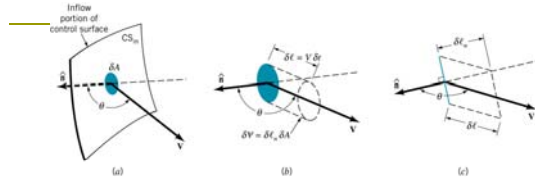
$$\delta \dot{B}_{out} = \lim_{\delta t \rightarrow 0} \frac{\rho b \delta V}{\delta t} = \lim_{\delta t \rightarrow 0} \frac{(\rho b V \cos \theta \delta t) \delta A}{\delta t} = \rho b V \cos \theta \delta A$$

$$\dot{B}_{out} = \int_{CS_{out}} \delta \dot{B}_{out} = \int_{CS_{out}} \rho b V \cos \theta dA$$

$$V \cos \theta = \vec{V} \cdot \hat{n}$$

$$\dot{B}_{out} = \int_{CS_{out}} \rho b \vec{V} \cdot \hat{n} dA$$

Generalizing Reynolds Transport Theorem

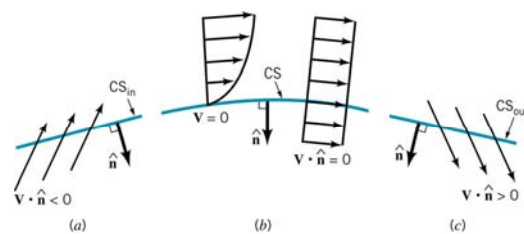


$$\dot{B}_{in} = - \int_{CS_{in}} \rho b V \cos \theta \delta A = - \int_{CS_{in}} \rho b \vec{V} \cdot \hat{n} \delta A$$

$$\dot{B}_{out} - \dot{B}_{in} = \int_{CS_{out}} \rho b \vec{V} \cdot \hat{n} \delta A - (- \int_{CS_{in}} \rho b \vec{V} \cdot \hat{n} dA)$$

$$= \int_{CS} \rho b \vec{V} \cdot \hat{n} \delta A$$

Generalizing Reynolds Transport Theorem



General form of Reynolds Transport Theorem

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \dot{B}_{out} - \dot{B}_{in}$$

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \int_{cs} \rho b \vec{V} \cdot \hat{n} \delta A$$

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \left(\int_{cv} \rho b dV \right) + \int_{cs} \rho b \vec{V} \cdot \hat{n} \delta A$$

EQ 4.19

What does it mean?

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \left(\int_{cv} \rho b dV \right) + \int_{cs} \rho b \vec{V} \cdot \hat{n} \delta A$$

The time rate of change of some arbitrary extensive property of a system (i.e. mass, momentum, energy), **DEPENDS** upon the choice of **B**.

Net flux of **B** over the control surface. Could be positive, negative or zero.

Rate of change of **B** within the control volume as the fluid flows through it.

Steady Effects

For steady flow, the amount of the property **B** within the **control volume** does **NOT** change with time.

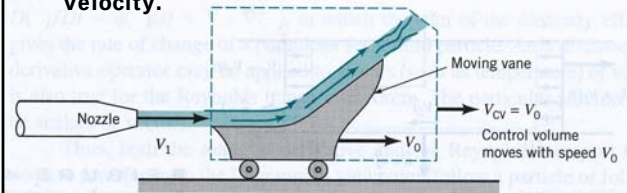
The amount of the property **B** associated with the **system** may or not may change with time depending on the property.

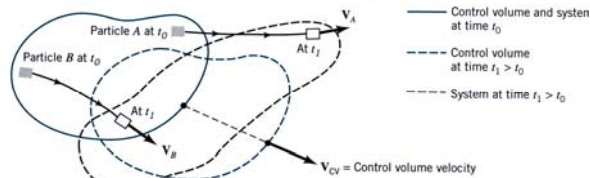
$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \left(\int_{cv} \rho b dV \right) + \int_{cs} \rho b \vec{V} \cdot \hat{n} \delta A$$

$$\frac{DB_{sys}}{Dt} = \int_{cs} \rho b \vec{V} \cdot \hat{n} \delta A$$

Moving Control Volumes

- Most problems will involve fixed control volumes.
- Some problems may be simplified if the c.v. is allowed to move or deform.
- Most of the time we use a nondeforming control volume that moves with a constant velocity.





■ FIGURE 4.21 Typical moving control volume and system.

The control volume translates at a constant velocity, V_{cv} .

IN general, the velocity of the control volume and the velocity of the fluid are not the same, so that there is flow of fluid through the moving control volume, just as in the stationary case.

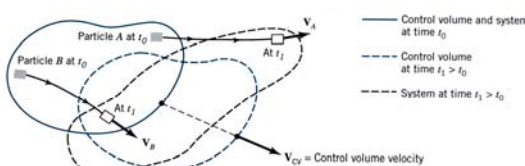
The MAIN difference between the fixed and the moving control volume cases is that the **relative velocity**, (W), that carries the fluid across the moving control surface, whereas it is the **absolute velocity** (V) that carries the fluid across the fixed control surface. BOTH V and W are vectors!

The **absolute velocity** (V) is the fluid velocity as seen by a stationary observer in a fixed coordinate system.

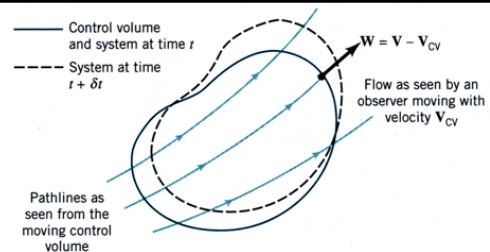
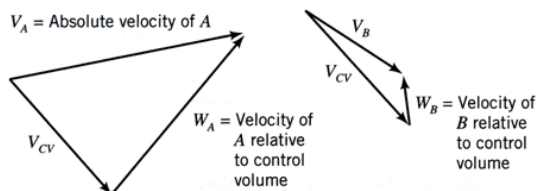
The **relative velocity** (W) is the fluid velocity relative to the moving control volume – the fluid velocity as seen by an observer riding along on the control volume.

$$V = V_{cv} + W$$

Or you may remember this as $V_A = V_B + V_{B \text{ relative to } A}$



■ FIGURE 4.21 Typical moving control volume and system.



The **absolute velocity** (V) is the fluid velocity as seen by a stationary observer in a fixed coordinate system.

The **relative velocity** (W) is the fluid velocity relative to the moving control volume – the fluid velocity as seen by an observer riding along on the control volume.

Reynolds transport theorem for a control volume moving with a constant velocity.

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \left(\int_{cv} \rho b dV \right) + \int_{cs} \rho b \vec{W} \cdot \hat{n} \delta A$$

Selecting control volumes

- Locate points for which we want parameters (ex. p, V, F) on the control surface.
- The control surface should be normal to the fluid velocity so θ in

$$V \cdot \hat{n} = V \cos \theta$$

is either zero or 180 degrees.



UTILIZING THE REYNOLDS TRANSPORT THEOREM TO DEVELOP CONSERVATION LAWS APPLICABLE TO FLUIDS

- CONSERVATION OF MASS
- CONSERVATION OF MOMENTUM
- CONSERVATION OF ENERGY

ALL OF WHICH LEAD TO FLOW IN PIPES, OPEN CHANNELS, TURBINES AND PUMPS AND ENGINEERING DESIGN OF THESE COMPONENTS.