

Chapter 4

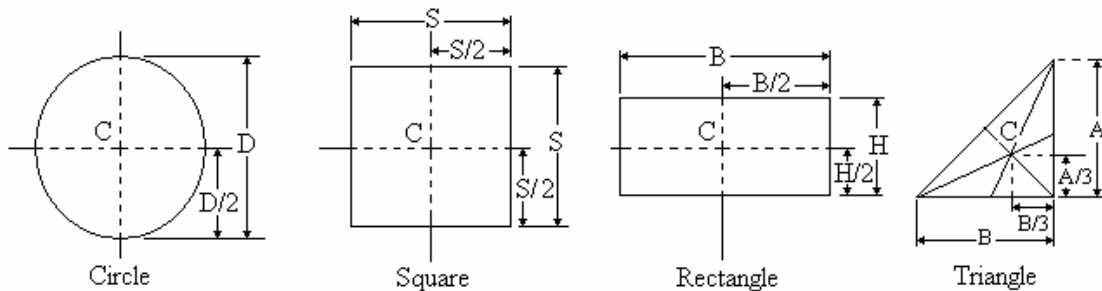
Pure Bending

- 1) Center of gravity and neutral axis.
- 2) Moment of inertia.
- 3) Theory of simple bending.
- 4) Bending of composite or flitched beam.
- 5) Combined bending and direct stress.

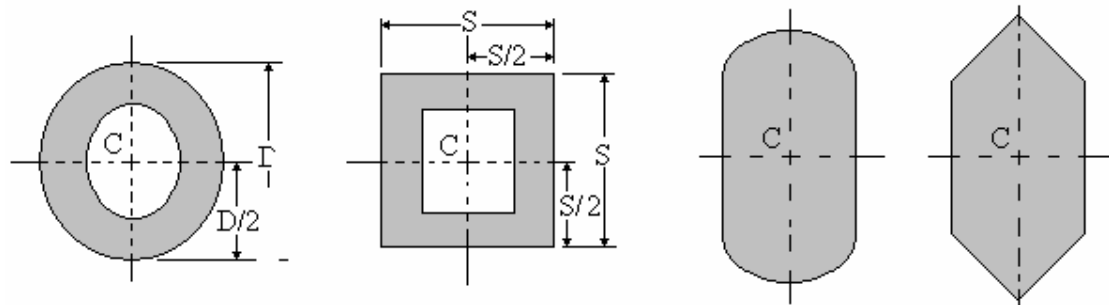
1) Center of Gravity

Centroid of an area is the point about which the area could be balanced if it was supported from that point.

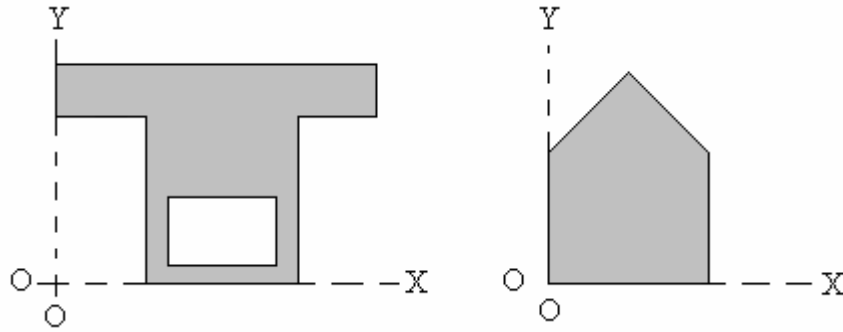
a) Simple shapes



b) Area has an axis of symmetry



c) Two axis of symmetry do not occur



$$A_T \bar{y} = \sum A_i y_i$$

$$\bar{y} = \frac{\sum A_i y_i}{A_T}$$

Where A_T = total area of the composite shape
 $\bar{y}$ = distance to the centroid of the composite shape measured from some reference axis OX.
 A_i = area of one component part the shape.
 y_i = distance the centroid at the component part the reference axis.

Or

$$A_T \bar{x} = \sum A_i x_i$$

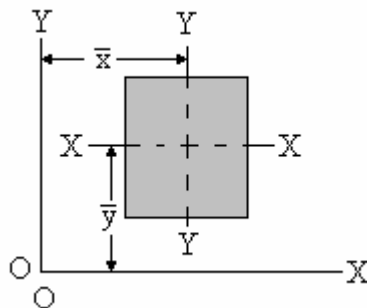
$$\bar{x} = \frac{\sum A_i x_i}{A_T}$$

Where $\bar{x}$ = distance to the centroid of the composite shape measured from some reference axis OY.
 x_i = distance the centroid at the component part the reference axis.

2) *Moment of Inertia*

Moment of inertia is a measure of the resistance of the section to applied moment or load that tends to bend it.

Moment of inertia depends on the shape and not material. It is a derived property.



$I_{xx} = \sum da.y^2 = \int y^2.dA$ is called the moment of inertia or the second moment of area about the axis XX.

$I_{yy} = \sum da.x^2 = \int x^2.dA$ is called the moment of inertia or the second moment of area about the axis YY.

Parallel axis Theorem

This theorem is mainly used to transfer the moment of inertia of a body from its individual axis to another reference line.

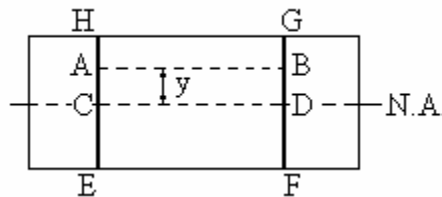
$$I_{OX} = I_{xx} + A.\bar{y}^2$$

$$I_{OY} = I_{yy} + A.\bar{x}^2$$

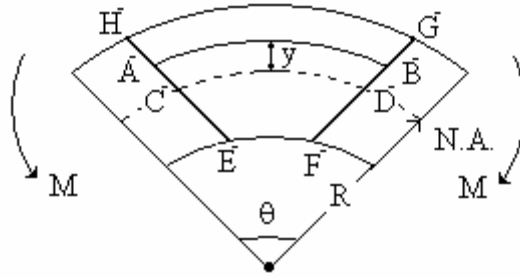
3) *Assumption for the Simple Bending Theory*

1. The beam is initially straight and unstressed.
2. The material is homogeneous, same density and elastic properties.
3. The elastic limit is nowhere exceeded.
4. Young's modulus for the materials is the same in tension and compression.
5. Plane cross-section remains plane before and after bending.
6. No resulted force perpendicular to any cross – section.

Bending theory



- If we apply a constant B.M.
- It will bend to radius R.
- The top fibers of the beam will be subjected to tension.
- The bottom fibers of the beam will be subjected to compression.
- Somewhere between the two there are points at which the stress is zero.
Locus of all points is termed the neutral axis.
The radius of curvature R is then measured to this axis.
The neutral axis will always pass through the centre of area of centroid.
- The maximum tension and compression will be at the outer surface or the farthest distance from the neutral axis.



Consider fiber AB distance y from the N.A. when the beam is bent this will stretch to $A'B'$.

$$\text{Strain in } AB = \frac{\text{Extension}}{\text{Original Length}} = \frac{A'B' - AB}{AB}$$

$$AB = CD$$

$$CD = C'D'$$

$$\text{Strain } \varepsilon = \frac{A'B' - C'D'}{C'D'}$$

$$C'D' = R\theta$$

$$A'B' = (R + y)\theta$$

$$\therefore \text{Strain } \varepsilon = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

Within the elastic region:

$$\text{Strain} = \frac{\text{Stress}}{\text{Young's Modulus}} = \frac{\sigma}{E}$$

$$\therefore \frac{\sigma}{E} = \frac{y}{R}$$

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = \frac{E}{R} y$$

If the strip of area δA

$$\sigma = \frac{E}{R} y$$

$$F = \sigma \cdot \delta A = \frac{E}{R} \cdot y \cdot \delta A$$

This force has a moment about the N.A of

$$F \cdot y = \frac{E}{R} \cdot y^2 \cdot \delta A$$

The total moment for the whole cross section is:

$$\sum F \cdot y = \sum \frac{E}{R} \cdot y^2 \cdot \delta A$$

$$M = \frac{E}{R} \sum y^2 \cdot \delta A$$

$\sum y^2 \cdot \delta A$ is the second moment of area of the cross-section.

$$\therefore M = \frac{E}{R} I$$

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

If the beam is of uniform section, the material of the beam is homogeneous and the applied moment is constant, I, E, M remain constant and hence the radius of curvature of the beam will also be constant.

$$R = \frac{E}{M} I$$

EI is known as the flexural rigidity.

- For larger value of $R \longrightarrow$ smaller deflection $\longrightarrow$ greater the rigidity.

$$\text{Maximum Stress } \sigma_{\max} = \frac{M}{I} y_{\max}$$

$$M = \frac{\sigma_{\max} I}{y_{\max}} = \frac{I}{y_{\max}} \sigma_{\max}$$

$\frac{I}{y_{\max}}$ is termed the section modulus Z .

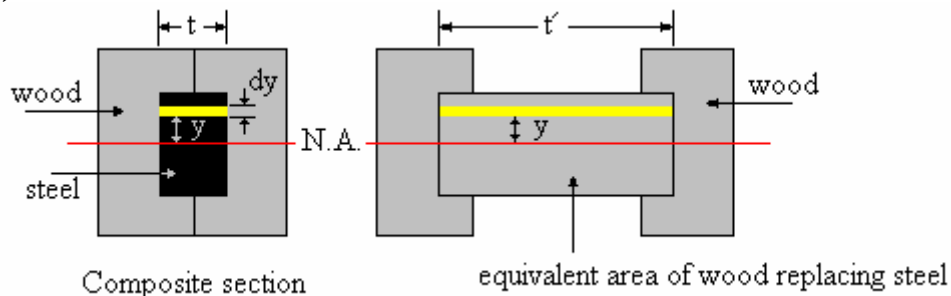
$$Z = \frac{I}{y_{\max}}$$

$$M = Z \sigma_{\max} \text{ for symmetrical beam}$$

$$\text{for unsymmetrical sections } Z_1 = \frac{I}{y_1} \text{ and } Z_2 = \frac{I}{y_2}$$

4) *Bending of Composite or Flitched Beams*

Composite beam is one which is constructed from a combination of material (flitched beam).



$$\text{Force}_{\text{steel}} = (\sigma \cdot dy \cdot t)_{\text{steel}} = (\sigma' \cdot dy \cdot t')_{\text{equivalent}}$$

Since the moment at any section must be the same in the equivalent section as in the original, so that the force at given dy in the equivalent beam must be equal to that at the strip it replaces.

$$\sigma \cdot t = \sigma' \cdot t'$$

$$\varepsilon \cdot E \cdot t = \varepsilon' \cdot E' \cdot t'$$

Strain must be equal

$$\varepsilon = \varepsilon'$$

$$E \cdot t = E' \cdot t'$$

$$\frac{t'}{t} = \frac{E}{E'}$$

$$t' = \frac{E}{E'} t$$

Thus to replace the steel strip by an equivalent wooden strip the thickness must be multiplied by the modular ratio $\frac{E}{E'}$.

$$\frac{\sigma}{\sigma'} = \frac{E}{E'} = \frac{t'}{t}$$

$$\sigma = \frac{E}{E'} \sigma'$$

Where: σ = the stress in the steel section.
 σ' = the stress in the equivalent section.
 t = the thickness of the steel section.
 t' = the thickness of the equivalent

5) *Combined Bending and Direct Stress – Eccentric Loading*

a) Eccentric loading on one axis

$$\sigma = \frac{P}{A} \mp \frac{M}{I_{xx}} y$$

$$M = P \cdot e$$

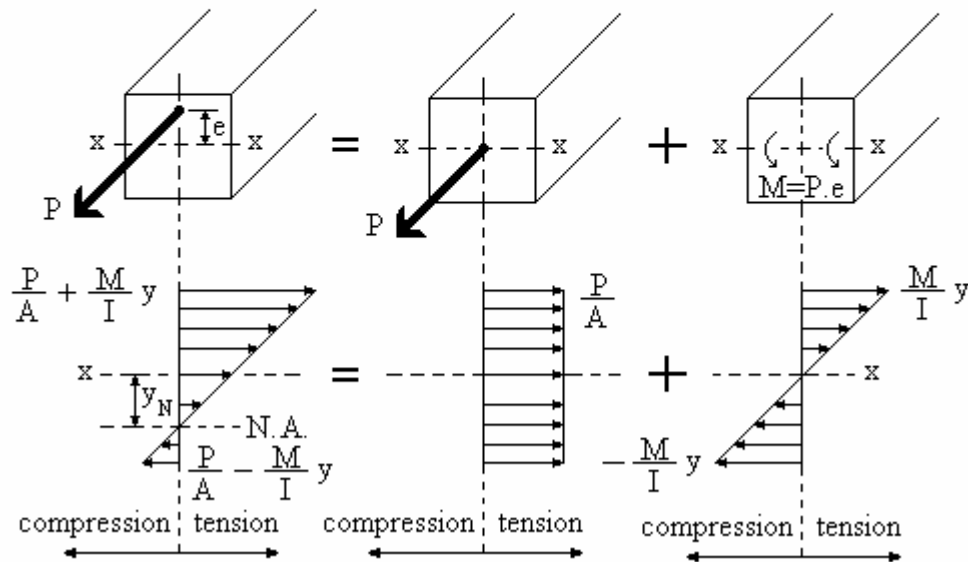
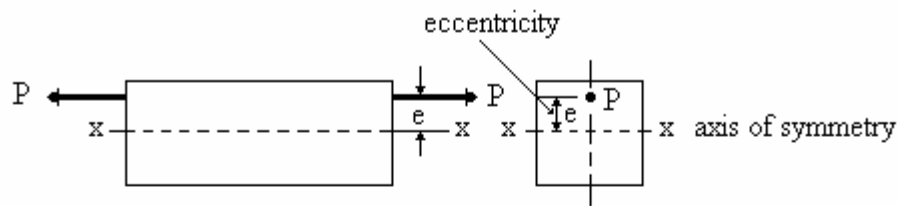
$$\sigma = \frac{P}{A} \mp \frac{P \cdot e}{I_{xx}} y$$

The equation of the N.A can be obtained by setting σ equal to zero.

$$0 = \frac{P}{A} \mp \frac{P \cdot e}{I_{xx}} y$$

$$\frac{P}{A} = \pm \frac{P \cdot e}{I_{xx}} y$$

$$y_N = \mp \frac{I_{xx}}{Ae}$$



The larger the eccentricity of the load the closer the N.A. will be to the axis of symmetry.

y_N is the distance between the N.A. and the axis of symmetry.

6) Eccentric loading on two axis

If the applied load will not be applied on either of the axis of symmetry, there will be a direct stress effect plus simultaneous bending about both axis.

The total stress at any point (x, y):

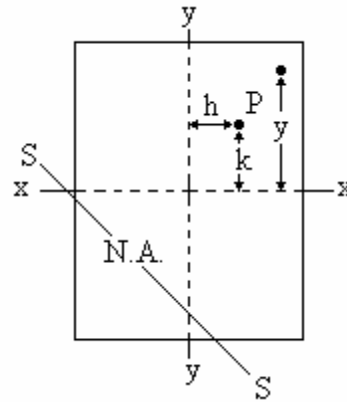
$$\sigma = \frac{P}{A} \mp \frac{P.h}{I_{yy}} x \mp \frac{P.k}{I_{xx}} y$$

Equation of the N.A. is obtained by equating to zero:

$$0 = \frac{P}{A} \mp \frac{P.h}{I_{yy}} x \mp \frac{P.k}{I_{xx}} y$$

$$1 = \pm \frac{A.h}{I_{yy}} x \pm \frac{A.k}{I_{xx}} y$$

This is linear equation in x and y. The line may or may not cut the section.



(Middle-Quarter) and (Middle-Third) Rules

For rectangular and circular cross sections, provided that the load is applied within certain defined areas, no tension will be produced whatever the magnitude of the applied compressive load.

