

FUNDAMENTALS OF FLUID MECHANICS

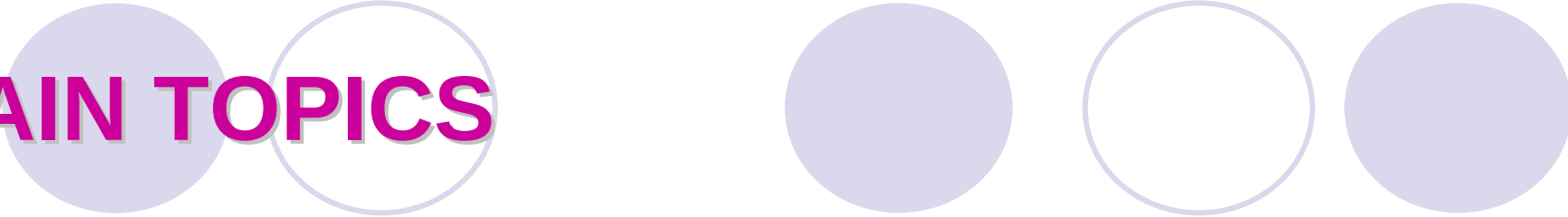
Chapter 5 Flow Analysis Using Control Volume

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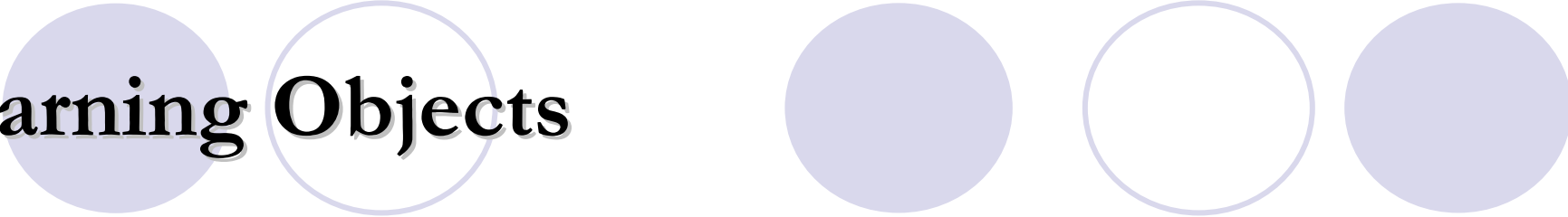
10/19/2009

MAIN TOPICS



- ❖ Conservation of Mass
- ❖ Newton's Second Law – The Linear Momentum Equations
- ❖ The Moment-of-Momentum Equations
- ❖ First Law of Thermodynamics – The Energy Equation
- ❖ Second Law of Thermodynamics – Irreversible Flow

Learning Objects

The title 'Learning Objects' is positioned on the left. To its right, there are five circles arranged horizontally. The first circle is solid light purple. The second circle is a light purple outline. The third circle is solid light purple. The fourth circle is a light purple outline. The fifth circle is solid light purple.

- ➔ Select an appropriate finite CV to solve a fluid mechanics problem.
- ➔ Apply basic laws to the contents of a finite CV to get important answers.

How to apply these basic laws?

How to express these basic laws based on CV method?

Review of Reynolds Transport Theorem

This is the fundamental relation between the rate of change of any arbitrary extensive property, B , of a system and the variations of this property associated with a control volume.

$$\begin{aligned}\frac{dB_{\text{sys}}}{dt} &= \frac{\partial B_{\text{CV}}}{\partial t} + \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n} dA \\ &= \frac{\partial}{\partial t} \int_{\text{CV}} \rho b dV + \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n} dA\end{aligned}$$

Conservation of Mass – The Continuity Equation 1/4

❖ Basic Law for Conservation of Mass

System method下，描述質量守恒的方程式

$$\left. \frac{dM}{dt} \right)_{\text{system}} = 0 \quad M_{\text{system}} = \int_{M(\text{system})} dm = \int_{V(\text{system})} \rho dV$$

❖ For a system and a fixed, nondeforming control volume that are coincident at an instant of time, the Reynolds Transport Theorem leads to

第一時間，兩者一致

B=M and b =1

$$\frac{d}{dt} \int_{\text{sys}} \rho dV \equiv \frac{\partial}{\partial t} \int_{\text{cv}} \rho dV + \int_{\text{cs}} \rho \vec{V} \cdot \vec{n} dA$$

Time rate of change
of the mass of the
coincident system

=

Time rate of change of the
mass of the content of the
coincident control volume

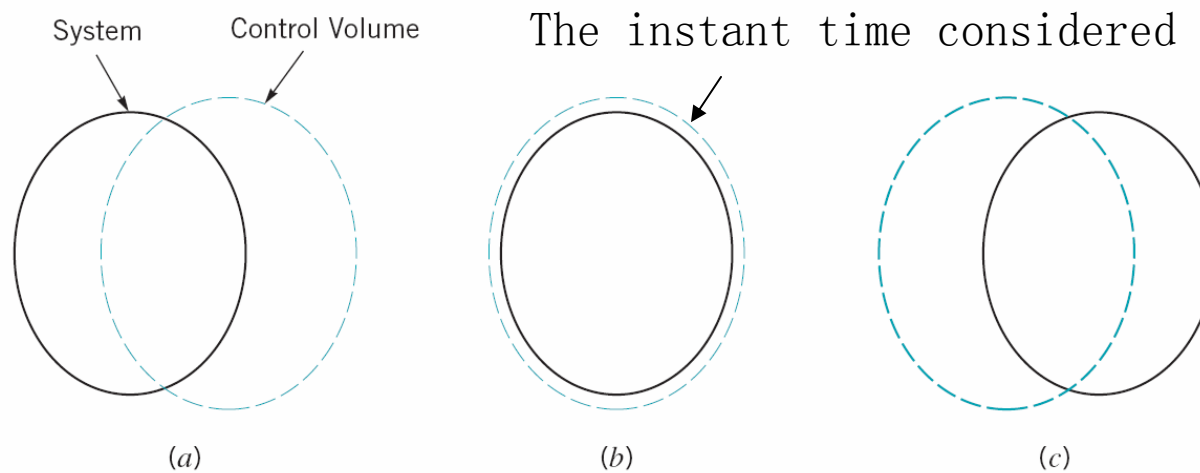
+

Net rate of flow of
mass through the
control surface

依據Chapter 4推導Reynolds transport theorem的步驟

Conservation of Mass – The Continuity Equation ^{2/4}

Chapter 4: system與CV在不同時間下的關係



System and control volume at three different instances of time.
(a) System and control volume at time $t - \delta t$. (b) System and control volume at time t , coincident condition. (c) System and control volume at time $t + \delta t$.

Conservation of Mass – The Continuity Equation 3/4

當CV是固定且不變形

❖ For a fixed, nondeforming control volume, the control volume formulation of the conservation of mass: The continuity equation

$$\left(\frac{dM}{dt} \right)_{\text{system}} \equiv \frac{\partial}{\partial t} \int_{\text{CV}} \rho dV + \int_{\text{CS}} \rho \vec{V} \cdot \vec{n} dA = 0$$

$\sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}}$

解讀結果

**Rate of increase
Of mass in CV** **==** **Net influx of
mass**

$$\frac{\partial}{\partial t} \int_{\text{CV}} \rho dV = - \int_{\text{CS}} \rho \vec{V} \cdot \vec{n} dA$$

Conservation of Mass – The Continuity Equation 4/4

Special case

❖ Incompressible Fluids

$$\rho \frac{\partial}{\partial t} \int_{CV} dV + \rho \int_{CS} \vec{V} \cdot \vec{n} dA = 0 \rightarrow \frac{\partial}{\partial t} \int_{CV} dV + \int_{CS} \vec{V} \cdot \vec{n} dA = 0$$

❖ For Steady flow

$$\sum \dot{m}_{out} - \sum \dot{m}_{in}$$

$$\int_{CS} \rho \vec{V} \cdot \vec{n} dA = 0$$

The mass flow rate into a control volume must be equal to the mass flow rate out of the control volume.

Other Definition

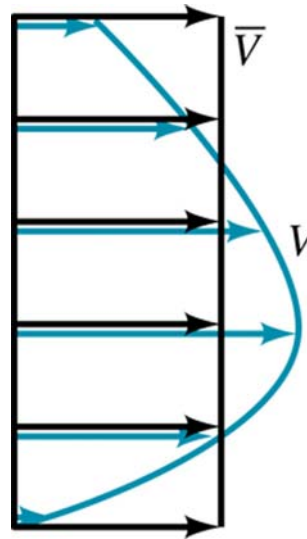
衍生定義

❖ Mass flowrate through a section of control surface

$$\dot{m} = \rho Q = \int_A \rho \vec{V} \cdot \vec{n} dA = \sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}}$$

❖ The average velocity

$$\bar{V} = \frac{\int_A \rho \vec{V} \cdot \vec{n} dA}{\rho A}$$



Fixed, Nondeforming Control Volume ^{1/2}

Special case

❖ When the flow is steady

$$\frac{\partial}{\partial t} \int_{cv} \rho dV = 0 \quad \longrightarrow \quad \sum \dot{m}_{out} = \sum \dot{m}_{in}$$

❖ When the flow is steady and incompressible

$$\sum \dot{Q}_{out} = \sum \dot{Q}_{in}$$

❖ When the flow is not steady

$$\frac{\partial}{\partial t} \int_{cv} \rho dV \neq 0$$

“+” : the mass of the contents of the control volume is increasing
“-” : the mass of the contents of the control volume is decreasing.

Fixed, Nondeforming Control Volume ^{2/2}

Special case

- ❖ When the flow is uniformly distributed over the opening in the control surface (one dimensional flow)

$$\dot{m} = \rho A V$$

- ❖ When the flow is nonuniformly distributed over the opening in the control surface

$$\dot{m} = \rho A \bar{V}$$

Example 5.1 Conservation of Mass – Steady, Incompressible Flow

- Seawater flows steadily through a simple conical-shaped nozzle at the end of a fire hose as illustrated in Figure E5.1. If the nozzle exit velocity must be at least 20 m/s, determine the minimum pumping capacity required in m^3/s .

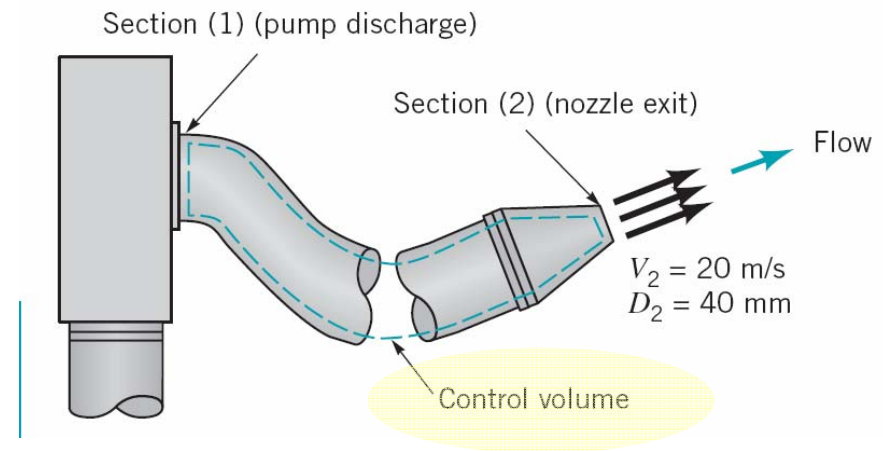


Figure E5.1

Example 5.1 Solution

The continuity equation

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \vec{V} \cdot \vec{n} dA = 0$$

Steady flow

$$\int_{cs} \rho \vec{V} \cdot \vec{n} dA = \dot{m}_2 - \dot{m}_1 = 0 \quad \text{or} \quad \dot{m}_2 = \dot{m}_1$$

$$\rho_1 Q_1 = \rho_2 Q_2$$

With incompressible condition

$$\rho_1 = \rho_2 \Rightarrow Q_1 = Q_2 = V_2 A_2 = \dots = 0.0251 \text{ m}^3 / \text{s}$$

求minimum pumping capacity

Example 5.2 Conservation of Mass – Steady, Compressible Flow

- Air flows steadily between two sections in a long, straight portion of 4-in. inside diameter as indicated in Figure E5.2. The uniformly distributed temperature and pressure at each section are given. If the average air velocity (Nonuniform velocity distribution) at section (2) is 1000ft/s, calculate the average air velocity at section (1).

求section (1) 平均速度

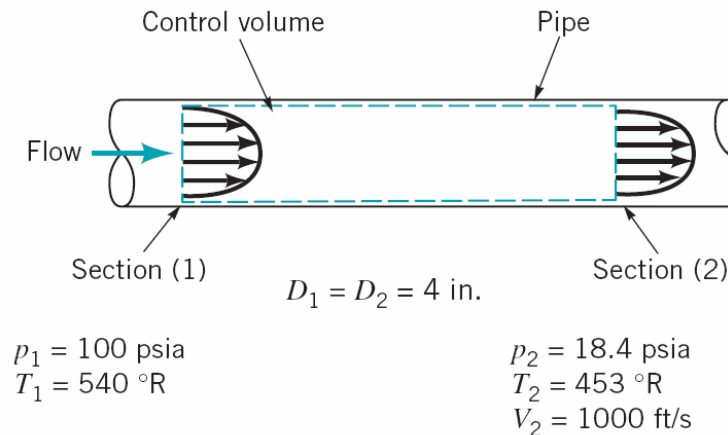


Figure E5.2

Example 5.2 Solution

The continuity equation

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \vec{V} \cdot \vec{n} dA = 0$$

Steady flow

$$\int_{cs} \rho \vec{V} \cdot \vec{n} dA = \dot{m}_2 - \dot{m}_1 = 0 \Rightarrow \dot{m}_2 = \dot{m}_1$$

$$\rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2$$

假設氣體為理想氣體

$$\text{Since } A_1 = A_2 \Rightarrow \bar{V}_1 = \frac{\rho_2}{\rho_1} \bar{V}_2 \Rightarrow \bar{V}_1 = \frac{p_2 T_1}{p_1 T_2} \bar{V}_2 = \dots 219 \text{ ft/s}$$

The ideal gas equation

$$\rho = \frac{p}{RT}$$

Example 5.3 Conservation of Mass – Two Fluids

- Moist air (a mixture of dry air and water vapor) enters a dehumidifier at the rate of 22 slugs/hr. Liquid water drains out of the dehumidifier at a rate of 0.5 slugs/hr. Determine the mass flowrate of the dry air and the water vapor leaving the dehumidifier.

求出口處的mass flowrate

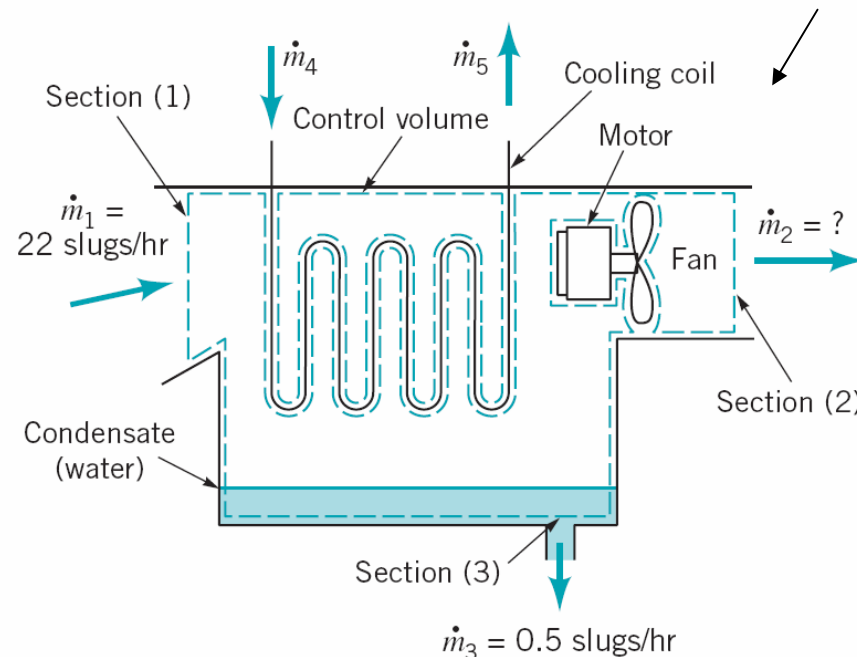
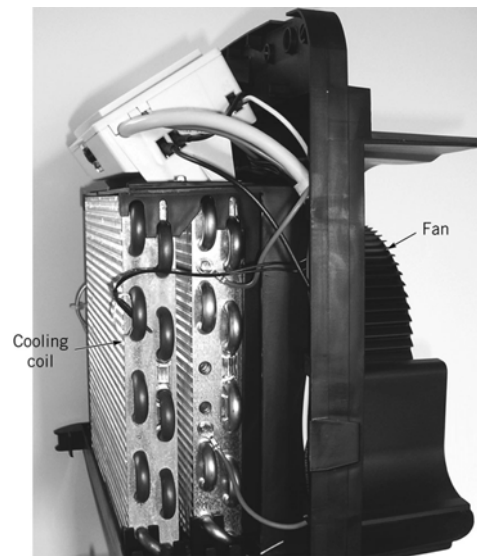


Figure E5.3 16

Example 5.3 Solution

The continuity equation

Steady flow

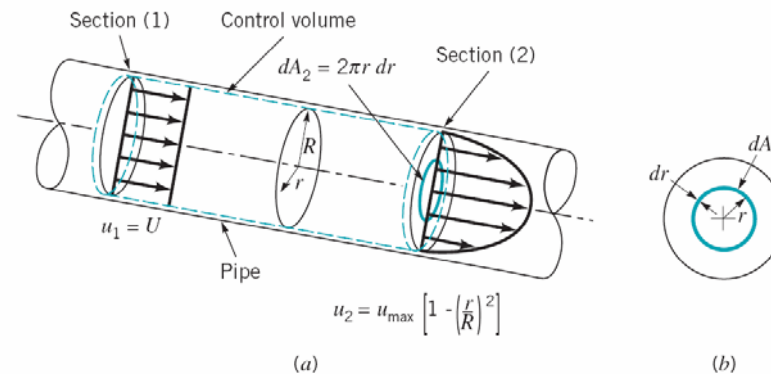
$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \vec{V} \cdot \vec{n} dA = 0$$

$$\rho \int_{cs} \vec{V} \cdot \vec{n} dA = -\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = 0$$

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 22 \text{ slugs / hr} - 0.5 \text{ slugs / hr} = 21.5 \text{ slugs / hr}$$

Example 5.4 Conservation of Mass – Nonuniform Velocity Profiles

- Incompressible, laminar water flow develops in a straight pipe having radius R as indicated in Figure E5.4. At section (1), the velocity profile is uniform; the velocity is equal to a constant value U and is parallel to the pipe axis everywhere. At section (2), the velocity profile is axisymmetric and parabolic, with zero velocity at the pipe wall and a maximum value of $u_{\max}$ at the centerline. How are U and $u_{\max}$ related? How are the average velocity at section (2), $\bar{V}_2$, and $u_{\max}$ related?



Example 5.4 Solution

The continuity equation

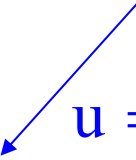
Steady flow

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \vec{V} \cdot \vec{n} dA = 0$$

$$-\rho_1 A_1 U + \int_{A_2} \rho \vec{V} \cdot \vec{n} dA = 0 \quad -\rho_1 A_1 U + \rho_2 \int_0^R u_2 2\pi r dr = 0$$

With incompressible condition $\rho_1 = \rho_2$


$$-A_1 U + 2\pi u_{\max} \int_0^R \left[1 - \left(\frac{r}{R} \right)^2 \right] r dr = 0$$


$$u = u_{\max} \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

$$u_{\max} = 2U \quad \bar{V}_2 = u_{\max} / 2$$

Example 5.5 Conservation of Mass – Unsteady Flow

- A bathtub is being filled with water from a faucet. The rate of flow from the faucet is steady at 9 gal/min. The tub volume is approximated by a rectangular space as indicate Figure E5.5a. Estimate the time rate of change of the depth of water in the tub, $\partial h / \partial t$, in in./min at any instant.

求水深的時間改變率

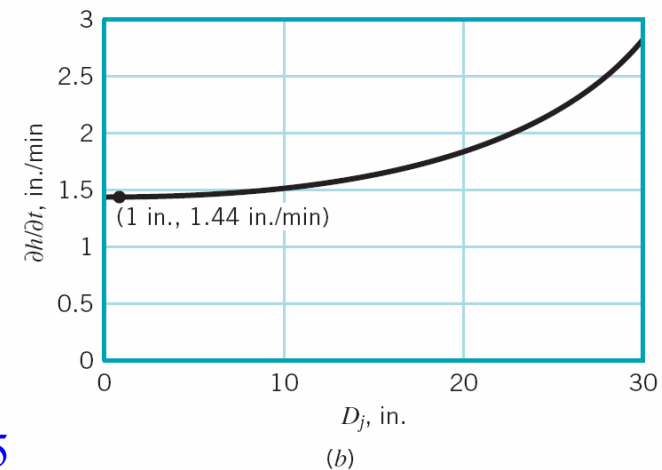
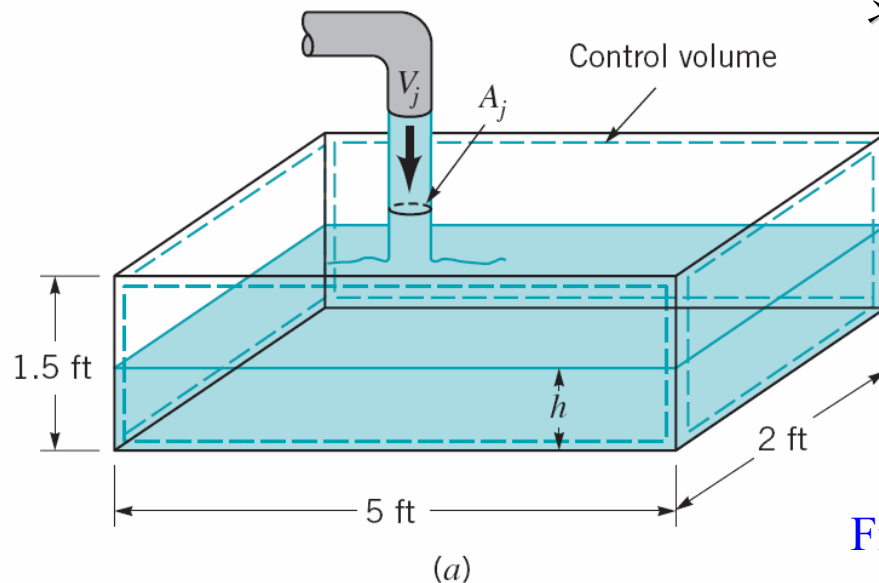


Figure E5.5

Example 5.5 Solution^{1/2}

The continuity equation

$$\frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{CS} \rho \vec{V} \cdot \vec{n} dA = 0$$

$$= \frac{\partial}{\partial t} \int_{\text{air volume}} \rho_{\text{air}} dV_{\text{air}} + \frac{\partial}{\partial t} \int_{\text{water volume}} \rho_{\text{water}} dV_{\text{water}} - \dot{m}_{\text{water}} + \dot{m}_{\text{air}}$$

$$\text{For air } \frac{\partial}{\partial t} \int_{\text{air volume}} \rho_{\text{air}} dV_{\text{air}} + \dot{m}_{\text{air}} = 0$$

Example 5.5 Solution^{2/2}

For water $\frac{\partial}{\partial t} \int_{\text{water volume}} \rho_{\text{water}} dV_{\text{water}} = \dot{m}_{\text{water}}$

CV內水的體積 $\int_{\text{water volume}} \rho_{\text{water}} dV_{\text{water}} = \rho_{\text{water}} [h(2\text{ft})(5\text{ft}) + (1.5\text{ft} - h)A_j]$

$$\Rightarrow \rho_{\text{water}} (10\text{ft}^2 - A_j) \frac{\partial h}{\partial t} = \dot{m}_{\text{water}}$$

$$\frac{\partial h}{\partial t} = \frac{Q_{\text{water}}}{(10\text{ft}^2 - A_j)} = \frac{(9\text{gal} / \text{min})(12\text{in.} / \text{ft})}{(7.48\text{gal} / \text{ft}^3)(10\text{ft}^2)}$$

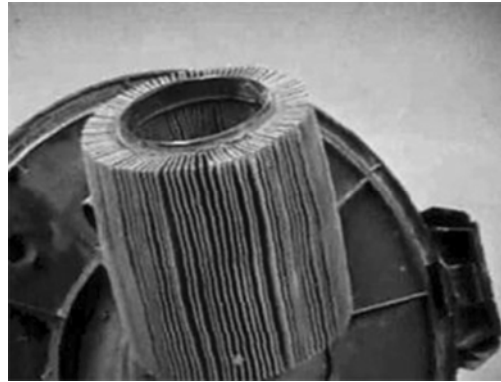
$A_j \ll 10\text{ft}^2$

Films

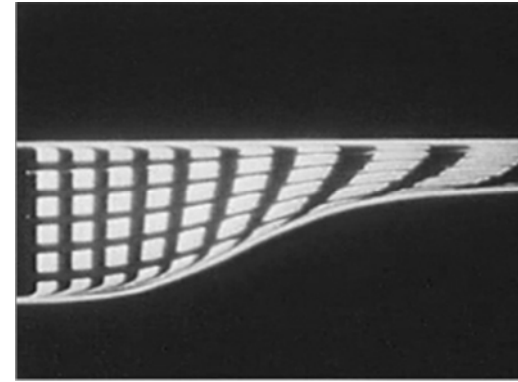
Sink flow



Vacuum filter



Flow through a contraction



Moving, Nondeforming Control Volume^{1/2}

CV是移動，不變形

❖ When a moving control volume is used, the fluid velocity relative to the moving control is an important variable.

⇒ $\mathbf{W}$ is the relative fluid velocity seen by an observer moving with the control volume. 觀察者站在CV上看到的流體速度

⇒ $\mathbf{V}_{cv}$ is the control volume velocity as seen from a fixed coordinate system. 站在固定座標看到的CV移動速度

⇒ $\mathbf{V}$ is the absolute fluid velocity seen by a stationary observer in a fixed coordinate system.

站在固定
座標看到
的流體速
度（絕對）

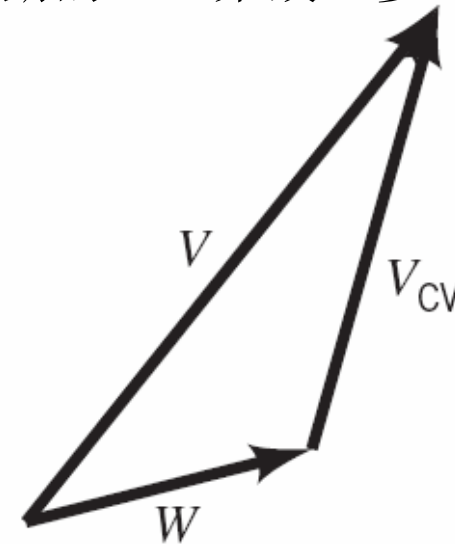
Moving, Nondeforming Control Volume^{2/2}

$$\vec{V} = \vec{W} + \vec{V}_{cv} \quad \frac{dM_{sys}}{dt} = \frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{c.s.} \rho \vec{W} \cdot \vec{n} dA$$

$$\Rightarrow \frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{c.s.} \rho \vec{W} \cdot \vec{n} dA = 0$$

在移動的CV上架設一參考座標系

Velocities seen from the control volume reference frame (relative velocities)



Example 5.6 Conservation of Mass - Compressible Flow with a Moving Control Volume

- An airplane moves forward at speed of 971 km/hr as shown in Figure E5.6a. The frontal intake area of the jet engine is 0.80 m^2 and the entering air density is 0.736 kg/m^3 . A stationary observer determines that relative to the earth, the jet engine exhaust gases move away from the engine with a speed of 1050 km/hr. The engine exhaust area is 0.558 m^2 , and the exhaust gas density is 0.515 kg/m^3 . Estimate the mass flowrate of fuel into the engine in kg/hr.

Determine the mass flowrate of fuel into the engine in kg/hr

CV選在移動的引擎

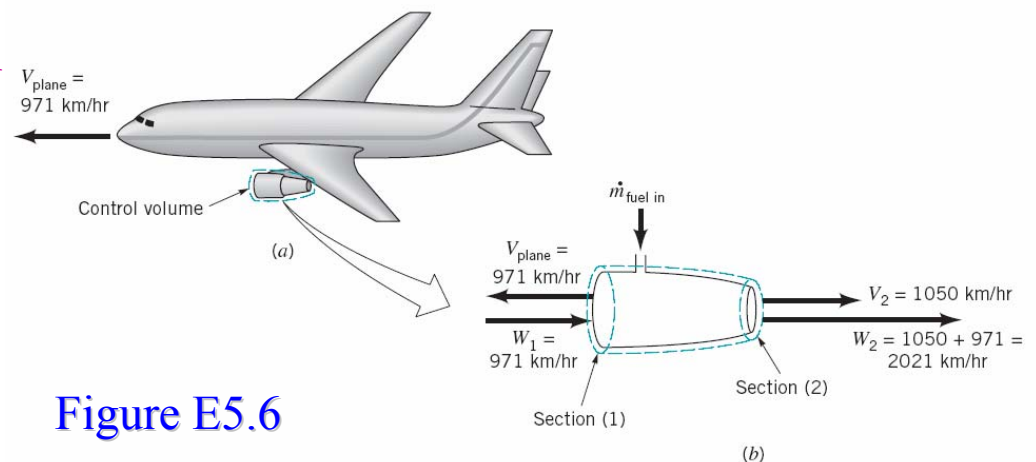


Figure E5.6

Example 5.6 Solution

The continuity equation

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{c.s.} \rho \vec{W} \cdot \vec{n} dA = 0$$

The intake velocity, W_1 , relative to the moving control volume. The exhaust velocity, W_2 , also needs to be measured relative to the moving control volume.

W_1 是從 CV 觀察到的進氣速度
 W_2 是從 CV 觀察到的排氣速度

Assuming one-dimensional flow

$$-\dot{m}_{\text{fuel in}} - \rho_1 A_1 W_1 + \rho_2 A_2 W_2 = 0$$

$$\rightarrow \dot{m}_{\text{fuel in}} = \rho_2 A_2 W_2 - \rho_1 A_1 W_1$$

$$W_2 = V_2 - V_{\text{plane}} = 1050 \text{ km/hr} + 971 \text{ km/hr} = 2021 \text{ km/hr}$$

$$\rightarrow \dot{m}_{\text{fuel in}} = (0.515 \text{ kg/m}^3)(0.558 \text{ m}^2)(2021 \text{ km/hr})(1000 \text{ m/km}) - \dots = 9100 \text{ kg/hr}$$

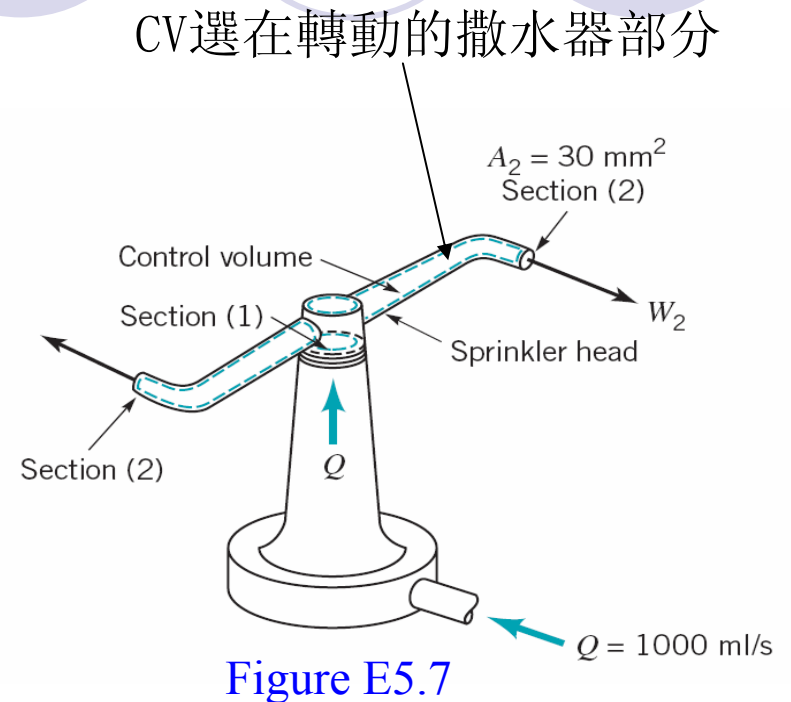
在地面觀察者看到的排氣離開引擎的速度

飛機飛行速度

W_2 是從 CV 觀察到的排氣速度

Example 5.7 Conservation of Mass - Relative Velocity

- Water enters a rotating lawn sprinkler through its base at the steady rate of 1000 ml/s as sketched in Figure E5.7. If the exit area of each of the two nozzles is 30 mm^2 , determine the average speed of the water leaving each nozzle, relative to the nozzle, if (a) the rotary sprinkler head is stationary, (b) the sprinkler head rotates at 60 rpm, and (c) the sprinkler head accelerates from 0 to 600 rpm.



Determine the average speed of the water leaving each nozzle, relative to the nozzle...

CV跟著轉動

Example 5.7 Solution

W是相對於轉動的Nozzle的速度

The continuity equation

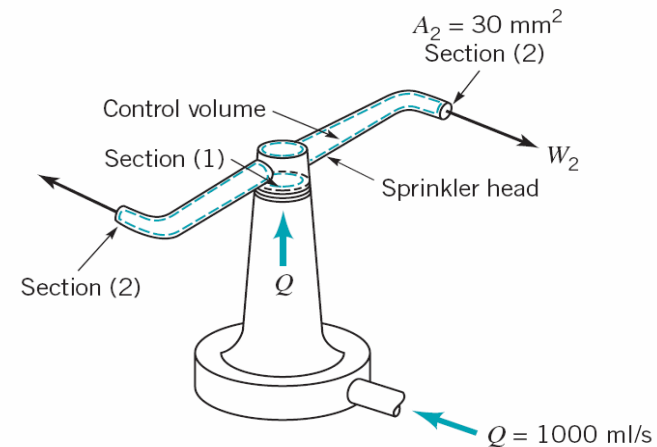
$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{c.s.} \rho \vec{W} \cdot \vec{n} dA = 0$$

$$\int_{c.s.} \rho \vec{W} \cdot \vec{n} dA = -\dot{m}_{in} + \dot{m}_{out} = 0$$

$$\dot{m}_{out} = 2\rho A_2 \overline{W}_2 = \dot{m}_{in} = \rho Q$$

$$\overline{W}_2 = \frac{Q}{2A_2} = \frac{(1000\text{ml/s})(0.001\text{m}^3/\text{liter})(10^6\text{mm}^2/\text{m}^2)}{(1000\text{ml/liter})(2)(30\text{mm}^2)} = 16.7\text{m/s} = W_2$$

The value of W_2 is independent of the speed of rotation of the sprinkler head and represents the average speed of the water exiting from each nozzle with respect to the nozzle for case (a), (b), (c).



Deforming Control Volume ^{1/2}

CV變形，CS當然就出現移動

- ❖ A deforming control volume involves changing volume size and control surface movement.
- ❖ The Reynolds transport theorem for a deforming control volume can be used for this case.

$$\frac{dM_{\text{sys}}}{dt} = \frac{\partial}{\partial t} \int_{\text{cv}} \rho dV + \int_{\text{c.s.}} \rho \vec{W} \cdot \vec{n} dA$$

$$\vec{V} = \vec{W} + \vec{V}_{\text{cs}}$$

V_{cs} is the velocity of the control surface as seen by a fixed observer.
 W is the relative velocity referenced to the control surface.

Deforming Control Volume 2/2

$$\frac{dM_{\text{sys}}}{dt} = \frac{\partial}{\partial t} \int_{\text{cv}} \rho dV + \int_{\text{c.s.}} \rho \vec{W} \cdot \vec{n} dA$$

通常不是零。因CV範圍與時俱變，必須妥善處理。

相對於變形的control volume表面的速度。CV表面速度並非每一點都相同。因此，相對速度自然較為複雜。

Example 5.8 Conservation of Mass – Deforming Control Volume ^{1/2}

- A syringe is used to inoculate a cow. The plunger has a face area of 500 mm^2 . If the liquid in the syringe is to be injected steadily at a rate of $300 \text{ cm}^3/\text{min}$, at what speed should the plunger be advanced? The leakage rate past the plunger is 0.01 times the volume flowrate out of the needle.

Determine the speed of the plunger be advanced

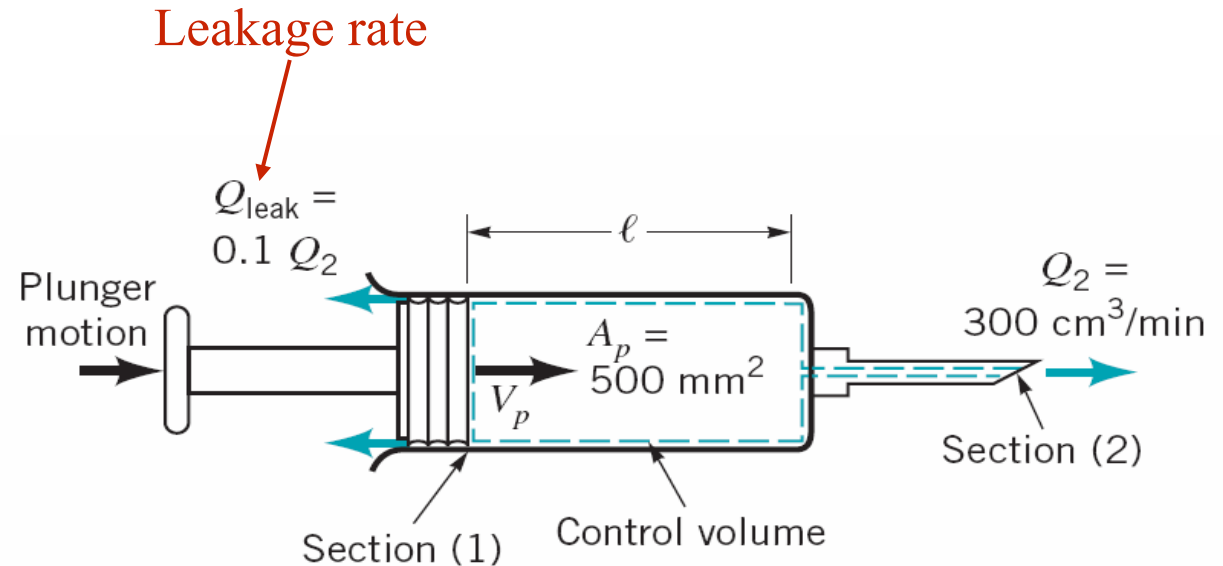


Figure E5.8

Example 5.8 Solution

$$A_1 \cong A_p$$

外漏加上從針頭注出者=貫穿CS進出CV的量

The continuity equation $\frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{C.S.} \rho \vec{W} \cdot \vec{n} dA = 0$

$$\rightarrow \frac{\partial}{\partial t} \int_{CV} \rho dV + \dot{m}_2 + \rho Q_{leak} = 0 \quad \int_{CV} \rho dV = \rho(\ell A_1 + V_{needle})$$

$$\rightarrow \frac{\partial}{\partial t} \int_{CV} \rho dV = \rho A_1 \frac{\partial \ell}{\partial t}$$

$$\text{Let } -\frac{\partial \ell}{\partial t} = V_p \Rightarrow -\rho A_1 V_p + \dot{m}_2 + \rho Q_{leak} = 0$$

$$\dot{m}_2 = \rho Q_2$$

$$-\rho A_1 V_p + \rho Q_2 + \rho Q_{leak} = 0$$

$$V_p = \frac{Q_2 + Q_{leak}}{A_1} = \dots = 660 \text{ mm/min}$$

The Linear Momentum Equations 1/4

- ❖ Newton's second law for a system moving relative to an inertial coordinate system.

利用System method描述Newton's second law

Time rate of change of
the linear momentum of
the **system** = Sum of external forces
acting on the system

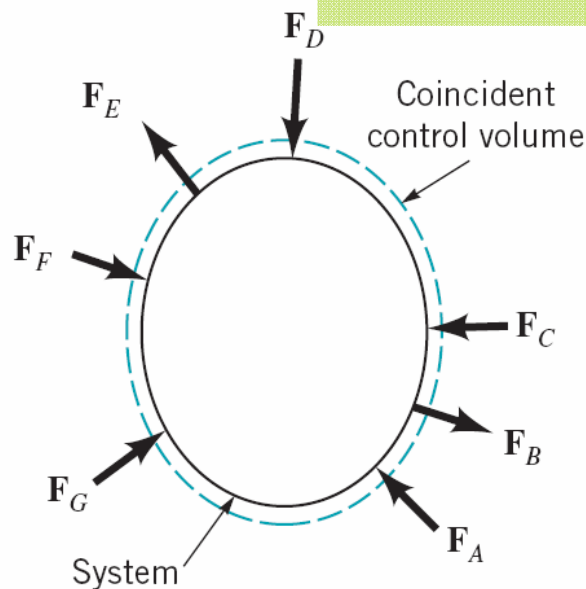
$$\sum \vec{F}_{\text{sys}} = \sum \vec{F}_S + \sum \vec{F}_B = \frac{d}{dt} \int_{\text{sys}} \vec{V} \rho dV = \frac{d\vec{P}}{dt} \Bigg)_{\text{system}}$$

$$\vec{P}_{\text{system}} = \int_{M(\text{system})} \vec{V} dm = \int_{V(\text{system})} \vec{V} \rho dV$$

The Linear Momentum Equations ^{2/4}

- ❖ When a control volume is coincident with a system at an instant of time, the force acting on the system and the force acting on the contents of the coincident control volume are instantaneously identical.

$$\sum \vec{F}_{\text{sys}} = \sum \vec{F}_{\text{contents of the coincident control volume}}$$



在時間 t 的瞬間，作用在CV上的external force等於作用在system者。透過這層連結，才能得到……

想一想，為何Continuity equation不需要這種連結？（原因在於：兩邊都等於0）

External forces acting on system and coincident control volume

Review of Reynolds Transport Theorem

This is the fundamental relation between the rate of change of any arbitrary extensive property, B , of a system and the variations of this property associated with a control volume.

$$\begin{aligned}\frac{dB_{\text{sys}}}{dt} &= \frac{\partial B_{\text{CV}}}{\partial t} + \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n} dA \\ &= \frac{\partial}{\partial t} \int_{\text{CV}} \rho b dV + \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n} dA\end{aligned}$$

The Linear Momentum Equations ^{3/4}

- ❖ For the system and a fixed, nondeforming control volume that are coincident at an instant of time, the Reynolds Transport Theorem leads to 依據Chapter 4推導Reynolds transport theorem的步驟

$$\underbrace{\mathbf{B}=\mathbf{P} \text{ and } b = \vec{V}}_{\text{解讀}} \Rightarrow \frac{d}{dt} \int_{\text{sys}} \vec{V} \rho dV \equiv \frac{\partial}{\partial t} \int_{\text{cv}} \vec{V} \rho dV + \int_{\text{cs}} \underbrace{\vec{V} \rho \vec{V} \cdot \vec{n} dA}_{\substack{\text{mass flow rate} \\ \Sigma \vec{F}_{\text{contents of the coincident control volume}}}}$$

因為質量進出所引發的動量進出

Time rate of change of the linear momentum of the coincident system

=

Time rate of change of the linear momentum of the content of the coincident control volume

+

Net rate of flow of linear momentum through the control surface

The Linear Momentum Equations 4/4

對固定、不變形的CV而言

❖ For a fixed and nondeforming control volume, the control volume formulation of Newton's second law

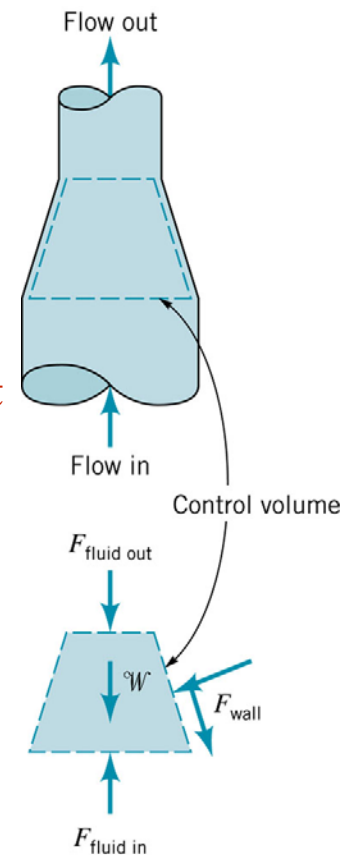
Linear momentum equation



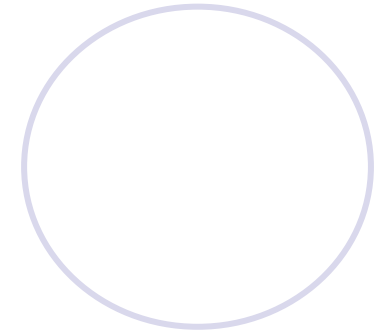
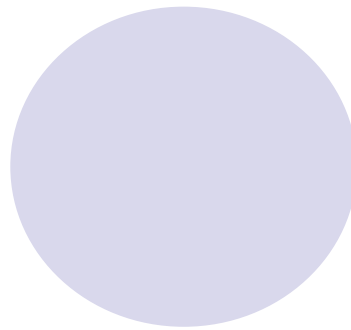
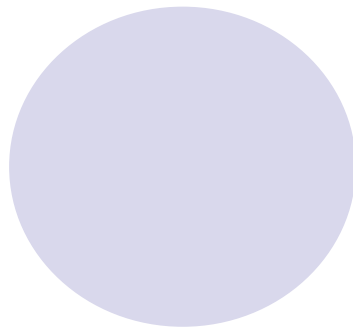
$$\frac{\partial}{\partial t} \int_{CV} \vec{V} \rho dV + \int_{CS} \underbrace{\vec{V} \rho \vec{V} \cdot \vec{n} dA}_{\text{mass flow rate}} = \sum \vec{F}$$

Contents of the coincident control volume

這是基於CV method, 描述牛頓第二定律的方程式



Linear momentum equation written for a moving control volume



Moving, Nondeforming Control Volume^{1/3}

對移動、不變形的CV而言

Chapter 4: Reynolds transport equation for a control volume moving with **constant velocity** is

$$\frac{dB_{\text{sys}}}{dt} = \frac{\partial}{\partial t} \int_{\text{cv}} b \rho dV + \underbrace{\int_{\text{cs}} b \rho \vec{W} \cdot \vec{n} dA}_{\text{mass flow rate}}$$

$$\frac{d}{dt} \int_{\text{sys}} \vec{V} \rho dV \equiv \frac{\partial}{\partial t} \int_{\text{cv}} \vec{V} \rho dV + \int_{\text{cs}} \vec{V} \rho \vec{W} \cdot \vec{n} dA$$

$$\left\{ \begin{array}{l} \frac{\partial}{\partial t} \int_{\text{cv}} \vec{V} \rho dV + \int_{\text{cs}} \vec{V} \rho \vec{W} \cdot \vec{n} dA = \sum \vec{F} \quad \text{Contents of the coincident control volume} \\ \vec{V} = \vec{W} + \vec{V}_{\text{cv}} \end{array} \right.$$

$$\Rightarrow \frac{\partial}{\partial t} \int_{\text{cv}} (\vec{W} + \vec{V}_{\text{cv}}) \rho dV + \int_{\text{cs}} (\vec{W} + \vec{V}_{\text{cv}}) \rho \vec{W} \cdot \vec{n} dA = \sum \vec{F} \quad \text{Contents of the coincident control volume}$$

Moving, Nondeforming Control Volume^{2/3}

特例

For a constant control volume velocity, V_{cv} , and **steady flow** in the control volume reference frame

$$\frac{\partial}{\partial t} \int_{cv} (\vec{W} + \vec{V}_{cv}) \rho dV = 0 \quad \text{STEADY FLOW}$$

$$\Rightarrow \int_{cs} (\vec{W} + \vec{V}_{cv}) \rho \vec{W} \cdot \vec{n} dA = \int_{cs} \vec{W} \rho \vec{W} \cdot \vec{n} dA + \vec{V}_{cv} \int_{cs} \rho \vec{W} \cdot \vec{n} dA = 0$$

For steady flow, continuity equation

$$\frac{dM_{sys}}{dt} = \frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{c.s.} \rho \vec{W} \cdot \vec{n} dA = 0$$

STEADY FLOW

$$\int_{c.s.} \rho \vec{W} \cdot \vec{n} dA = 0$$

Moving, Nondeforming Control Volume^{3/3}

⇒ For a **moving**, nondeforming **control volume**, the linear momentum equation of **steady flow**

$$\int_{CS} \underbrace{\vec{W} \rho \vec{W} \cdot \vec{n} dA}_{\text{mass flow rate}} = \sum \vec{F}_{\text{Contents of the coincident control volume}}$$

Vector Form of Momentum Equation

- ❖ The sum of all forces (surface and body forces) acting on a Non-accelerating control volume is equal to the sum of the rate of change of momentum inside the control volume and the net rate of flux of momentum out through the control surface.

解讀

$$\sum \vec{F}_{\text{contents of the coincident control volume}} = \sum \vec{F}_S + \sum \vec{F}_B$$

$$= \frac{\partial}{\partial t} \int_{CV} \vec{V} \rho dV + \int_{CS} \vec{V} \rho \vec{V} \cdot \vec{n} dA$$

$$\vec{F}_B = \int \vec{B} dm = \int_{CV} \vec{B} \rho dV$$

$$\vec{F}_S = \int_A -p d\vec{A}$$

Where the velocities are measured
Relative to the control volume.

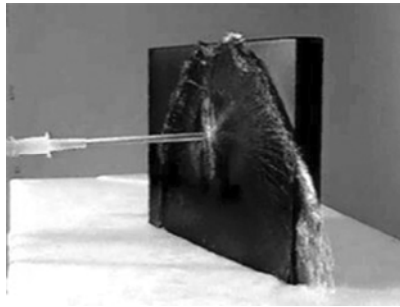
FILMS

煙囪雲煙

Smokestack
plume momentum



Force due to
a water jet

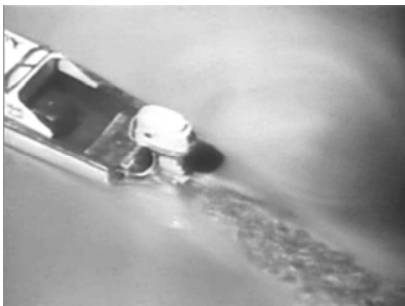


Fire hose

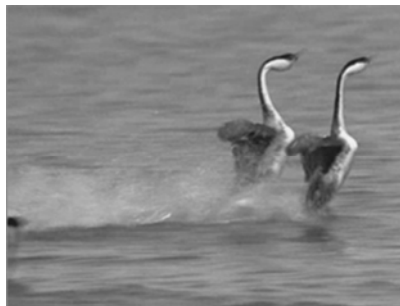


船舶推力

Marine propulsion



Running on
water



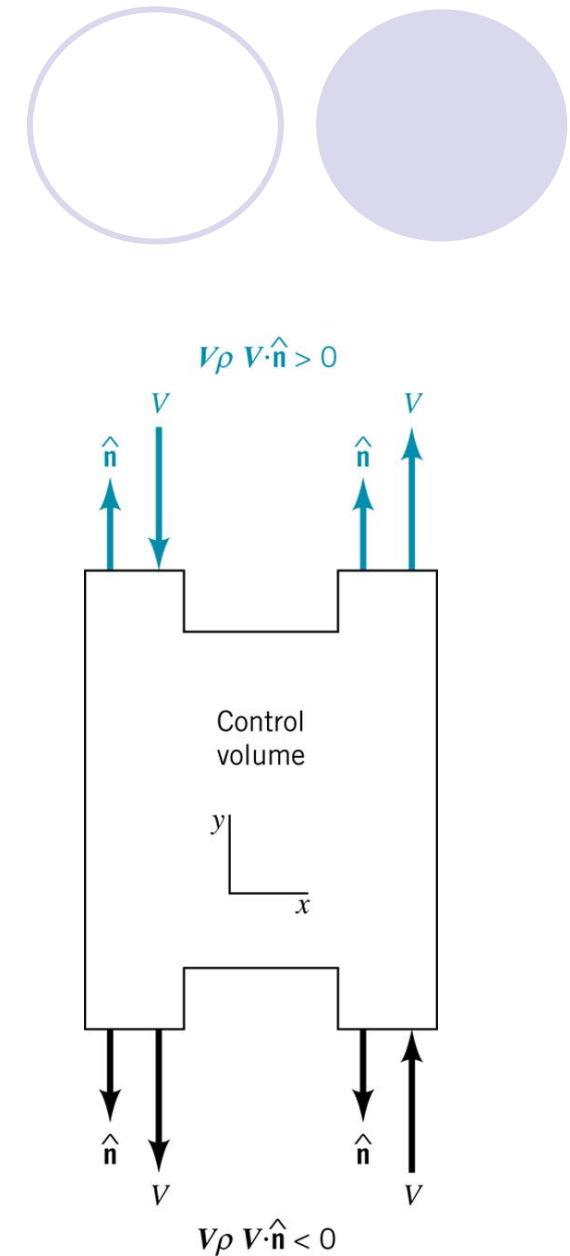
Jelly fish
水母



Linear Momentum Equations

應用及注意事項

- ❖ 線動量、力具方向性，其正負要與選用的座標系統相符。
- ❖ 流體進出CV，要注意流體速度與表面法向向量， $\vec{V} \cdot \vec{n}$ （+ for flow out of the CV，- for flow into the CV）。



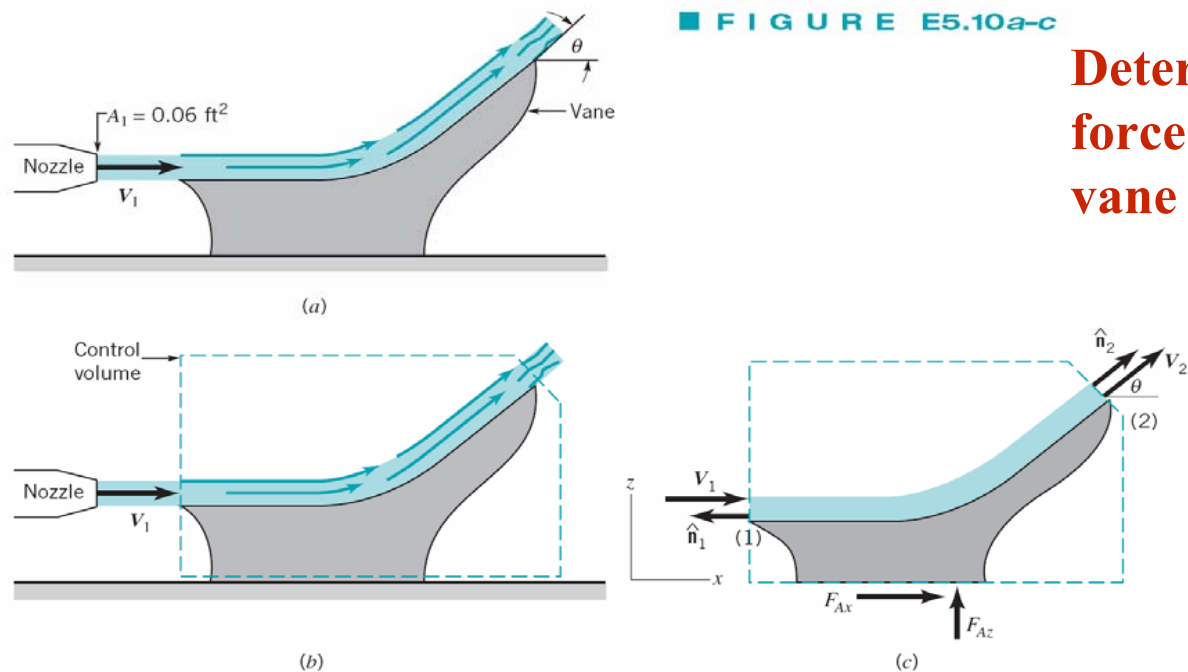


Application for **FIXING CV**

5.10~5.16

Example 5.10 Linear Momentum – Change in Flow Direction

- As shown in Figure E5.10a, a horizontal jet of water exits a nozzle with a uniform speed of $V_1 = 10$ ft/s, strike a vane, and is turned through an angle θ . Determine the anchoring force needed to hold the vane stationary. Neglect gravity and viscous effects.



Determine the anchoring force needed to hold the vane stationary.

Example 5.10 Solution

The x and z direction components of linear momentum equation

$$\frac{\partial}{\partial t} \int_{cv} u \rho dV + \int_{cs} u \rho \vec{V} \cdot \vec{n} dA = \sum F_x$$

$$\vec{V} = u \vec{i} + w \vec{k}$$

$$\frac{\partial}{\partial t} \int_{cv} w \rho dV + \int_{cs} w \rho \vec{V} \cdot \vec{n} dA = \sum F_z$$

$$V_1 \rho (-V_1) A_1 + V_1 \cos \theta \rho (V_1) A_2 = F_{Ax}$$

$$(0) \rho (-V_1) A_1 + V_1 \sin \theta \rho (V_1) A_2 = F_{Az}$$

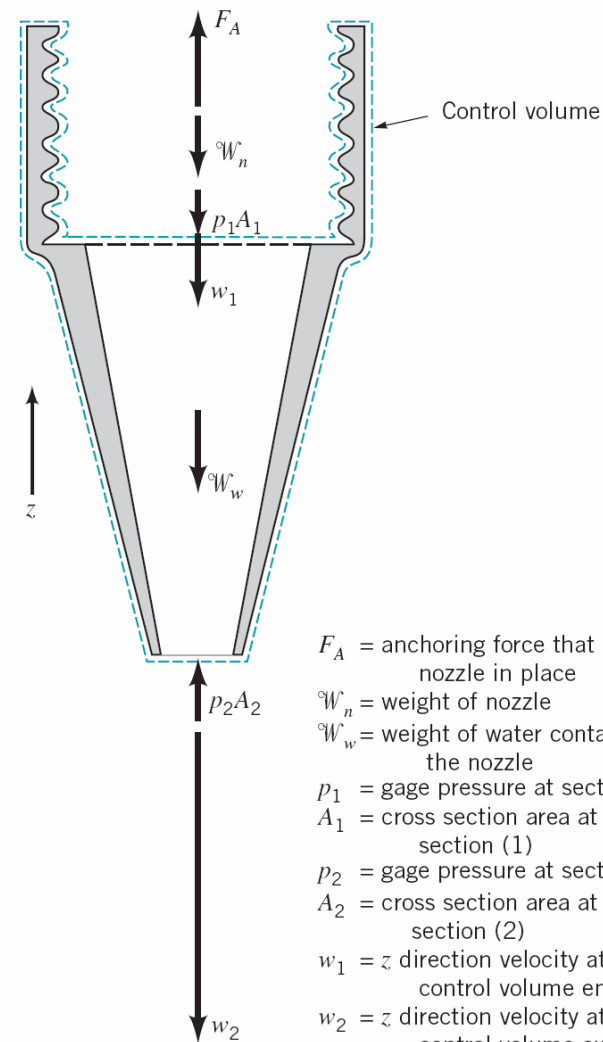
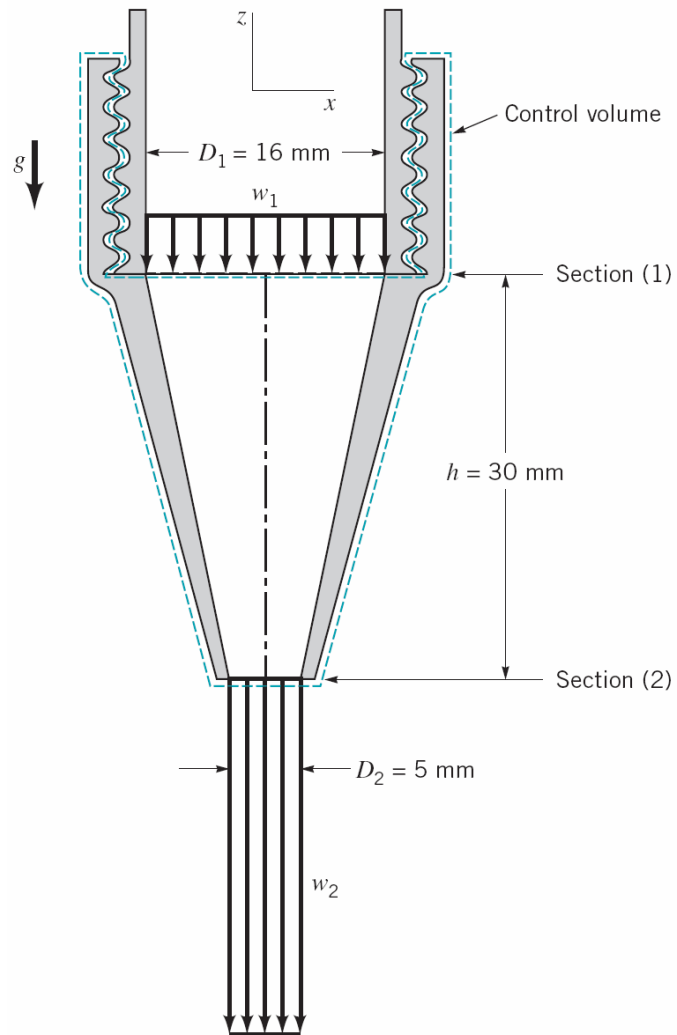
$$\rightarrow F_{Ax} = -\rho V_1^2 A_1 (1 - \cos \theta) = \dots = -11.64 (1 - \cos \theta) \text{ lb}$$

$$\rightarrow F_{Az} = \rho V_1^2 A_1 \sin \theta = \dots = 11.64 \sin \theta \text{ lb}$$

Example 5.11 Linear Momentum – Weight, pressure, and Change in Speed

- Determine the anchoring force required to hold in place a conical nozzle attached to the end of a laboratory sink faucet when the water flowrate is 0.6 liter/s. The nozzle mass is 0.1kg. The nozzle inlet and exit diameters are 16mm and 5mm, respectively. The nozzle axis is vertical and the axial distance between section (1) and (2) is 30mm. The pressure at section (1) is 464 kPa. to hold the vane stationary. Neglect gravity and viscous effects.

Example 5.11 Solution^{1/3}



F_A = anchoring force that holds nozzle in place
 $\mathcal{W}_n$ = weight of nozzle
 $\mathcal{W}_w$ = weight of water contained in the nozzle
 p_1 = gage pressure at section (1)
 A_1 = cross section area at section (1)
 p_2 = gage pressure at section (2)
 A_2 = cross section area at section (2)
 w_1 = z direction velocity at control volume entrance
 w_2 = z direction velocity at control volume exit

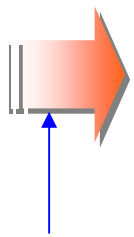
Example 5.11 Solution^{2/3}

The z direction component of linear moment equation

$$\frac{\partial}{\partial t} \int_{CV} w \rho dV + \int_{CS} w \rho \vec{V} \cdot \vec{n} dA = F_A - W_n - p_1 A_1 - W_w + p_2 A_2$$

$$\vec{V} \cdot \vec{n} dA = \pm |w| dA$$

With the “+” used for flow out of the control volume and “-” used for flow in.



$$(-\dot{m}_1)(-w_1) + \dot{m}_2(-w_2) = -W_n - p_1 A_1 - W_w + p_2 A_2$$

$$F_A = \dot{m}(w_1 - w_2) + W_n + p_1 A_1 + W_w - p_2 A_2$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m} \quad \dot{m}_1 = \dot{m}_2 = \dot{m} = \rho w_1 A_1 = \rho Q = \dots = 0.599 \text{ kg/s}$$

Example 5.11 Solution^{3/3}

$$w_1 = \frac{Q}{A_1} = \frac{Q}{\pi(D_1^2/4)} = \dots = 2.98 \text{ m/s}$$

$$w_2 = \frac{Q}{A_2} = \frac{Q}{\pi(D_2^2/4)} = \dots = 30.6 \text{ m/s}$$

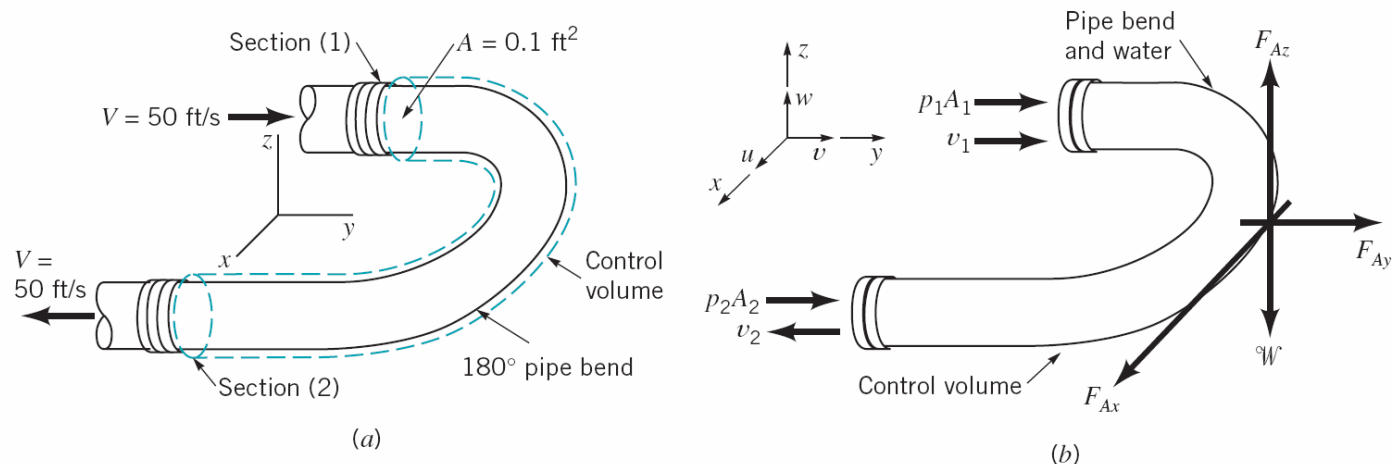
$$W_n = m_n g = (0.1 \text{ kg})(9.81 \text{ m/s}^2) = 0.981 \text{ N}$$

$$W_w = \rho V_w g = \rho \left[\frac{1}{12} \pi h (D_1^2 + D_2^2 + D_1 D_2) \right] V_w g = \dots = 0.0278 \text{ N}$$


$$\begin{aligned} F_A &= \dot{m}(w_1 - w_2) + W_n + p_1 A_1 + W_w - p_2 A_2 \\ &= (0.599 \text{ kg/s})(\dots) = \dots = 77.8 \text{ N} \end{aligned}$$

Example 5.12 Linear Momentum – Pressure, Change in Speed, and Friction

- Water flows through a horizontal, 180° pipe bend. The flow cross-section area is constant at a value of 0.1 ft^2 through the bend. The magnitude of the flow velocity everywhere in the bend is axial and 50 ft/s . The absolute pressure at the entrance and exit of the bend are 30 psia and 24 psia , respectively. Calculate the horizontal (x and y) components of the anchoring force required to hold the bend in place.



Example 5.12 Solution^{1/2}

The x direction component of linear momentum equation

$$\frac{\partial}{\partial t} \int_{cv} u \rho dV + \int_{cs} u \rho \vec{V} \cdot \vec{n} dA = F_{Ax}$$

At section (1) and (2), the flow is in the y direction and therefore $u=0$ at both sections.

$$F_{Ax} = 0$$

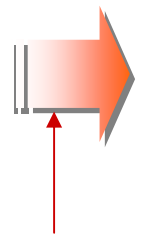
The y direction component of linear momentum equation

$$\frac{\partial}{\partial t} \int_{cv} v \rho dV + \int_{cs} v \rho \vec{V} \cdot \vec{n} dA = F_{Ay} + p_1 A_1 + p_2 A_2$$

Example 5.12 Solution^{2/2}

For one-dimensional flow

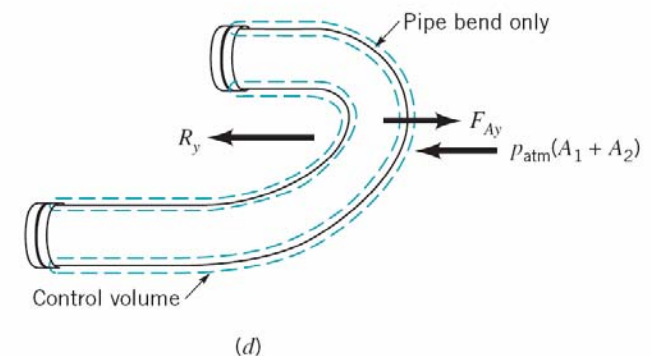
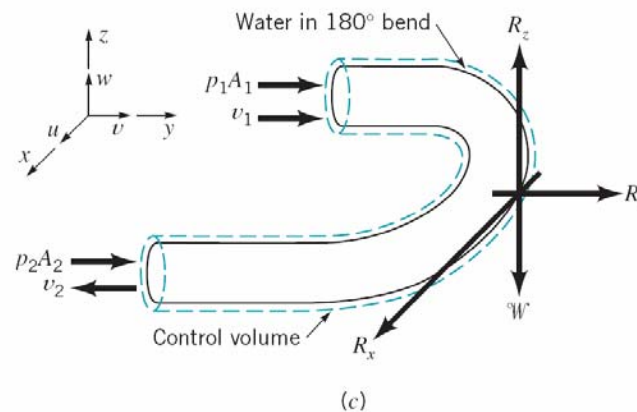
$$(+v_1)(-\dot{m}_1) + (-v_2)(+\dot{m}_2) = F_{Ay} + p_1A_1 + p_2A_2$$



$$-\dot{m}(v_1 + v_2) = F_{Ay} + p_1A_1 + p_2A_2$$

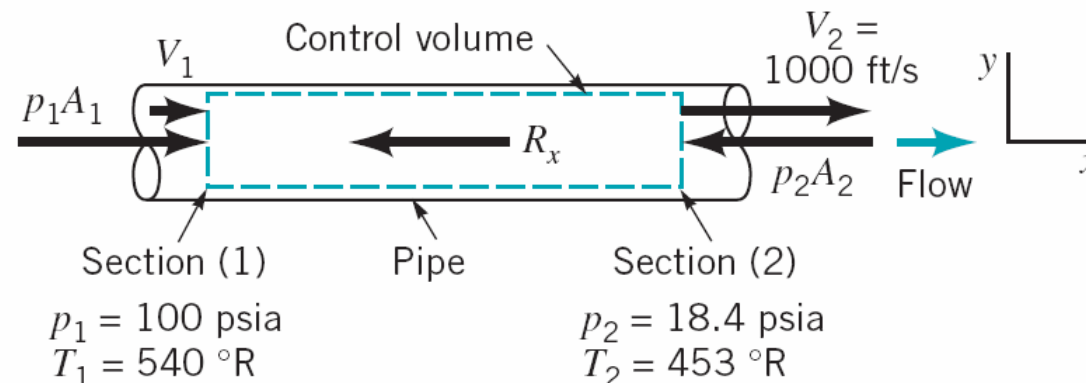
$$F_{Ay} = -\dot{m}(v_1 + v_2) - p_1A_1 - p_2A_2 = \dots = -1324\text{lb}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m} = \rho v_1 A_1 = \dots = 9.70\text{slugs/s}$$



Example 5.13 Linear Momentum – Weight, pressure, and Change in Speed

- Air flows steadily between two cross sections in a long, straight portion of 4-in. inside diameter pipe as indicated in Figure E5.13, where the uniformly distributed temperature and pressure at each cross section are given. If the average air velocity at section (2) is 1000 ft/s, we found in Example 5.2 that the average air velocity at section (1) must be 219 ft/s. Assuming uniform velocity distributions at sections (1) and (2), determine the frictional force exerted by the pipe wall on the air flow between sections (1) and (2).

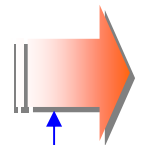


Example 5.13 Solution^{1/2}

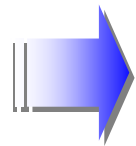
The axial component of linear moment equation

$$\frac{\partial}{\partial t} \int_{cv} u \rho dV + \int_{cs} u \rho \vec{V} \cdot \vec{n} dA = -R_x + p_1 A_1 - p_2 A_2$$

$$(+u_1)(-\dot{m}_1) + (+u_2)(+\dot{m}_2) = -R_x + p_1 A_1 - p_2 A_2$$



$$\dot{m}(u_2 - u_1) = -R_x + A_2(p_1 - p_2)$$

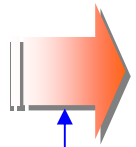


$$R_x = A_2(p_1 - p_2) - \dot{m}(u_2 - u_1)$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m} = \left(\frac{p_2}{RT_2} \right) \left(\frac{\pi D_2^2}{4} \right) u_2 = \dots = 0.297 \text{ slugs/s}$$

Example 5.13 Solution^{2/2}

$$R_x = A_2(p_1 - p_2) - \dot{m}(u_2 - u_1)$$



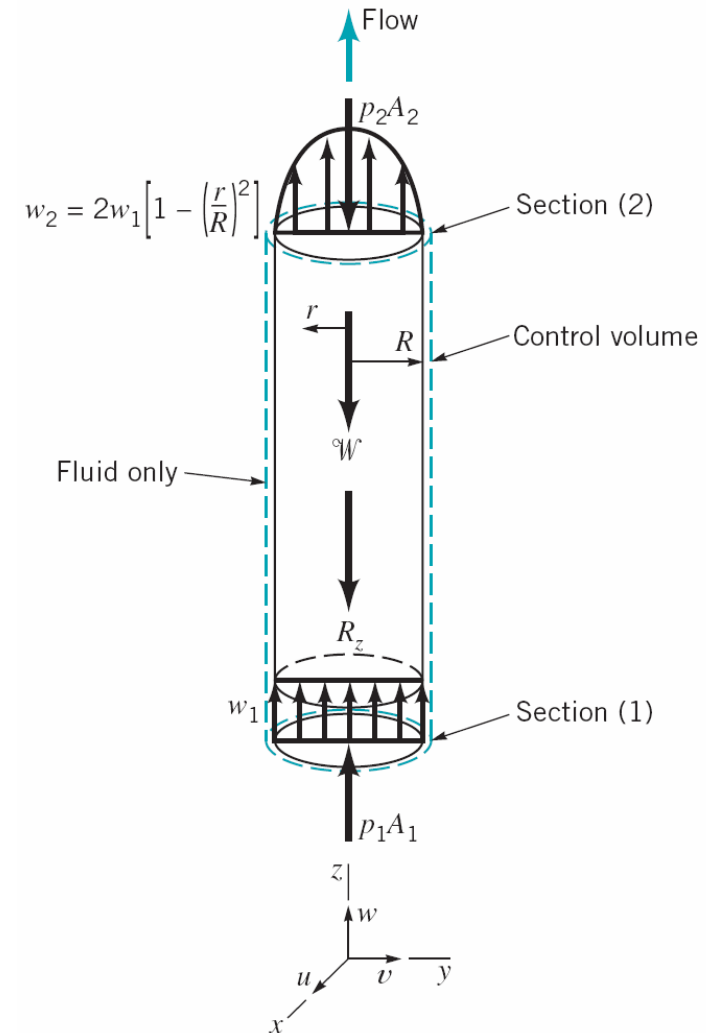
$$R_x = A_2(p_1 - p_2) - \dot{m}(u_2 - u_1) = \dots = 7931\text{lb}$$

$$\rho_2 = \frac{p_2}{RT_2}$$

$$A_2 = \frac{\pi D_2^2}{4}$$

Example 5.14 Linear Momentum – Weight, Pressure,...

- If the flow of Example 5.4 is vertically upward, develop an expression for the fluid pressure drop that occurs between sections (1) and (2).



Example 5.14 Solution

The axial component of linear moment equation

$$\frac{\partial}{\partial t} \int_{cv} w \rho dV + \int_{cs} w \rho \vec{V} \cdot \vec{n} dA = p_1 A_1 - R_z - W - p_2 A_2$$

$$\Rightarrow (+w_1)(-\dot{m}_1) + \int_{cs} (+w_2) \rho (+w_2 dA_2) = p_1 A_1 - R_z - W - p_2 A_2$$

$$w_2 = 2w_1 \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

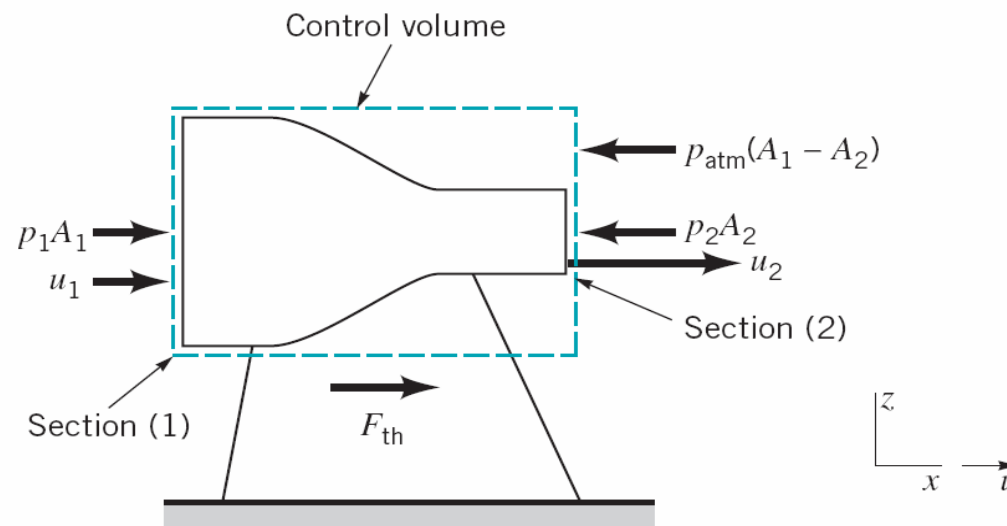
$$\int_{cs} (+w_2) \rho (+w_2 dA_2) = \rho \int_0^R w_2^2 2\pi r dr = 4\pi \rho w_1^2 \frac{R^2}{3}$$

$$-w_1^2 \rho \pi R^2 + \frac{4}{3} w_1^2 \rho \pi R = p_1 A_1 - R_z - W - p_2 A_2$$

$$\Rightarrow p_1 - p_2 = \frac{\rho w_1^2}{3} + \frac{R_z}{A_1} + \frac{W}{A_1}$$

Example 5.15 Linear Momentum - Thrust

- A static thrust as sketched in Figure E5.15 is to be designed for testing a jet engine. The following conditions are known for a typical test: Intake air velocity = 200 m/s; exhaust gas velocity = 500 m/s; intake cross-section area = 1 m²; intake static pressure = 78.5 kPa (abs); intake static temperature = 268 K; exhaust static pressure = 101 kPa (abs). Estimate the normal thrust for which to design.



Example 5.15 Solution

The x direction component of linear moment equation

$$\frac{\partial}{\partial t} \int_{cv} u \rho dV + \int_{cs} u \rho \vec{V} \cdot \vec{n} dA = p_1 A_1 + F_{th} - p_2 A_2 - p_{atm} (A_1 - A_2)$$

$$\Rightarrow (+u_1)(-\dot{m}_1) + (+u_2)(+\dot{m}_2) = (p_1 - p_{atm})A_1 - (p_2 - p_{atm})A_2 + F_{th}$$

$$\dot{m} = \dot{m}_1 = \rho_1 A_1 u_1 = \dot{m}_2 = \rho_2 A_2 u_2$$

$$\Rightarrow \dot{m}(u_2 - u_1) = p_1 A_1 - p_2 A_2 + F_{th}$$

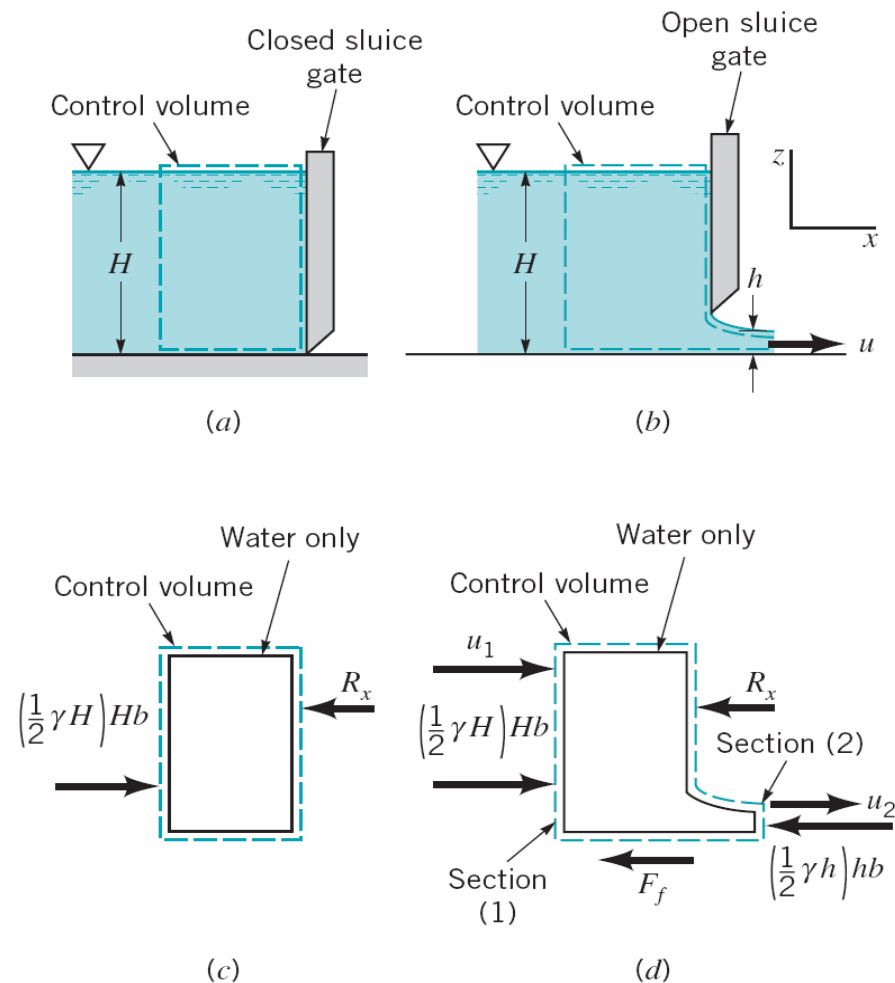
$$F_{th} = -\rho_1 A_1 u_1 + \rho_2 A_2 u_2 + \dot{m}(u_2 - u_1) = \dots = 83700 \text{ N}$$

$$\rho_1 = \frac{p_1}{RT_1}$$

$$\dot{m} = \rho_1 A_1 u_1 = \dots = 204 \text{ kg/s}$$

Example 5.16 Linear Momentum – Nonuniform Pressure

- A sluice gate across a channel of width b is shown in the closed and open position in Figure 5.16a and b. Is the anchoring force required to hold the gate in place larger when the gate is closed or when it is open?



Example 5.16 Solution

When the gate is closed, the horizontal forces acting on the contents of the control volume are identified in Figure E5.16c.

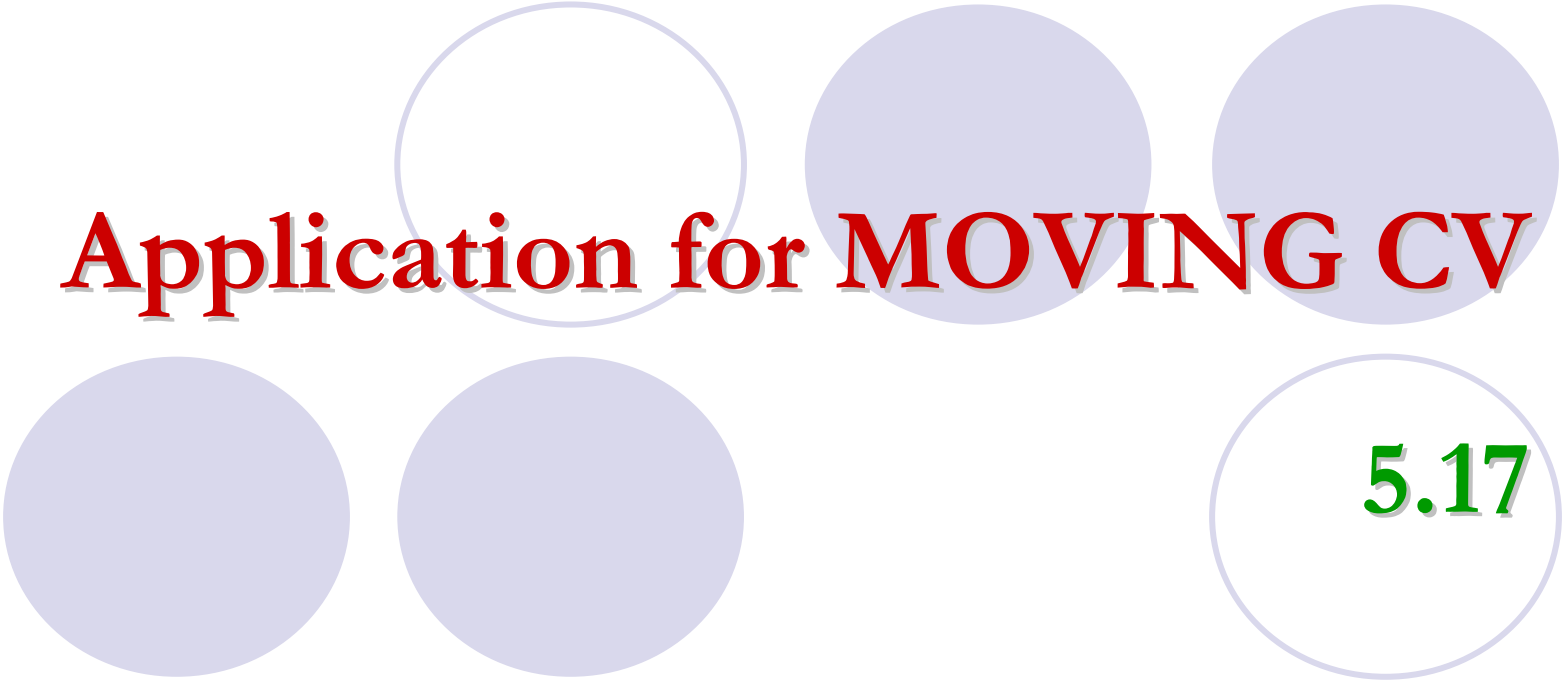
$$\int_{CS} u \rho \vec{V} \cdot \vec{n} dA = \frac{1}{2} \gamma H^2 b - R_x \Rightarrow R_x = \frac{1}{2} \gamma H^2 b$$

When the gate is open, the horizontal forces acting on the contents of the control volume are identified in Figure E5.16d.

$$\int_{CS} u \rho \vec{V} \cdot \vec{n} dA = \frac{1}{2} \gamma H^2 b - R_x - \frac{1}{2} \gamma h^2 b - F_f$$

$$-\rho u_1^2 H b + \rho u_2^2 h b = \frac{1}{2} \gamma H^2 b - R_x - \frac{1}{2} \gamma h^2 b - F_f$$

$$\text{For } H \gg h \text{ and } u_1 \ll u_2 \Rightarrow R_x = \frac{1}{2} \gamma H^2 b - \frac{1}{2} \gamma h^2 b - F_f - \rho u_2^2 h b$$

The slide features a decorative arrangement of six circles. Three circles are arranged in a horizontal row at the top, and three are arranged in a horizontal row at the bottom. The top row consists of an empty circle with a light blue outline on the left, and two solid light blue circles on the right. The bottom row consists of two solid light blue circles on the left, and an empty circle with a light blue outline on the right. The text 'Application for MOVING CV' is centered horizontally between the two rows of circles.

Application for MOVING CV

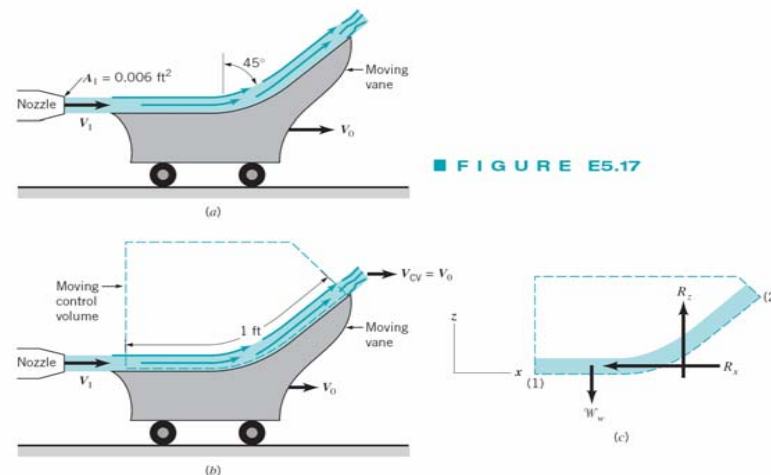
5.17

Example 5.17 Linear Momentum - Moving Control Volume ^{1/2}

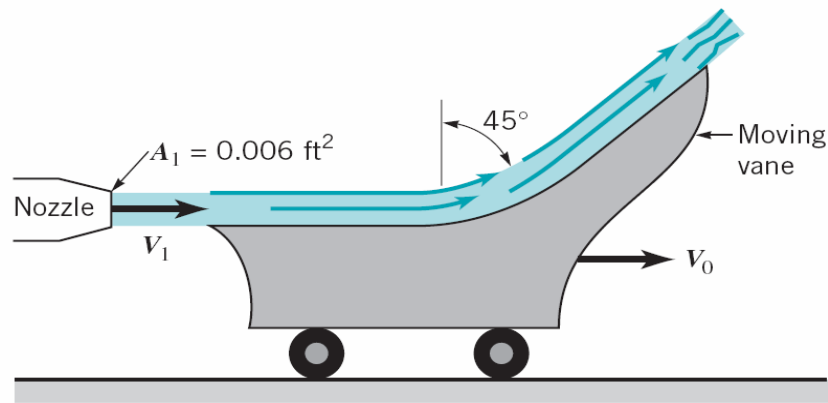
- A vane on wheels move with a constant velocity V_0 when a stream of water having a nozzle exit velocity of V_1 is turned 45° by the vane as indicated in Figure E5.17a. Note that this is the same moving vane considered in Section 4.4.6 earlier. Determine the magnitude and direction of the force, F , exerted by the stream of water on the vane surface. The speed of the water jet leaving the nozzle is 100 ft/s, and the vane is moving to the right with a constant speed of 20 ft/s.

CV移動速度 V_0

V_1 是流體離開Nozzle的絕對速度

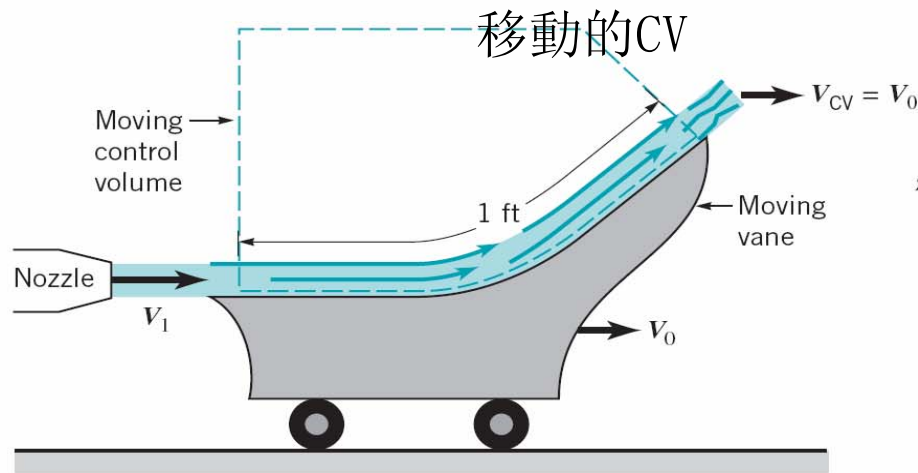


Example 5.17 Linear Momentum - Moving Control Volume ^{2/2}

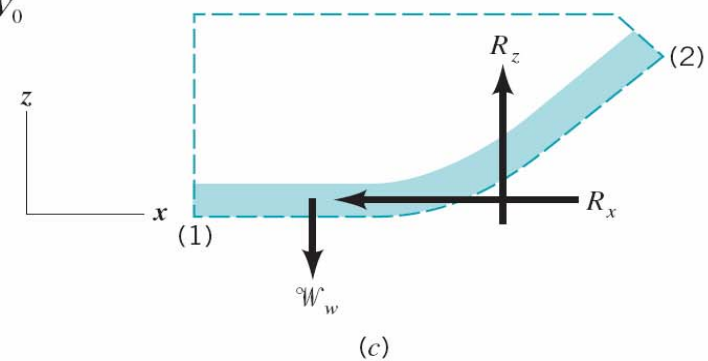


(a)

FIGURE E5.17



(b)



(c)

Example 5.17 Solution^{1/2}

The x direction component of linear momentum equation

$$\int_{CS} W_x \rho \vec{W} \cdot \vec{n} dA = -R_x$$

$$\Rightarrow (+W_1)(-\dot{m}_1) + (+W_2 \cos 45^\circ)(+\dot{m}_2) = -R_x$$



$$\dot{m}_1 = \rho_1 W_1 A_1 \quad \dot{m}_2 = \rho_2 W_2 A_2$$

The z direction component of linear momentum equation

$$\int_{CS} W_z \rho \vec{W} \cdot \vec{n} dA = R_z - W_w$$

$$\Rightarrow (+W_2 \sin 45^\circ)(+\dot{m}_2) = R_z - W_w$$



$$\dot{m}_1 = \rho_1 W_1 A_1 = \dot{m}_2 = \rho_2 W_2 A_2$$

$$W_1 = W_2 = V_1 - V_0 = \dots$$

Example 5.17 Solution^{2/2}

$$R_x = \rho W_1^2 A_1 (1 - \cos 45^\circ) = \dots = 21.8 \text{ lb}$$

$$R_z = \rho W_1^2 A_1 \sin 45^\circ + W_w = \dots = 53 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_z^2} = \dots = 57.3 \text{ lb}$$

$$W_w = \rho g A_1 \ell$$

$$\alpha = \tan^{-1} \frac{R_z}{R_x}$$

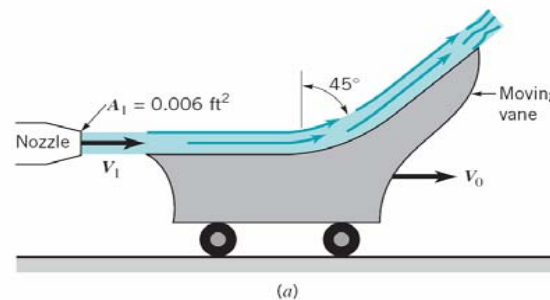
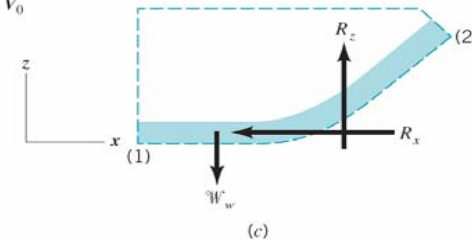
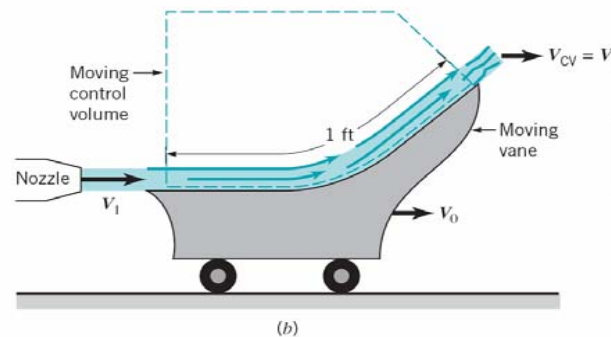


FIGURE E5.17



From the Proceeding Examples

- ❖ **A flowing fluid can be forced** to change direction, Speed up or slow down, have a velocity profile change, do only some or all of the above, do none of the above.
- ❖ **A net force** on the fluid is required for achieving any or all of the first four above. The forces on a flowing fluid balance out with no net force for the fifth.
- ❖ **Typical force** considered include pressure, friction, weight.

Moment-of-Momentum Equation^{1/4}

❖ Applying Newton's second law of motion to a particle of fluid

牛頓第二定律

針對一個Particle

$$\frac{d}{dt}(\vec{V}\rho\delta V) = \delta\vec{F}_{\text{particle}}$$

The velocity measured in an inertial reference system

Acting on the particle

❖ Taking moment of each side with respect to the origin of an inertial coordinate system

r是質點與座標原點的距離

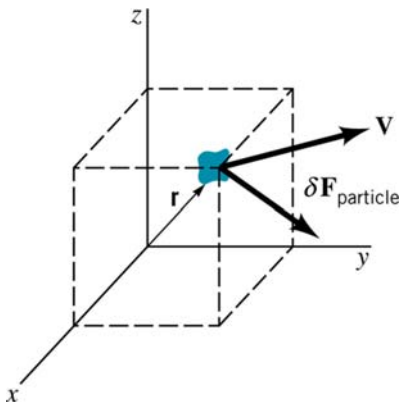
$$\vec{r} \times \frac{d}{dt}(\vec{V}\rho\delta V) = \vec{r} \times \delta\vec{F}_{\text{particle}}$$

$$\frac{d\vec{r}}{dt} = \vec{V} \quad \vec{V} \times \vec{V} = 0$$

$$\frac{d}{dt}[(\vec{r} \times \vec{V})\rho\delta V] = \frac{d\vec{r}}{dt} \times \vec{V}\rho\delta V + \vec{r} \times \frac{d}{dt}(\vec{V}\rho\delta V)$$

$$\Rightarrow \frac{d}{dt}[(\vec{r} \times \vec{V})\rho\delta V] = \vec{r} \times \delta\vec{F}_{\text{particle}}$$

對座標原點的力矩



Moment-of-Momentum Equation^{2/4}

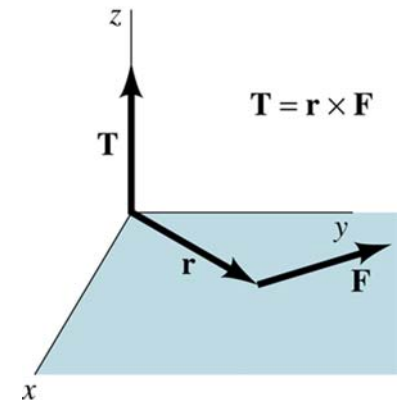
$$\frac{d}{dt} [(\vec{r} \times \vec{V}) \rho \delta V] = \vec{r} \times \delta \vec{F}_{\text{particle}}$$

適用到所有particles

積分 $\Rightarrow \int_{\text{sys}} \frac{d}{dt} [(\vec{r} \times \vec{V}) \rho dV] = \frac{d}{dt} \int_{\text{sys}} (\vec{r} \times \vec{V}) \rho dV$

$$\Rightarrow \frac{d}{dt} \int_{\text{sys}} (\vec{r} \times \vec{V}) \rho dV = \sum (\vec{r} \times \vec{F})_{\text{sys}}$$

適用所有質點，擴及整個System



**The time rate of change of the
Moment-of-momentum of the system**

**Sum of external torques
Acting on system**

$$\Rightarrow \frac{d}{dt} \int_{\text{sys}} (\vec{r} \times \vec{V}) \rho dV = \sum_{\text{sys}} (\vec{r} \times \vec{F})$$

Based on system method

Moment-of-Momentum Equation^{3/4}

- ❖ When a control volume is coincident with a system at an instant of time, the torque acting on the system and the torque acting on the contents of the coincident control volume are instantaneously identical

$$\sum (\vec{r} \times \vec{F})_{\text{sys}} = \sum (\vec{r} \times \vec{F})_{\text{cv}}$$

在時間 t 的瞬間，作用在CV上的 external force所產生的力矩等於作用在system者。透過這層連結，才能得到……

- ❖ For fixed and nondeforming control volume, the moment-of-momentum equation:

$$\frac{\partial}{\partial t} \int_{\text{CV}} (\vec{r} \times \vec{V}) \rho dV + \int_{\text{CS}} (\vec{r} \times \vec{V}) \rho \vec{V} \cdot \vec{n} dA = \sum (\vec{r} \times \vec{F})$$

Contents of the coincident control volume

Based on Control Volume

Review of Reynolds Transport Theorem

This is the fundamental relation between the rate of change of any arbitrary extensive property, B , of a system and the variations of this property associated with a control volume.

$$\begin{aligned}\frac{dB_{\text{sys}}}{dt} &= \frac{\partial B_{\text{CV}}}{\partial t} + \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n} dA \\ &= \frac{\partial}{\partial t} \int_{\text{CV}} \rho b dV + \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n} dA\end{aligned}$$

Moment-of-Momentum Equation^{4/4}

依據 Chapter 4 推導 Reynolds transport theorem 的步驟

❖ For the system and the contents of the coincident control volume that is fixed and nondeforming, The Reynolds transport theorem leads to 選擇一固定且不變形的CV = SYSTEM

$$\frac{d}{dt} \int_{\text{sys}} (\vec{r} \times \vec{V}) \rho dV = \frac{\partial}{\partial t} \int_{\text{cv}} (\vec{r} \times \vec{V}) \rho dV + \int_{\text{cs}} (\vec{r} \times \vec{V}) \rho \vec{V} \cdot \vec{n} dA$$

解讀

Time rate of change
of the moment-of-
momentum of the
system

=

Time rate of change of the
moment-of-momentum of
the content of the
coincident control volume

+

Net rate of flow of
moment-of-momentum
through the control
surface

$$\mathbf{b} = \vec{r} \times \vec{V}$$

$$\mathbf{B} = b m \Rightarrow \vec{r} \times \vec{V} m$$

Application^{1/8}

CV是固定的，注意所涵蓋範圍



❖ Consider the rotating sprinkler.

假
設
條
件

⇒ The flows are one-dimensional.

⇒ The flows are steady or steady-in-the-mean.

⇒ Using the axial component of the moment-of-momentum equation to analyze this flow

$$\frac{\partial}{\partial t} \int_{CV} (\vec{r} \times \vec{V}) \rho dV = 0$$

Using the fixed and non-deforming control volume which contains within its boundaries the spinning or stationary sprinkler head and the portion of the water flowing through the sprinkler contained in the control volume.

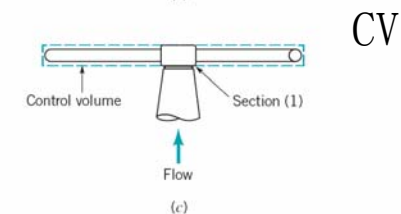
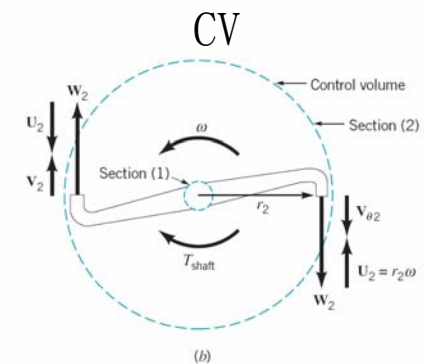
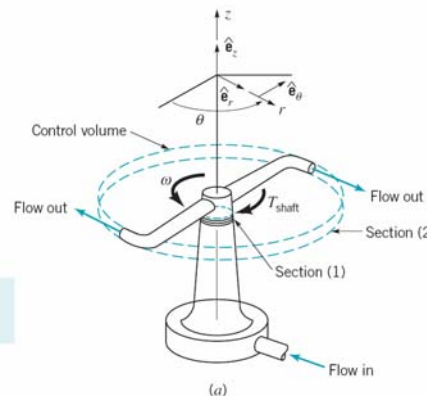


FIGURE 5.4 (a) Rotary water sprinkler. (b) Rotary water sprinkler, plane view. (c) Rotary water sprinkler, side view.

Application^{2/8}

$$\frac{\partial}{\partial t} \int_{CV} (\vec{r} \times \vec{V}) \rho dV + \int_{CS} (\vec{r} \times \vec{V}) \rho \vec{V} \cdot \vec{n} dA = \sum (\vec{r} \times \vec{F})$$

$$\Rightarrow \int_{CS} (\vec{r} \times \vec{V}) \rho \vec{V} \cdot \vec{n} dA$$

This term can be nonzero only where fluid is crossing the control surface. Everywhere else on the control surface this term will be zero because

解構此項的內容

At section (1) $\vec{r} \times \vec{V} = 0 \Rightarrow$ There is no axial moment-of-momentum flow in section (1)

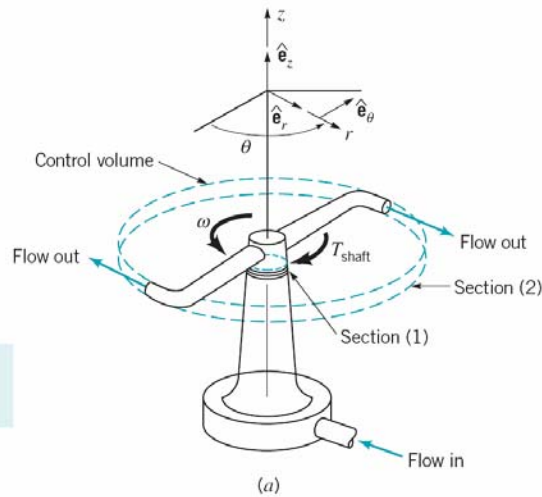
At section (2) $\vec{r} \times \vec{V} = r_2 V_{\theta 2}$

轉軸到噴嘴中心的距離

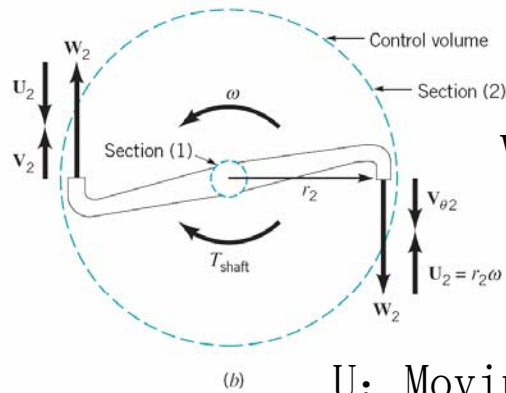
流體流出噴嘴的絕對切線速度

r_2 is the radius from the axis of rotation to the nozzle centerline and $V_{\theta 2}$ is the tangential component of the velocity of the flow exiting each nozzle as observed from a frame of reference attached to the fixed and nondeforming control volume.

Application^{3/8}



W: 流體相對Moving nozzle的相對速度



V: 流體流出噴嘴的絕對速度

U: Moving nozzle相對於固定座標的速度

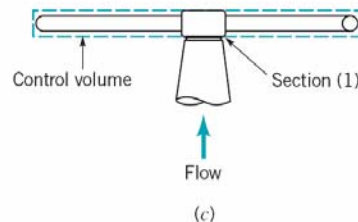


FIGURE 5.4 (a) Rotary water sprinkler. (b) Rotary water sprinkler, plane view. (c) Rotary water sprinkler, side view.

$$\vec{V} = \vec{W} + \vec{U}$$

U is the velocity of the moving nozzle as measured relative to the fixed control surface.
W is relative velocity of exit flow as viewed from the nozzle
V is the absolute velocity of exit flow relative to a fixed control surface.

Application^{4/8}

$$\int_{CS} (\vec{r} \times \vec{V}) \rho \vec{V} \cdot \vec{n} dA$$

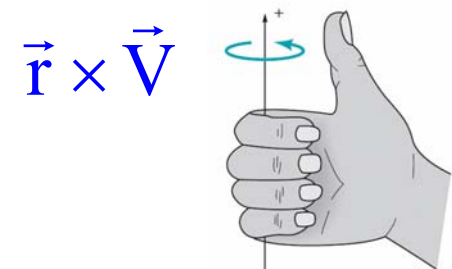
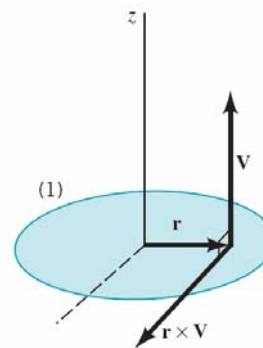
$$\vec{V} \cdot \vec{n} \quad \begin{array}{l} \text{"-" for flow into} \\ \text{"+" for flow out} \end{array}$$

$$\int_{CS} (\vec{r} \times \vec{V}) \rho \vec{V} \cdot \vec{n} dA \Big|_{\text{axial}} = (-r_2 V_{\theta 2})(+ \dot{m})$$

Where $\dot{m}$ is the total mass flowrate through both nozzles. The mass flowrate is the same whether the sprinkler rotates or not.

如何判斷正負

$\vec{r} \times \vec{V}$ “+” or “-” ascertained by using the right-hand rule



右手定則

Application^{5/8}

Turbine (渦輪機) 的Torque 為『負』

The torque term $\sum (\vec{r} \times \vec{F})_{\text{content of the control volume}}$

$$\sum (\vec{r} \times \vec{F})_{\text{content of the CV}} \Big|_{\text{axial}} = T_{\text{shaft}} = -r_2 V_{\theta 2} \dot{m}$$

$$\vec{r} \times \vec{V}$$

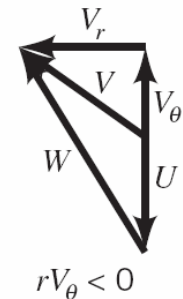
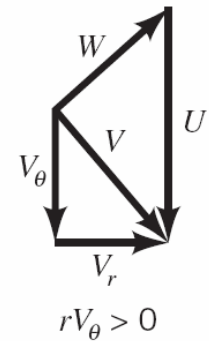
Acting on the shaft

『負』表示Torque與旋轉方向相反

The correct algebraic sign of the axial component of $\vec{r} \times \vec{V}$ can be easily remembered in the following way:

If V_{θ} and U are in the same direction, use +

If V_{θ} and U are in opposite direction, use -



Application^{6/8}

Shaft power?

Acting on the CV

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = -r_2 V_{\theta 2} \dot{m} \omega \quad w_{\text{shaft}} = \dot{W}_{\text{shaft}} \div \dot{m} = -U_2 V_{\theta 2}$$

Sprinkler speed $U = r_2 \omega$

Negative shaft work is work out of the control volume, that is, work done by the fluid on the rotor and thus its shaft.

Torque與旋轉方向相反→『負』的力矩→導致『負』的軸功：表示shaft work is out of the CV，是水『做』功在渦輪機的轉子上！

Application^{7/8}

$$\frac{\partial}{\partial t} \int_{CV} (\vec{r} \times \vec{V}) \rho dV + \int_{CS} (\vec{r} \times \vec{V}) \rho \vec{V} \cdot \vec{n} dA = \sum (\vec{r} \times \vec{F})$$

Contents of the
Control volume

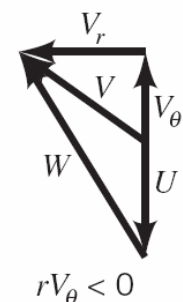
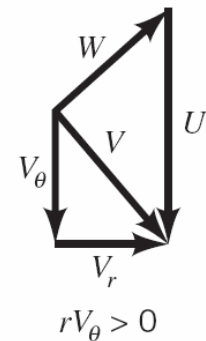


$$T_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm r_{\text{in}} V_{\theta \text{in}}) + (\dot{m}_{\text{out}})(\pm r_{\text{out}} V_{\theta \text{out}})$$

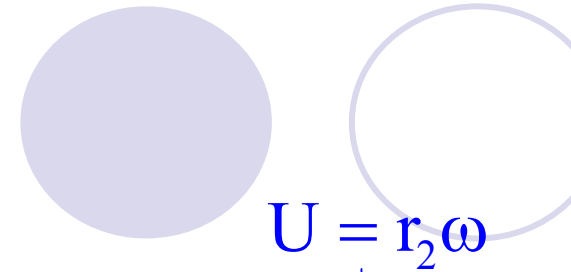
The “-” is used with mass flowrate into the control volume, $\dot{m}_{\text{in}}$, and the “+” is used with mass flowrate out of the control volume, $\dot{m}_{\text{out}}$, to account for the sign of the dot product $\vec{V} \cdot \vec{n}$.

The “+” or “-” is used with the rV_{θ} product depends on the direction of $(\vec{r} \times \vec{V})_{\text{axial}}$

If V_{θ} and U are in the same direction, use +
If V_{θ} and U are in opposite direction, use -



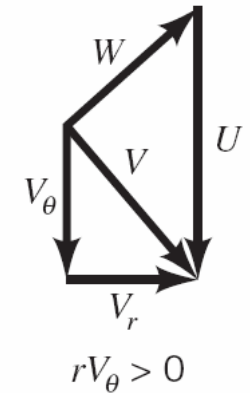
Application^{8/8}



The shaft power

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = (-\dot{m}_{\text{in}})(\pm r_{\text{in}} \omega V_{\theta \text{in}}) + (\dot{m}_{\text{out}})(\pm r_{\text{out}} \omega V_{\theta \text{out}})$$

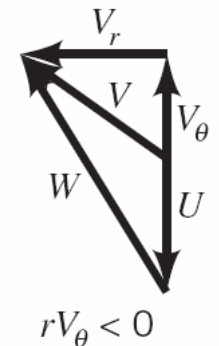
$$\Rightarrow \dot{W}_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm U_{\text{in}} V_{\theta \text{in}}) + (\dot{m}_{\text{out}})(\pm U_{\text{out}} V_{\theta \text{out}})$$



質量守衡

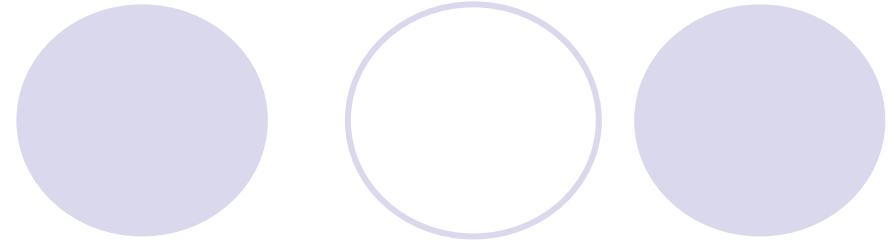
$$\dot{m} = \dot{m}_{\text{in}} = \dot{m}_{\text{out}}$$

⇒ $W_{\text{shaft}} = -(\pm U_{\text{in}} V_{\theta \text{in}}) + (\pm U_{\text{out}} V_{\theta \text{out}})$



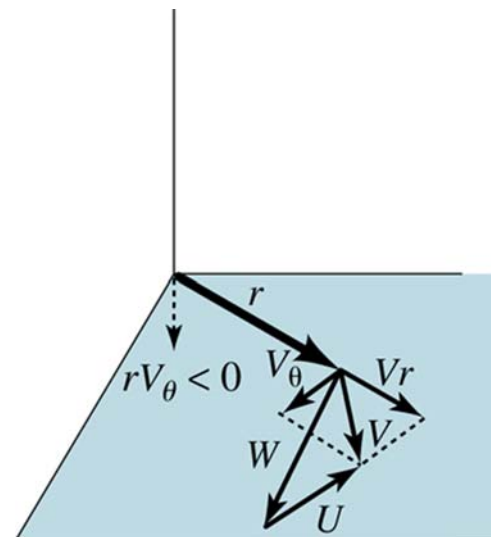
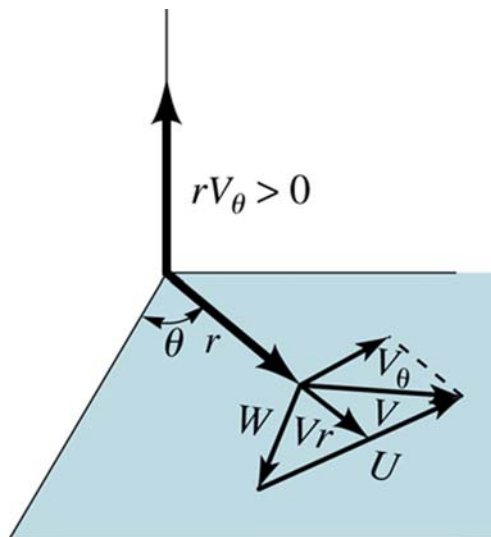
When shaft torque and shaft rotation are in the same (opposite) direction, power is into (out of) the fluid.

判斷 rV_θ 正負

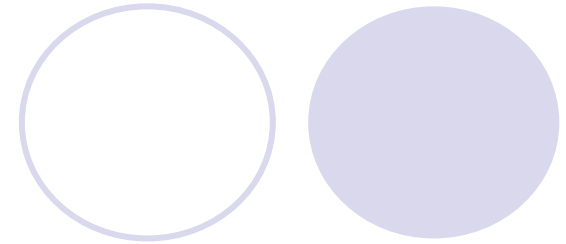


A simple way to determine the sign of the rV_θ product is to compare the direction of V_θ and the blade speed U .

If V_θ and U are in the same direction, the product rV_θ is positive.
If V_θ and U are in opposite direction, the product rV_θ is negative.

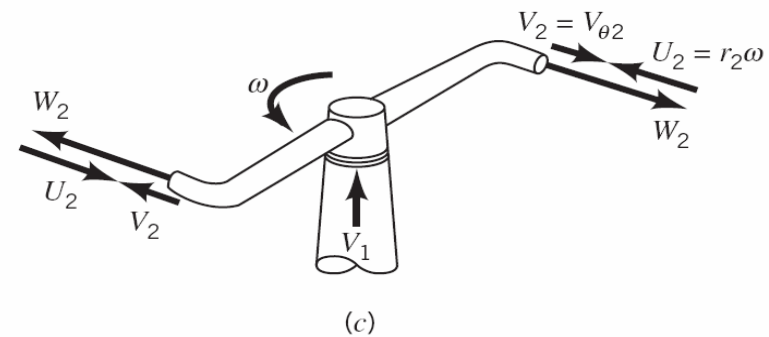
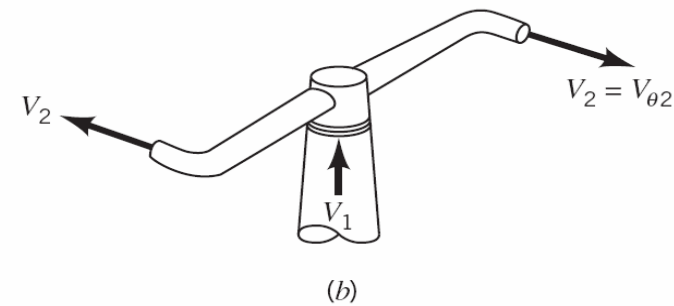
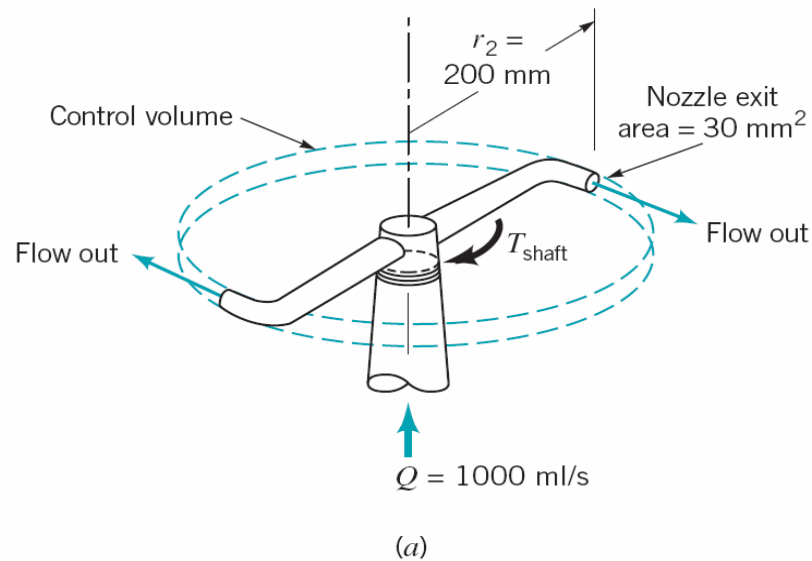


Example 5.18 Moment of Momentum – Torque ^{1/2}



- Water enters a rotating lawn sprinkler through its base at the steady rate of 1000 ml/s as sketched in Figure E5.18. The exit area of each nozzle is in the tangential direction. The radius from the axis of rotation to the centerline of each nozzle is 200mm. **(a) The resisting torque required to hold the sprinkler head stationary. (b) The resisting torque associated with the sprinkler rotating with a constant speed of 500rev/min. (c) The speed of the sprinkler if no resisting torque is applied.**

Example 5.18 Moment of Momentum – Torque ^{2/2}



■ FIGURE E5.18a–c

Example 5.18 Solution^{1/2}

(a) $T_{shaft} = -r_2 V_{\theta 2} \dot{m} \quad V_{\theta 2} = V_2$

$\Rightarrow T_{shaft} = -r_2 V_2 \dot{m} \quad \text{where } V_2 = 16.7 \text{ m/s from Example 5.7}$

$$\dot{m} = Q\rho = \frac{(1000 \text{ ml/s})(10^{-3} \text{ m}^3 / \text{liter})(999 \text{ kg/m}^3)}{(1000 \text{ ml/liter})} = 0.999 \text{ kg/s}$$

$$T_{shaft} = \frac{(200 \text{ mm})(16.7 \text{ m/s})(0.999 \text{ kg/s})[1(\text{N/kg})/(\text{m/s}^2)]}{(1000 \text{ ml/liter})} = -3.34 \text{ N} \cdot \text{m}$$

(b) $V_2 = W_2 - U_2$

where $W_2 = 16.7 \text{ m/s} \quad U_2 = r_2 \omega$

$$V_2 = 16.7 \text{ m/s} - \frac{(200 \text{ mm})(500 \text{ rev/min})(2\pi \text{ rad/rev})}{(1000 \text{ mm/m})(60 \text{ s/min})} = 6.2 \text{ m/s}$$

Example 5.18 Solution^{2/2}

$$T_{\text{shaft}} = -r_2 V_2 \dot{m}$$

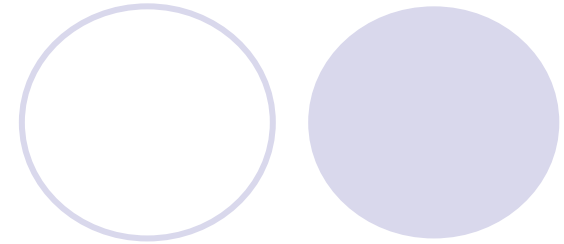
$$T_{\text{shaft}} = - \frac{(200\text{mm})(6.2\text{m/s})(0.999\text{kg/s})[1(\text{N/kg})/(\text{m/s}^2)]}{(1000\text{ml/liter})} = -1.24\text{N} \cdot \text{m}$$

(c)

$$T_{\text{shaft}} = -r_2 (W_2 - r_2 \omega) \dot{m} = 0$$

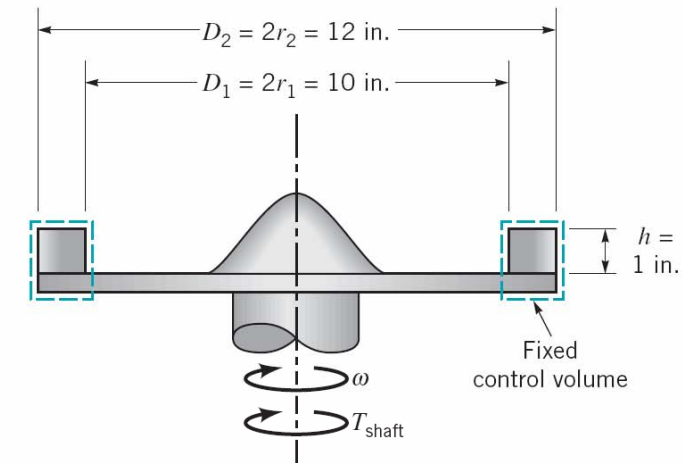
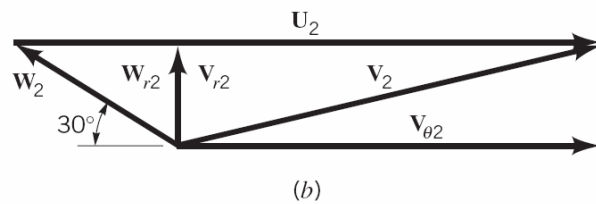
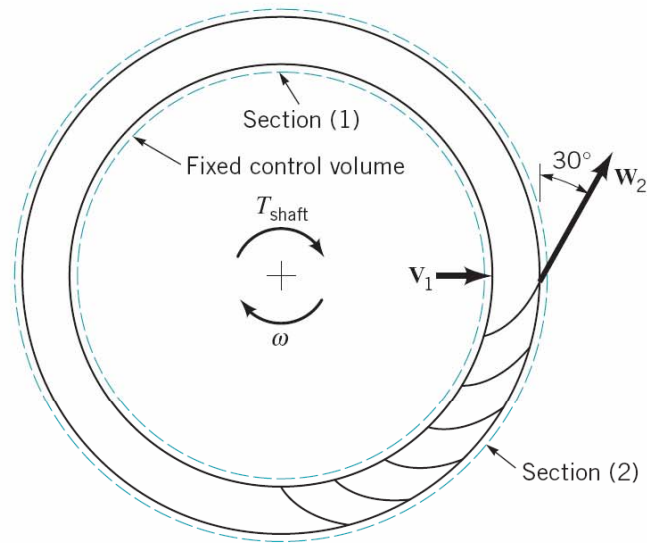
$$\omega = \frac{W_2}{r_2} = \frac{(16.7\text{m/s})(1000\text{mm/m})}{(200\text{mm})} = 83.5 \text{ rad/s} = 797\text{rpm}$$

Example 5.19 Moment of Momentum – Power ^{1/2}



- An air fan has a bladed rotor of 12-in. outside diameter and 10-in. inside diameter as illustrated in Figure E5.19a. The height of each rotor is constant at 1 in. from blade inlet to outlet. The flowrate is steady, on a time-average basis, at 230 ft³/min, and the absolute velocity of the air at blade inlet, V_1 , is radial. The blade discharge angle is 30° from the tangential direction. If the rotor rotates at a constant speed of 1725 rpm, **estimate the power required to run the fan.**

Example 5.19 Moment of Momentum – Power ^{2/2}



(a)

FIGURE E5.19

Example 5.19 Solution

0 (V_1 is radial)

$$\dot{W}_{\text{shaft}} = (-\dot{m}_1)(\pm U_1 V_{\theta 1}) + (\dot{m}_2)(\pm U_2 V_{\theta 2})$$

$$\dot{m} = \rho Q = \dots = 0.00912 \text{ slug/s}$$

$$U_2 = r_2 \omega = \frac{(6 \text{ in.})(1725 \text{ rpm})(2\pi \text{ rad/rev})}{(12 \text{ in./ft})(60 \text{ s/min})} = 90.3 \text{ ft/s}$$

$$V_2 = W_2 + U_2 \quad V_{\theta 2} = U_2 - W_2 \cos 30^\circ \quad W_2 \cos 30^\circ = V_{r2}$$

$$\dot{m} = \rho Q = \rho A_2 V_{r2} = \rho 2\pi r_2 h V_{r2}$$

$$\gg W_2 = \dot{m} / (\rho 2\pi r_2 h \sin 30^\circ) = \dots = 29.3 \text{ ft/s}$$

$$\dot{W}_{\text{shaft}} = \dot{m} U_2 V_{\theta 2} = \dots = 0.972 \text{ hp}$$

First Law of Thermodynamics – The Energy Equation^{1/5}

Based on system method

❖ The first law of thermodynamics **for a system** is

Time rate of increase of the total stored energy of the system = Net time rate of energy addition by heat transfer into the system + Net time rate of energy addition by work transfer into the system

$$\frac{d}{dt} \int_{\text{sys}} e \rho dV = \left(\sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} \right)_{\text{sys}} + \left(\sum \dot{W}_{\text{in}} - \sum \dot{W}_{\text{out}} \right)_{\text{sys}} = \left(\dot{Q}_{\text{net/in}} + \dot{W}_{\text{net/in}} \right)_{\text{sys}}$$

$$\text{or } \frac{d}{dt} \int_{\text{sys}} e \rho dV = \left(\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} \right)_{\text{sys}}$$

注意『正』、『負』

“+” going into system

“-” coming out

$$e = \hat{u} + \frac{V^2}{2} + gz$$

Total stored energy per unit mass for each particle in the system

The net rate of work transfer into the system

The net rate of heat transfer into the system

First Law of Thermodynamics –

The Energy Equation^{2/5}

- ❖ For the control volume that is coincident with the system at an instant of time. 在時間 t 的瞬間，進出 CV 的 Q 與 W 等於進出 system 者。透過這層連結，才能得到……

$$(\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}})_{\text{sys}} = (\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}})_{\text{coincident control volume}}$$

Review of Reynolds Transport Theorem

This is the fundamental relation between the rate of change of any arbitrary extensive property, B , of a system and the variations of this property associated with a control volume.

$$\begin{aligned}\frac{dB_{\text{sys}}}{dt} &= \frac{\partial B_{\text{CV}}}{\partial t} + \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n} dA \\ &= \frac{\partial}{\partial t} \int_{\text{CV}} \rho b dV + \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n} dA\end{aligned}$$

First Law of Thermodynamics – The Energy Equation^{3/5}

依據 Chapter 4 推導 Reynolds transport theorem 的步驟

- ❖ For the system and the contents of the coincident control volume that is fixed and nondeforming -- Reynolds Transport Theorem leads to 選擇一固定且不變形的CV = SYSTEM

$$\frac{d}{dt} \int_{\text{sys}} e \rho dV = \frac{\partial}{\partial t} \int_{\text{cv}} e \rho dV + \int_{\text{c.s.}} e \rho \vec{V} \cdot \vec{n} dA$$

解
讀

Time rate of increase
of the total stored
energy of the system

=

Net time rate of increase
of the total stored energy
of the contents of the
control volume

+

The net rate of flow of the
total stored energy out of
the control volume through
the control surface

$$b = e$$

$$B = e \cdot m$$

First Law of Thermodynamics –

The Energy Equation^{4/5}

❖ The control volume formula for the first law of thermodynamics:

$$\frac{\partial}{\partial t} \int_{cv} e \rho dV + \int_{cs} e \rho \vec{V} \cdot \vec{n} dA = (\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}})_{cv}$$

Based on Control Volume

NEXT PAGE

Rate of Work done by CV

$$\dot{W} = \dot{W}_{\text{Shaft}} + \dot{W}_{\text{normal}} + \dot{W}_{\text{shear}} + \dot{W}_{\text{other}}$$

❖ Shaft work $\dot{W}_{\text{Shaft}}$: the rate of work transferred into through the CS by the shaft work (negative for work transferred out, positive for work input required) 藉由shaft傳遞的功

❖ Work done by normal stresses at the CS:

$$\dot{W}_{\text{normal}} = \delta \vec{F}_{\text{normal}} \cdot \vec{V} = \int_{\text{CS}} \sigma_{nn} \vec{V} \cdot \vec{n} dA = - \int_{\text{CS}} p \vec{V} \cdot \vec{n} dA$$

❖ Work done by shear stresses at the CS:

$$\dot{W}_{\text{shear}} = + \int_{\text{CS}} \tau \vec{V} \cdot \vec{n} dA \quad \text{Negligibly small}$$

❖ Other work ✕

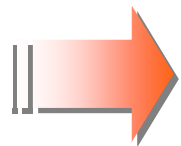
+輸入系統者，-輸出系統者

$$\Rightarrow \frac{\partial}{\partial t} \int_{\text{cv}} e \rho dV + \int_{\text{CS}} e \rho \vec{V} \cdot \vec{n} dA = \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} - \int_{\text{CS}} p \vec{V} \cdot \vec{n} dA$$

First Law of Thermodynamics –

The Energy Equation^{5/5}

$$\frac{\partial}{\partial t} \int_{cv} e \rho dV + \int_{cs} e \rho \vec{V} \cdot \vec{n} dA = \dot{Q}_{net \text{ in}} + \dot{W}_{Shaf \text{ net in}} - \int_{cs} p \vec{V} \cdot \vec{n} dA$$

 **Energy equation**

$$\frac{\partial}{\partial t} \int_{cv} e \rho dV + \int_{cs} \left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \vec{V} \cdot \vec{n} dA = \dot{Q}_{net / in} + \dot{W}_{Shaft / in}$$

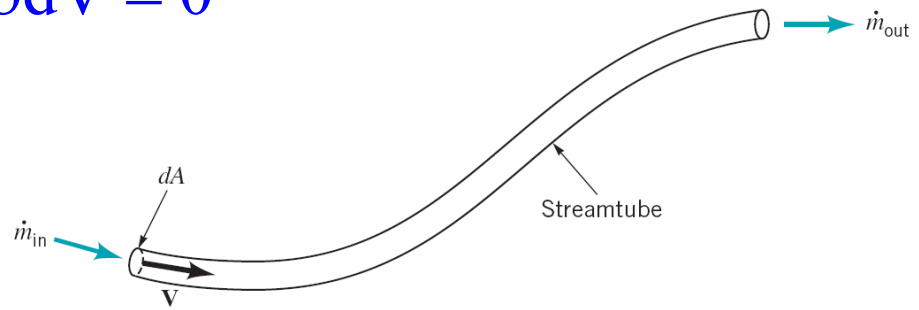
$$e = \hat{u} + \frac{V^2}{2} + gz$$

Application of Energy Equation^{1/3}

When the flow is steady $\frac{\partial}{\partial t} \int_{cv} e \rho dV = 0$

The integral of

$$\int_{cs} \left[\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right] \rho \vec{V} \cdot \vec{n} dA \quad ???$$



Special & simple case **Uniformly distribution**

$$\int_{cs} \left[\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right] \rho \vec{V} \cdot \vec{n} dA = \sum_{out} \left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \dot{m} - \sum_{in} \left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \dot{m}$$

**Only one stream
entering and leaving**

$$\begin{aligned} & \int_{cs} \left[\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right] \rho \vec{V} \cdot \vec{n} dA \\ &= \left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{out} \dot{m}_{out} - \left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{in} \dot{m}_{in} \end{aligned}$$

Special & simple case

Application of Energy Equation^{2/3}

If shaft work is involved.... 當 shaft work 包括進來

$$\dot{m} \left[\hat{u}_{\text{out}} - \hat{u}_{\text{in}} + \left(\frac{p}{\rho} \right)_{\text{out}} - \left(\frac{p}{\rho} \right)_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right]$$

$$= \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}}$$

← One-dimensional energy equation
for steady-in-the-mean flow

Enthalpy $\hat{h} = \hat{u} + \frac{p}{\rho}$

The energy equation is written in terms
of enthalpy.

$$\dot{m} \left[\hat{h}_{\text{out}} - \hat{h}_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net/in}} + \dot{W}_{\text{shaft net/in}}$$

Application of Energy Equation^{3/3}

If shaft work is zero....

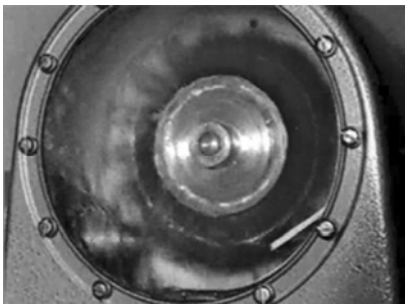
沒有shaft work

$$\dot{m} \left[\hat{u}_{\text{out}} - \hat{u}_{\text{in}} + \left(\frac{p}{\rho} \right)_{\text{out}} - \left(\frac{p}{\rho} \right)_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right]$$

$$= \dot{Q}_{\text{net in}}$$

One-dimensional energy equation
for steady-in-the-mean flow

$$\dot{m} \left[\hat{h}_{\text{out}} - \hat{h}_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net/in}}$$



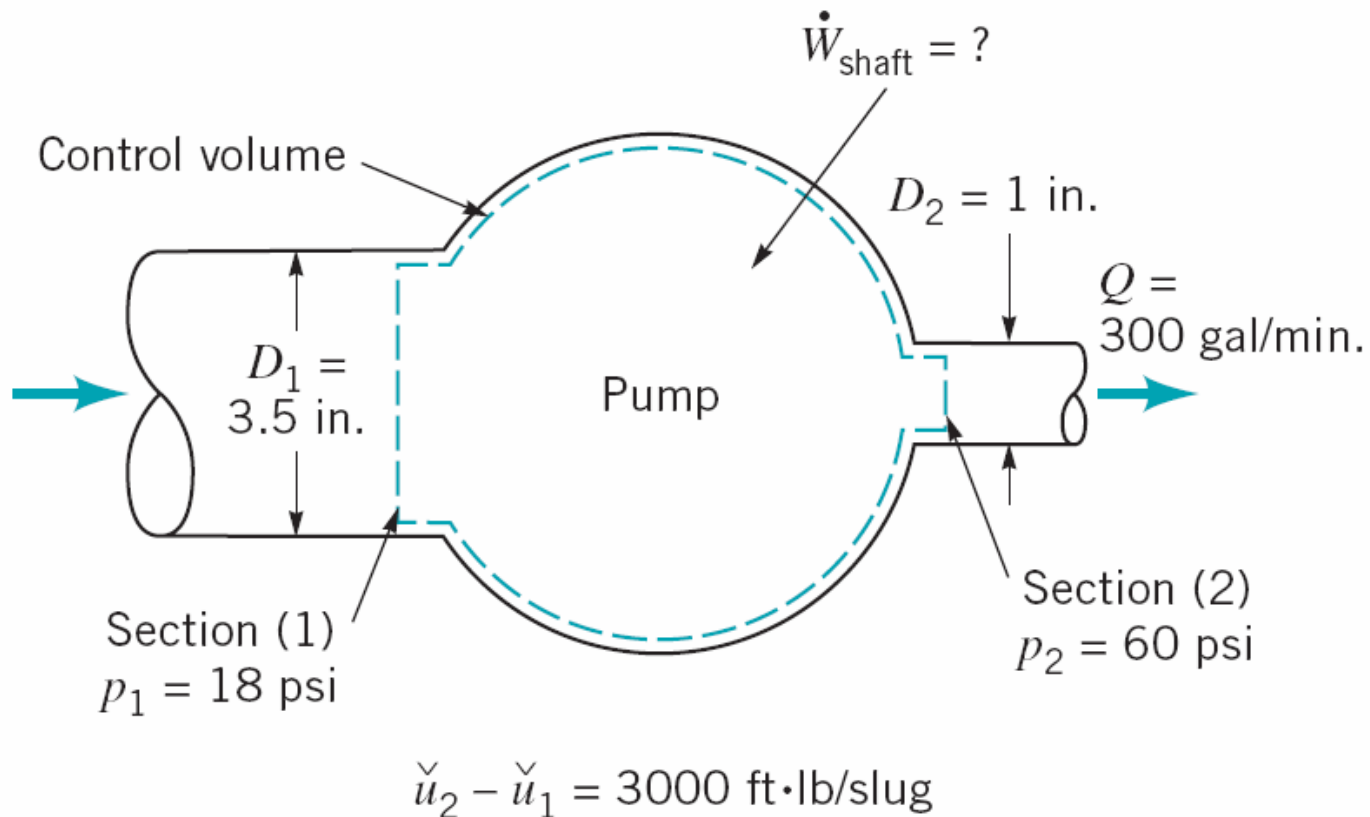
The Pelton wheel is among the most efficient types of water turbines. It was invented by Lester Allan Pelton (1829-1908) in the 1870s



Example 5.20 Energy – Pump Power ^{1/2}

- A pump delivers water at a steady rate of 300 gal/min as shown in Figure E5.20. Just upstream of the pump [section(1)] where the pipe diameter is 3.5 in., the pressure is 18 psi. Just downstream of the pump [section (2)] where the pipe diameter is 1 in., the pressure is 60 psi. The change in water elevation across the pump is zero. The rise in internal energy of water, $u_2 - u_1$, associated with a temperature rise across the pump is 3000 ft·lb/slug. If the pumping process is considered to be adiabatic, determine the power (hp) required by the pump. 絕熱條件：沒有 Q 進出 計算輸入功

Example 5.20 Energy – Pump Power ^{2/2}



Example 5.20 Solution

One-dimensional energy equation for steady-in-the-mean flow

$$\dot{m} \left[\hat{u}_2 - \hat{u}_1 + \left(\frac{p}{\rho} \right)_2 - \left(\frac{p}{\rho} \right)_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$

= 0 (Adiabatic flow)

$$= \dot{Q}_{\text{net/in}} + \dot{W}_{\text{shaft net/in}}$$

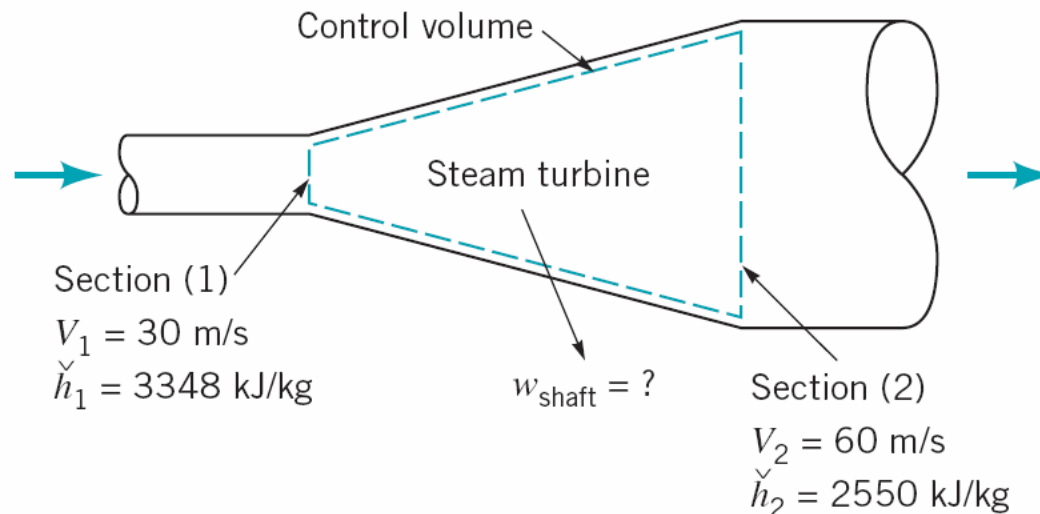
$$\dot{m} = \rho Q = \frac{(1.94 \text{ slug/ft}^3)(300 \text{ gal/min})}{(7.48 \text{ gal/ft}^3)(60 \text{ s/min})} = 1.30 \text{ slugs/s}$$

$$V = \frac{Q}{A} = \frac{Q}{\pi D^2 / 4} \quad V_1 = \frac{Q}{A_1} = \dots = 10.0 \text{ ft/s} \quad V_2 = \frac{Q}{A_2} = \dots = 123 \text{ ft/s}$$

$$\dot{W}_{\text{shaft net in}} = (1.30 \text{ slugs/s})[\dots] = 32.3 \text{ hp}$$

Example 5.21 Energy – Turbine Power per Unit Mass of Flow

- Steam enters a turbine with a velocity of 30 m/s and enthalpy, h_1 , of 3348 kJ/kg. The steam leaves the turbine as a mixture of vapor and liquid having a velocity of 60 m/s and an enthalpy of 2550 kJ/kg. If the flow through the turbine is adiabatic and changes in elevation are negligible, determine the work output involved per unit mass of steam through-flow. 絕熱條件：沒有 Q 進出 計算輸出功



Example 5.21 Solution

The energy equation in terms of enthalpy.

$$\dot{m} \left[\hat{h}_2 - \hat{h}_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = \dot{Q}_{\text{net/in}} + \dot{W}_{\text{shaft net/in}}$$

=0 (Adiabatic flow)

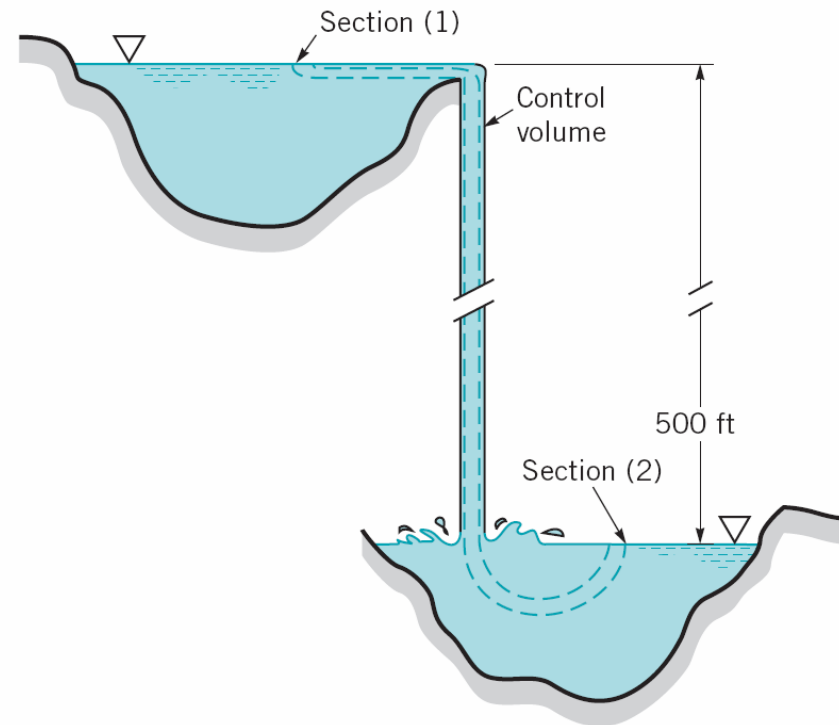
$$\dot{W}_{\text{shaft net in}} = \frac{\dot{W}_{\text{shaft net in}}}{\dot{m}} = \hat{h}_2 - \hat{h}_1 + \frac{V_2^2 - V_1^2}{2}$$

$$\dot{W}_{\text{shaft net out}} = -\dot{W}_{\text{shaft net in}}$$

$$\dot{W}_{\text{shaft net out}} = \hat{h}_1 - \hat{h}_2 + \frac{V_1^2 - V_2^2}{2} = \dots = 797 \text{ kJ/kg}$$

Example 5.22 Energy – Temperature Change

- A 500-ft waterfall involves steady flow from one large body of water to another. Determine the temperature change associated with this flow.



Example 5.22 Solution

The temperature change is related to the change of internal energy of the water

$$T_2 - T_1 = \frac{\hat{u}_2 - \hat{u}_1}{\check{c}}$$

where $\check{c} = 1 \text{ Btu}/(\text{lbm} \cdot ^\circ\text{R})$ is the specific heat of water

One-dimensional energy equation for steady-in-the-mean flow without shaft work

$$\dot{m} \left[\hat{u}_2 - \hat{u}_1 + \left(\frac{p_2}{\rho} \right) - \left(\frac{p_1}{\rho} \right) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = \dot{Q}_{\text{net in}}$$

$V_2 = V_1$
 $= 0$ (Adiabatic flow)

$$T_2 - T_1 = \frac{g(z_2 - z_1)}{\check{c}} = \frac{(32.2 \text{ ft/s}^2)(500 \text{ ft})}{[778 \text{ ft} \cdot \text{lb}/(\text{lbm} \cdot ^\circ\text{R})][32.2 (\text{lbm} \cdot \text{ft})/(\text{lb} \cdot \text{s}^2)]} = 0.643 ^\circ\text{R}$$

Energy Equation vs. Bernoulli Equation 1/4

$$\dot{m} \left[\hat{u}_{\text{out}} - \hat{u}_{\text{in}} + \left(\frac{p}{\rho} \right)_{\text{out}} - \left(\frac{p}{\rho} \right)_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}}$$

$$e = \hat{u} + \frac{V^2}{2} + gz$$

For steady, incompressible flow... One-dimensional energy equation

$$\dot{m} \left[\hat{u}_{\text{out}} - \hat{u}_{\text{in}} + \left(\frac{p_{\text{out}}}{\rho} \right) - \left(\frac{p_{\text{in}}}{\rho} \right) + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net in}}$$

沒有 shaft work
有 normal stress 做的功

$$\div \dot{m} \Rightarrow \frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} - (\hat{u}_{\text{out}} - \hat{u}_{\text{in}} - q_{\text{net in}})$$

where $q_{\text{net in}} = \dot{Q}_{\text{net in}} / \dot{m}$

For steady, incompressible, **frictionless flow**...

參照 chapter 3

$$p_{\text{out}} + \frac{\rho V_{\text{out}}^2}{2} + \gamma z_{\text{out}} = p_{\text{in}} + \frac{\rho V_{\text{in}}^2}{2} + \gamma z_{\text{in}}$$

Bernoulli equation

$$\hat{u}_{\text{out}} - \hat{u}_{\text{in}} - q_{\text{net in}} = 0$$

Frictionless flow...
沒有摩擦損失

Energy Equation & Bernoulli Equation 2/4

For steady, incompressible, **frictional flow...**

$$\hat{u}_{\text{out}} - \hat{u}_{\text{in}} - q_{\text{net in}} > 0 \quad \text{Frictional flow...}$$

Defining “useful or available energy”... $\frac{p}{\rho} + \frac{V^2}{2} + gz$ Loss發生在in
→out過程中

Defining “loss of useful or available energy”... $\hat{u}_{\text{out}} - \hat{u}_{\text{in}} - q_{\text{net in}} = \text{loss}$



$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}}^{\text{下游}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}}^{\text{上游}} - \text{loss}$$

因為1→2有摩擦損失， $\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$ 自然而然就低於 $\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1$

Energy Equation & Bernoulli Equation 3/4

For steady, incompressible flow with friction and shaft work...

有摩擦損失有軸功進來

$$\dot{m} \left[\hat{u}_{\text{out}} - \hat{u}_{\text{in}} + \left(\frac{p_{\text{out}}}{\rho} \right) - \left(\frac{p_{\text{in}}}{\rho} \right) + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}}$$

在 in $\rightarrow$ out 注入

$$\div \dot{m} \Rightarrow \frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\text{shaft net in}} - (\hat{u}_{\text{out}} - \hat{u}_{\text{in}} - q_{\text{net in}})$$

$$\Rightarrow \frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\text{shaft net in}} - \text{loss}$$

在 in $\rightarrow$ out 注入

$$\div g \Rightarrow \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_s - h_L$$

$$\text{Shaft head } h_s = \frac{w_{\text{shaft net/in}}}{g} \equiv \frac{\dot{W}_{\text{shaft net/in}}}{\dot{m}g} = \frac{\dot{W}_{\text{shaft net/in}}}{\gamma Q} \quad \text{Head loss } h_L = \frac{\text{loss}}{g}$$

Energy Equation & Bernoulli Equation 4/4

$$\frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_s - h_L$$

in → out 輸出

❖ For turbine $h_s = -h_T$ ($h_T > 0$) **h_T is turbine head**

❖ For pump $h_s = h_p$ **h_p is pump head** in → out 輸入

❖ The actual head drop across the turbine

$$h_T = -(h_s + h_L)_T$$

想像：讓loss擴大

❖ The actual head drop across the pump

$$h_p = (h_s - h_L)_p$$

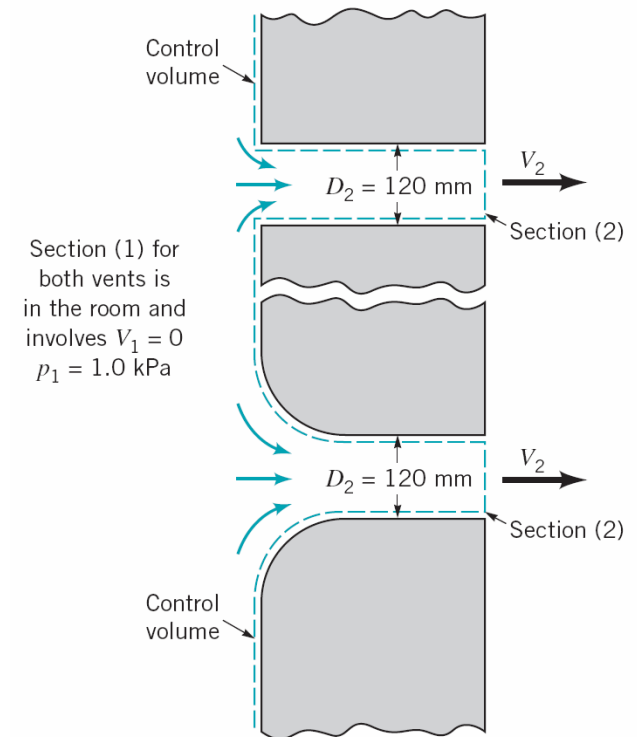
想像：讓loss減緩

Energy transfer



Example 5.23 Energy – Effect of Loss of Available Energy

- Compare the volume flowrates associated with two different vent configurations, a cylindrical hole in the wall having a diameter of 120 mm and the same diameter cylindrical hole in the wall but with a well-rounded entrance (see Figure E5.23a). The room pressure is held constant at 0.1 kPa above atmospheric pressure. Both vents exhaust into the atmosphere. As discussed in Section 8.4.2, the loss in available energy associated with flow through the cylindrical bent from the room to the vent exit is $0.5V_2^2/2$ where V_2 is the uniformly distributed exit velocity of air. The loss in available energy associated with flow through the rounded entrance vent from the room to the vent exit is $0.05V_2^2/2$, where V_2 is the uniformly distributed exit velocity of air.



Example 5.23 Solution

For steady, incompressible flow with friction, the energy equation

$V_1=0$ No elevation change

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1} - \text{loss}_2$$

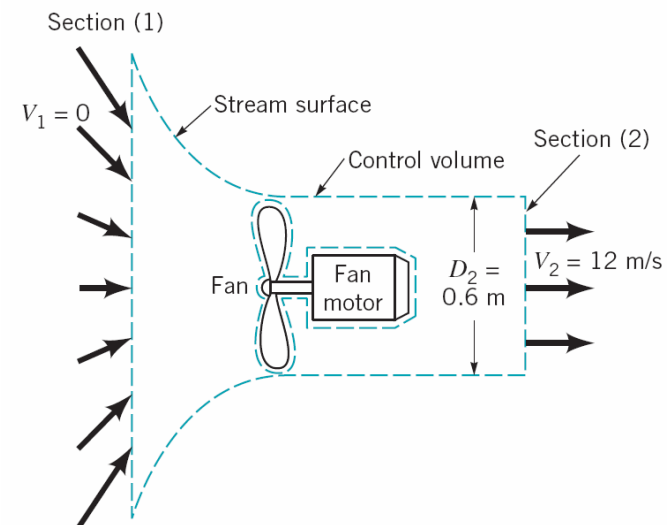
$$V_2 = \sqrt{2 \left[\left(\frac{p_1 - p_2}{\rho} \right) - \text{loss}_2 \right]} \quad \text{loss}_2 = K_L \frac{V_2^2}{2}$$

$$V_2 = \sqrt{\frac{p_1 - p_2}{\rho [1 + K_L / 2]}}$$

$$Q = A_2 V_2 = \frac{\pi D_2^2}{4} \sqrt{\frac{p_1 - p_2}{\rho [1 + K_L / 2]}}$$

Example 5.24 Energy – Fan Work and Efficiency

- An axial-flow ventilating fan driven by a motor that delivers 0.4 kW of power to the fan blades produces a 0.6-m-diameter axial stream of air having a speed of 12 m/s. The flow upstream of the fan involves negligible speed. Determine how much of the work to the air actually produces a useful effects, that is, a rise in available energy and estimate the fluid mechanical efficiency of this fan.



Example 5.24 Solution

For steady, incompressible flow with friction and shaft work...

$$W_{\text{shaft net in}} - \text{loss} = \left(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 \right) - \left(\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \right)$$

$p_1 = p_2 = \text{atmospheric pressure}$, $V_1 = 0$, no elevation change

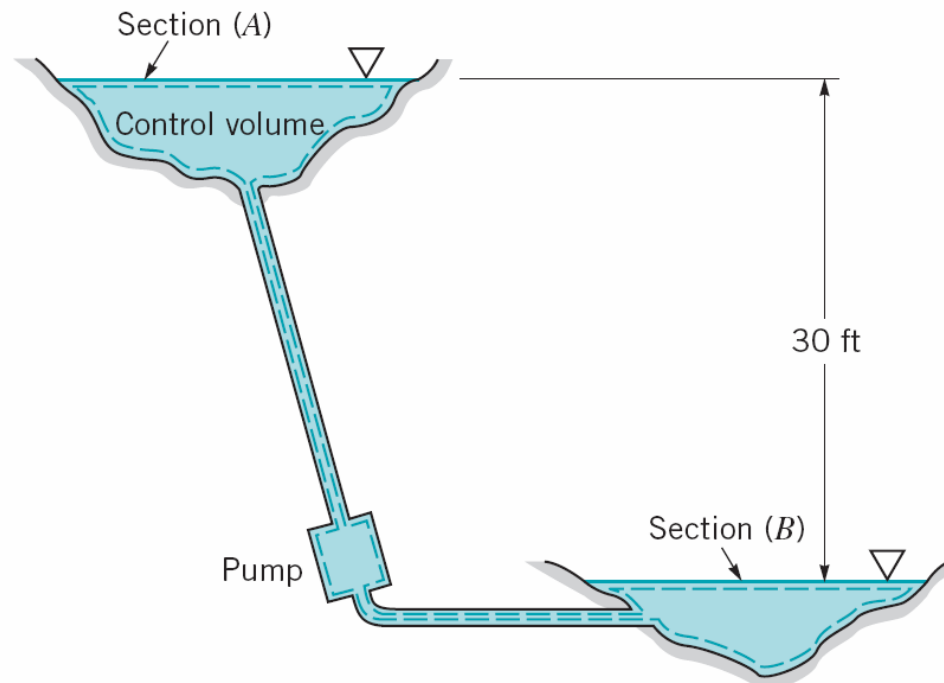

$$W_{\text{shaft net in}} - \text{loss} = \frac{V_2^2}{2} = 72.0 \text{ N} \cdot \text{m} / \text{kg}$$

Efficiency $\eta = \frac{W_{\text{shaft net in}} - \text{loss}}{W_{\text{shaft net in}}}$

$$W_{\text{shaft net in}} = \frac{\dot{W}_{\text{shaft net in}}}{\dot{m}} = \frac{\dot{W}_{\text{shaft net in}}}{\rho AV} = 95.8 \text{ N} \cdot \text{m} / \text{kg}$$

Example 5.25 Energy – Head Loss and Power Loss

- The pump shown in Figure E5.25 adds 10 horsepower to the water as it pumps water from the lower lake to the upper lake. The elevation difference between the lake surfaces is 30 ft and the head loss is 15 ft. Determine the flowrate and power loss associated with this flow.



Example 5.25 Solution

The energy equation

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_s - h_L$$

$$p_A = p_B = 0 \quad V_A = V_B = 0$$

The pump head

$$h_s = h_L + z_A - z_B = \frac{\dot{W}_{\text{shaft net/in}}}{\gamma Q} = 88.1 / Q$$

Power loss $\dot{W}_{\text{loss}} = \gamma Q h_L = \dots$

Application of Energy Equation to Nonuniform Flows ^{1/2}

If the velocity profile at any section where flow crosses the control surface is not uniform...

$$\int_{\text{C.S.}} \frac{V^2}{2} \rho \vec{V} \cdot \vec{n} dA$$

????

當進出CS的流體速度分布不是uniform

For one stream of fluid entering and leaving the control volume....

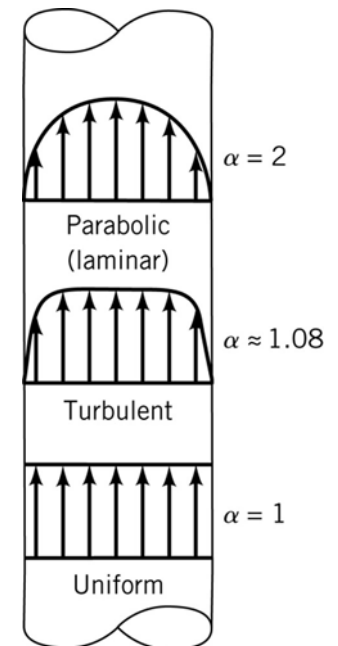
以平均值取代

$$\int_{\text{CS}} \frac{V^2}{2} \rho \vec{V} \cdot \vec{n} dA = \dot{m} \left(\frac{\alpha_{\text{out}} \tilde{V}_{\text{out}}^2}{2} - \frac{\alpha_{\text{in}} \tilde{V}_{\text{in}}^2}{2} \right)$$

Where α is the kinetic energy coefficient and V is the average velocity

新增參數

$$\alpha = \frac{\int_A \left(\frac{V^2}{2} \right) \rho \vec{V} \cdot \vec{n} dA}{\dot{m} \left(\frac{\bar{V}^2}{2} \right)} \geq 1$$



Application of Energy Equation to Nonuniform Flows ^{2/2}

For nonuniform velocity profile.....

$$\frac{p_{\text{out}}}{\rho} + \frac{\alpha_{\text{out}} \bar{V}_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{\alpha_{\text{in}} \bar{V}_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\text{shaft net in}} - \text{loss}$$

$\times \rho$ 

$$p_{\text{out}} + \frac{\rho \alpha_{\text{out}} \bar{V}_{\text{out}}^2}{2} + \gamma z_{\text{out}} = p_{\text{in}} + \frac{\rho \alpha_{\text{in}} \bar{V}_{\text{in}}^2}{2} + \gamma z_{\text{in}} + \rho w_{\text{shaft net in}} - \rho(\text{loss})$$

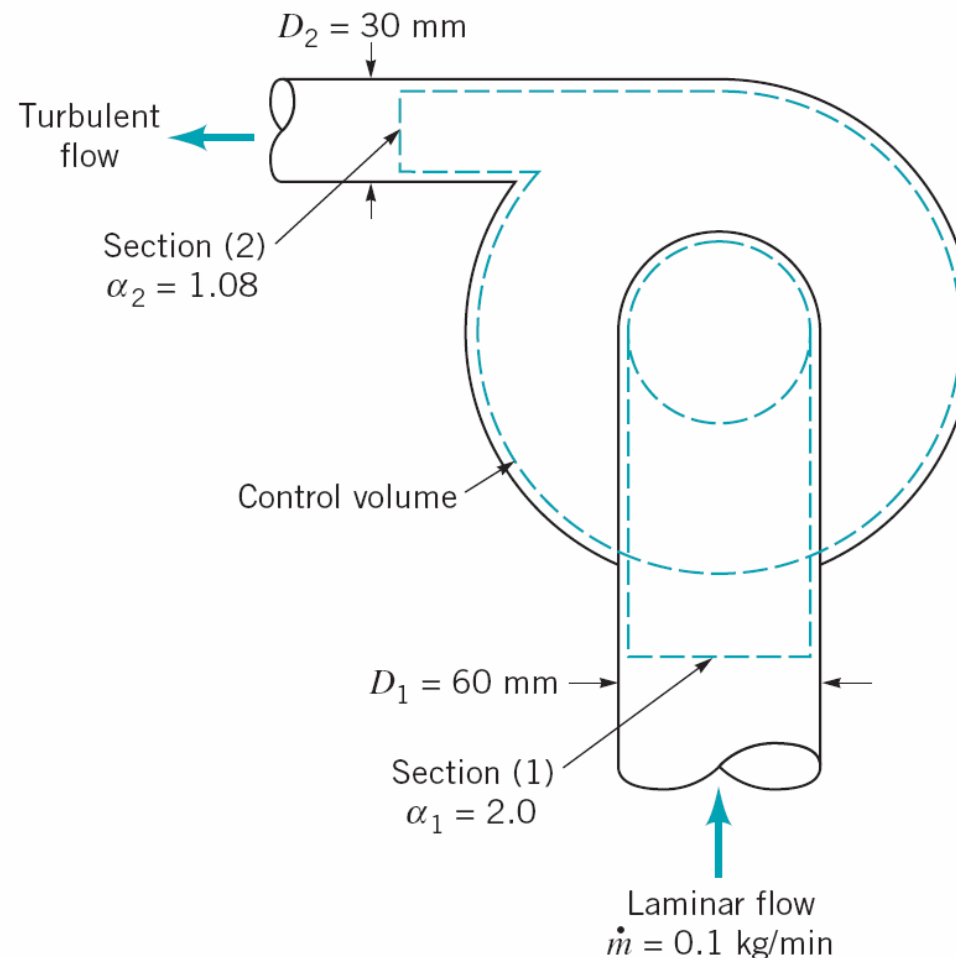
$\div g$ 

$$\frac{p_{\text{out}}}{\gamma} + \frac{\alpha_{\text{out}} \bar{V}_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{\alpha_{\text{in}} \bar{V}_{\text{in}}^2}{2g} + z_{\text{in}} + \frac{w_{\text{shaft net in}}}{g} - h_L$$

Example 5.26 Energy – Effect of Nonuniform Velocity Profile ^{1/2}

- The small fan shown in Figure E5.26 moves air at a mass flowrate of 0.1 kg/min. Upstream of the fan, the pipe diameter is 60 mm, the flow is laminar, the velocity distribution is parabolic, and the kinetic energy coefficient, α_1 , is equal to 2.0. Downstream of the fan, the pipe diameter is 30 mm, the flow is turbulent, the velocity profile is quite uniform, and the kinetic energy coefficient, α_2 , is equal to 1.08. If the rise in static pressure across the fan is 0.1 kPa and the fan motor draws 0.14 W, compare the value of loss calculated: (a) assuming uniform velocity distributions, (2) considering actual velocity distribution.

Example 5.26 Energy – Effect of Nonuniform Velocity Profile ^{2/2}



Example 5.26 Solution^{1/2}

The energy equation for nonuniform velocity profile.....

$$\frac{p_2}{\rho} + \frac{\alpha_2 \bar{V}_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{\alpha_1 \bar{V}_1^2}{2} + gz_1 + w_{\text{shaft net/in}} - \text{loss}$$


$$\text{loss} = w_{\text{shaft net in}} - \left(\frac{p_2 - p_1}{\rho} \right) + \frac{\alpha_1 \bar{V}_1^2}{2} - \frac{\alpha_2 \bar{V}_2^2}{2}$$

$$w_{\text{shaft net/in}} = \frac{\text{power to fan motor}}{\dot{m}}$$
$$= \frac{(0.14 \text{ W})[(1 \text{ N} \cdot \text{m/s}) / \text{W}]}{0.1 \text{ kg/min}} (60 \text{ s/min}) = 84.0 \text{ N} \cdot \text{m/kg}$$

$$\bar{V}_1 = \frac{\dot{m}}{\rho A_1} = \dots = 0.479 \text{ m/s}$$

$$\bar{V}_2 = \frac{\dot{m}}{\rho A_2} = \dots = 1.92 \text{ m/s}$$

Example 5.26 Solution^{1/2}


$$\text{loss} = w_{\text{shaft net/in}} - \left(\frac{p_2 - p_1}{\rho} \right) + \frac{\alpha_1 \bar{V}_1^2}{2} - \frac{\alpha_2 \bar{V}_2^2}{2}$$
$$= 0.975 \text{ N} \cdot \text{m} / \text{kg} (\alpha_1 = \alpha_2 = 1)$$


$$\text{loss} = w_{\text{shaft net/in}} - \left(\frac{p_2 - p_1}{\rho} \right) + \frac{\alpha_1 \bar{V}_1^2}{2} - \frac{\alpha_2 \bar{V}_2^2}{2}$$
$$= 0.940 \text{ N} \cdot \text{m} / \text{kg} (\alpha_1 = 2, \alpha_2 = 1.08)$$

Example 5.28 Energy – Fan Performance

- For the fan of Example 5.19, show that only some of the shaft power into the air is converted into a useful effect. Develop a meaningful efficiency equation and a practical means for estimating lost shaft energy.

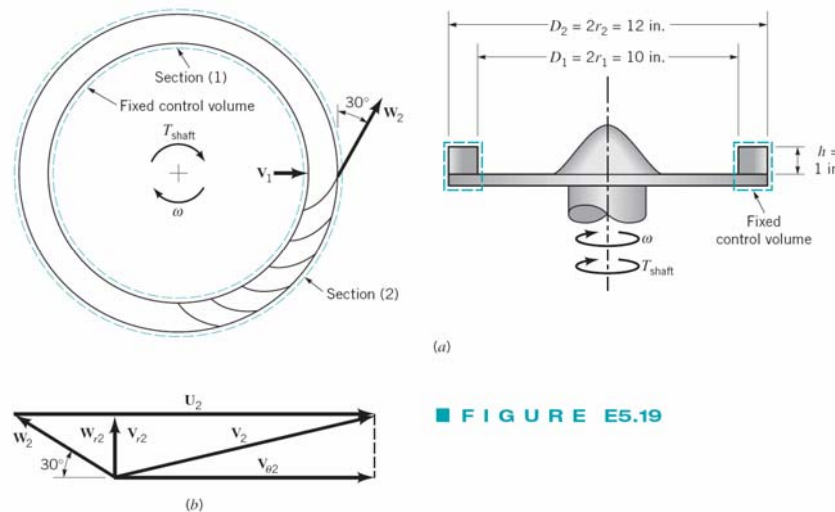


FIGURE E5.19

Example 5.28 Solution^{1/2}

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 + w_{\text{shaft net in}} - \text{loss} \quad (1)$$

$$\text{useful effect} = w_{\text{shaft net/in}} - \text{loss}$$

$$= \left(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 \right) - \left(\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \right) \quad (2)$$

$$\text{Efficiency} \quad \eta = \frac{w_{\text{shaft net in}} - \text{loss}}{w_{\text{shaft net in}}} \quad (3)$$

$$w_{\text{shaft net in}} = +U_2 V_{\theta 2} \quad (4)$$

Example 5.28 Solution^{2/2}

(2)+(3)+(4)

$$\eta = \{[(p_2 / \rho) + (V_2^2 / 2) + gz_2] - [(p_1 / \rho) + (V_1^2 / 2) + gz_1]\} / U_2 V_{\theta 2}$$

(2)+(4)

$$\eta = U_2 V_{\theta 2} - [(p_2 / \rho + V_2^2 / 2 + gz_2) - (p_1 / \rho + V_1^2 / 2 + gz_1)]$$

First Law of Thermodynamics – For Semi-infinitesimal CV ^{1/2}

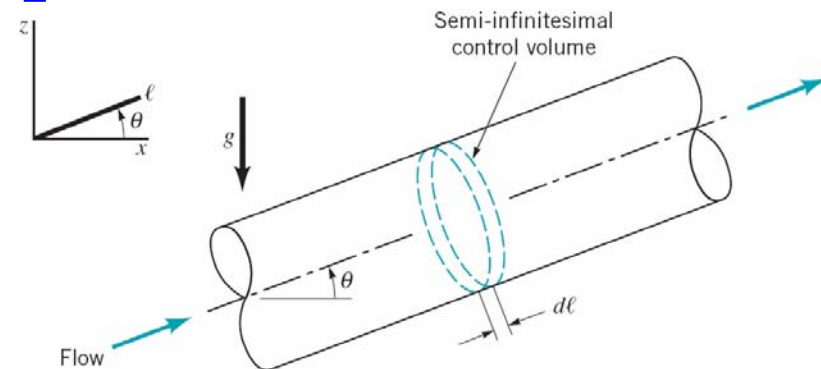
- ❖ Applying the one-dimensional, steady flow energy equation to the content of a semi-infinitesimal control volume

$$\dot{m} \left[\hat{u}_{\text{out}} - \hat{u}_{\text{in}} + \left(\frac{p_{\text{out}}}{\rho} \right) - \left(\frac{p_{\text{in}}}{\rho} \right) + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net in}}$$


$$\dot{m} \left[d\hat{u} + d\left(\frac{p}{\rho} \right) + d\left(\frac{V^2}{2} \right) + g(dz) \right] = \delta \dot{Q}_{\text{net in}}$$

semi-infinitesimal control volume

CV非有限大，也非無限小，故將 in 與 out 間的變化以 difference來表達



First Law of Thermodynamics – For Semi-infinitesimal CV ^{2/2}

$$\oplus Tds = d\tilde{u} + pd\left(\frac{1}{\rho}\right) \quad \text{For all pure substances including common engineering working fluids, such as air, water, oil, and gasoline}$$


$$\dot{m} \left[Tds - pd\left(\frac{1}{\rho}\right) + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g(dz) \right] = \delta \dot{Q}_{\text{net in}}$$



Semi-infinitesimal control volume statement of the energy equation

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + gdz = -(Tds - \delta q_{\text{net in}})$$

Second Law of Thermodynamics – Irreversible Flow 1/3

❖ A general statement of the second law of thermodynamics

The time rate of increase of the entropy of a system $\geq$

Sum of the ratio of net heat transfer rate into system to absolute temperature for each particle of mass in the system receiving heat from surroundings

$$\frac{d}{dt} \int_{\text{sys}} s \rho dV \geq \sum \left[\frac{\delta \dot{Q}_{\text{net in}}}{T} \right]_{\text{sys}}$$

Based on system method

依據 Chapter 4 推導 Reynolds transport theorem 的步驟

❖ For the system and the contents of the coincident control volume that is fixed and nondeforming -- Reynolds

Transport Theorem leads to 選擇一固定且不變形的CV = SYSTEM

$$\frac{d}{dt} \int_{\text{sys}} s \rho dV = \frac{\partial}{\partial t} \int_{\text{cv}} s \rho dV + \int_{\text{c.s.}} s \rho \vec{V} \cdot \vec{n} dA$$

Second Law of Thermodynamics – Irreversible Flow 2/3

- ❖ For the system and control volume at the instant when system and control volume are coincident

$$\sum \left[\frac{\delta \dot{Q}_{\text{net in}}}{T} \right]_{\text{sys}} = \sum \left[\frac{\delta \dot{Q}_{\text{net in}}}{T} \right]_{\text{cv}}$$

在時間 t 的瞬間，進出 CV 的 Q 等於進出 system 者。透過這層連結，才能得到……

- ❖ The control volume formula for the **second law of thermodynamics**

$$\frac{\partial}{\partial t} \int_{\text{cv}} s \rho dV + \int_{\text{cs}} s \rho \vec{V} \cdot \vec{n} dA \geq \sum \left(\frac{\delta \dot{Q}_{\text{net in}}}{T} \right)_{\text{cv}}$$

Based on Control Volume

Second Law of Thermodynamics – Irreversible Flow ^{3/3}

For one stream of fluid entering and leaving the control volume.... Special & simple case

$$\dot{m}(s_{\text{out}} - s_{\text{in}}) \geq \sum \frac{\delta \dot{Q}_{\text{net in}}}{T} \quad \text{Steady flow}$$



Semi-infinitesimal thin CV

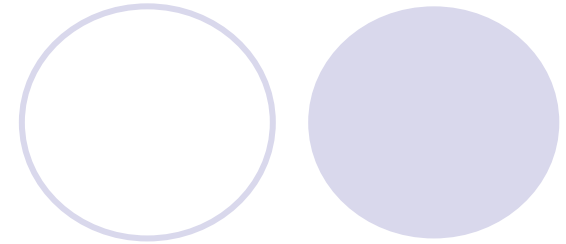
$$\dot{m}ds \geq \sum \frac{\delta \dot{Q}_{\text{net in}}}{T}$$



Uniform temperature

$$Tds - \delta q_{\text{net in}} \geq 0$$

First and Second Law of Thermodynamics 1/4



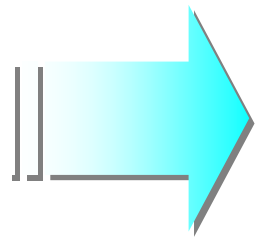
CV
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Semi-infinitesimal control volume statement of the energy equation

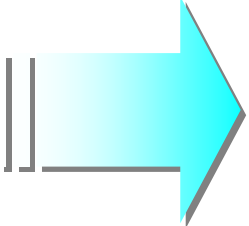
$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + gdz = -(Tds - \delta q_{\text{net in}})$$

Semi-infinitesimal CV of the second law of thermodynamics

$$Tds - \delta q_{\text{net in}} \geq 0$$


$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + gdz\right] \geq 0$$

First and Second Law of Thermodynamics 2/4


$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2} \right) + g dz \right] = \delta(\text{loss}) = (T ds - \delta q_{\text{net in}})$$

For steady frictionless flow

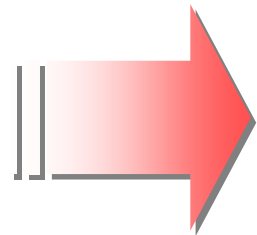
$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2} \right) + g dz \right] = 0$$

The shaft work is involved

$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2} \right) + g dz \right] = \delta(\text{loss}) - \delta w_{\text{shaft net in}}$$

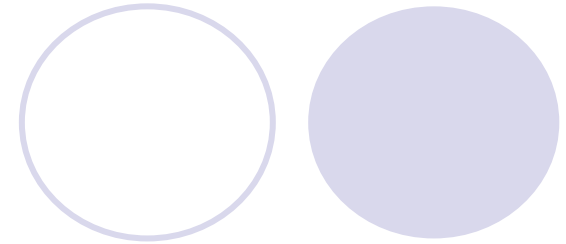
First and Second Law of Thermodynamics ^{3/4}

$$\left\{ \begin{array}{l} - \left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g dz \right] = \delta(\text{loss}) = (T ds - \delta q_{\text{net in}}) \\ \oplus T ds = d\tilde{u} + p d\left(\frac{1}{\rho}\right) \end{array} \right.$$


$$d\tilde{u} + p d\left(\frac{1}{\rho}\right) - \delta q_{\text{net in}} = \delta(\text{loss})$$

For incompressible flow $d\tilde{u} - \delta q_{\text{net in}} = \delta(\text{loss})$

First and Second Law of Thermodynamics 4/4



When control volume is finite

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} + \int_{\text{in}}^{\text{out}} p d\left(\frac{1}{\rho}\right) - q_{\text{net in}} = \text{loss}$$

For incompressible flow $\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}} = \text{loss}$

Application of the Loss Form^{1/2}

$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + gdz \right] = \delta(\text{loss}) - \delta W_{\text{shaft net in}}$$

Frictionless, loss=0, no shaft work, incompressible

||  Integrating $\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1$ Bernoulli equation

Frictionless, loss=0, no shaft work, compressible

||  Integrating $\int_1^2 \frac{dp}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{V_1^2}{2} + gz_1$

Application of the Loss Form^{2/2}

For adiabatic flow of an ideal gas

$$\frac{p}{\rho^k} = \text{const}$$



$$\int_1^2 \frac{dp}{\rho} = \frac{k}{k-1} \left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} \right)$$



$$\frac{k}{k-1} \frac{p_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 = \frac{k}{k-1} \frac{p_1}{\rho_1} + \frac{V_1^2}{2} + gz_1$$