Water flowing in a PVC sprinkler pipe

The page concerns water flowing in any pipe, but we imagine it to be sprinkler pipe.

Imagine a PVC pipe whose center line lies along the z axis of a coordinate system. Inside the pipe, the distance to a point in the flowing water from that center line is called r. If the pipe has an inside radius b, then clearly $0 \le r \le b$.

1. As seems intuitive, all particles of water in the pipe flow in the z direction, so $\mathbf{v} = v\hat{\mathbf{z}}$ where **v** is the velocity vector saying how every particle of water flows, and v is the speed of the water.

2. In general, this velocity **v** *could* be a function of both r and z. But in a pipe, it is constant as you move along the pipe in z, and varies only in r. So one has then **v**(r), and not **v**(r,z). The water velocity does not slow down as you move along the pipe along some line. Rather, it stays constant. But the velocity of the water varies radially in the pipe.

3. The velocity profile **v**(r) is parabolic in shape going across the pipe. It is a maximum at r = 0, on the center line, and it is 0 at the walls of the pipe, at r = b. We return to this shape in a moment. This is similar to the flow of water in a river which is maximal on the center line and zero at the edges. The river case is called Plane Poiseuille flow (pwa-zay'), while the pipe case is called Hagen-Poiseuille flow.

4. In general, the water pressure p *could* be a function of both r and z. But in a pipe, pressure depends only on z, and is therefore constant on any cross sectional disk of the pipe. So we have p(z) and not p(r,z).

5. If the pipe is of length L and if there is pressure p_1 at one end and p_2 at the other, with $p_1 > p_2$, then there is a "pressure gradient" α as you go along the pipe (assume in the z direction) which is this

$$\alpha \equiv - dp(z)/dz = (p_1 - p_2)/L > 0$$ α is the same anywhere along the pipe

For example, one might have p_1 = 200 psi, and p_2 = 14.7 psi if water dumps out into the atmosphere. The pressure decreases linearly along the pipe, dropping linearly from p_1 at the driving end to p_2 at the far end. If you plot the pressure p(z) along the pipe, it is a straight line with slope -α.

6. Water has a viscosity which is represented by a parameter μ. Inside the pipe, imagine a set of very thin concentric layers of water each shaped like a cylindrical shell. Imagine a particular such shell. The shell just outside of it tries to slow that shell's flow down due to the frictional drag of this viscosity at the boundary between the shells. The outermost shell is slowed down by the walls of the pipe which are moving at 0. That is why the water speed varies radially and is maximum along the center line.

7. The equation which describes the velocity of water in a PVC pipe is this

$$v(r) = (\alpha/4\mu)(b^2 - r^2)$$ "parabolic shape"

where b is the pipe's inside radius, α is the pressure gradient, μ is the viscosity, and r is the radial coordinate which ranges from 0 to b. To find the velocity of water on the center line, we set r = 0 to find that $v_{center} = (\alpha/4\mu)b^2$. You can see that, at the walls of the pipe, v(b) = 0.

8. The total water flow Q (say, gallons/min) through such a pipe is given by

$Q = (\pi/128)(\alpha/\mu)\, d^4$ d = 2b = inside diameter of the pipe

where again α is the pressure gradient, μ is the viscosity of water. Notice that if you double the pressure gradient α, you double v(r) everywhere and you double Q.

9. The average velocity of water flowing in the pipe (averaged across the cross section) turns out to be exactly 1/2 the velocity at the center, so $v_{average} = (\alpha/8\mu)b^2$.

10. Example: Standard sprinkler PVC pipe has an inside diameter of roughly d = 2 cm. Suppose the pipe is 10 meters long (about 30 feet) and has a pressure difference of p_1-p_2 = 50 psi, so then α = 50 psi/30 feet. Using this α number and the known value of μ for water, one finds that Q = 21 gallons/minute. This means that, more generally, for a PVC sprinkler pipe of length L, one can say

$Q(\text{gallons/min}) \approx 21 * (p_1-p_2)/50 * (30/L) * (d/2)^4$ p in psi, L in feet, d in cm

In Torrey area where lots of irrigation is done, they use PVC pipe of diameter larger than 2 cm. Notice from above that if you double the pipe diameter d, Q becomes 16 times larger, a result that is probably not intuitively obvious. This is also an issue with fire hoses.

11. A fluid which has a viscosity μ which does not vary with the nature of the flow is called a Newtonian fluid. Ketchup is a non-Newtonian fluid, water is a Newtonian fluid. In either case, μ varies with temperature.

12. The set of equations which govern the flow of all fluids are called the Navier-Stokes equations. These are partial differential equations in which **v** is the unknown, and x,y,z,t are the coordinates. The equations are quite complicated, but when applied to water flowing through a pipe, you get the relatively simple and practical results shown above.