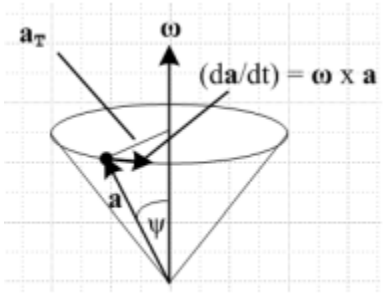
Page 134 confusion in Goldstein PhL 11.20.16

I am in the midst of a review of Goldstein, and I am now starting into a very difficult section (for me) which starts on page 134 and culminates in (4-103) which concerns a vector **ω** expressed in terms of Euler angle quantities. I will now look into how **ω** appears in frames doc.

1. What is **ω**?

I start by talking about what it means to rotate an arbitrary vector **a** about some axis and I get:



(a)

$$d\mathbf{a} = d\boldsymbol{\varphi} \times \mathbf{a} \qquad (1.22)$$

Dividing by dt gives

$$(d\mathbf{a}/dt) = \boldsymbol{\omega} \times \mathbf{a} \qquad \text{where } \boldsymbol{\omega} \equiv d\boldsymbol{\varphi}/dt \quad . \qquad (1.23)$$

This is nothing more than a description of the picture. Here a could be any vector. I could if I wanted attach a rotating frame S' to this vector, so that $(d\mathbf{a}/dt)_{S'} = 0$. Then the above equation can be interpreted as

$(d\mathbf{a}/dt)_S = \boldsymbol{\omega}\mathbf{x}\mathbf{a}$ for a vector **a** which is fixed in frame S'

so this is the G-rule with the first term vanishing

$$(d\mathbf{a}/dt)_S = (d\mathbf{a}/dt)_{S'} + \boldsymbol{\omega}\mathbf{x}\mathbf{a}$$

The angle dφ is relative to the ω rotation axis $\boldsymbol{\omega} = \omega\hat{\mathbf{n}}$ and in fact ω = dφ/dt. Thus the vector **ω** appears in my frames doc.

More generally in my opening frames doc picture, which is drawn in Frame S, you see the origin of Frame S' rotating about some arbitrary axis $\hat{\boldsymbol{\omega}}$ at rate ω, so this is my general meaning of **ω**.

2. What does this have to do with any Euler Angles?

Early on I write

$$\mathbf{e}_n = R\,\mathbf{e}'_n \qquad n = 1,2,3 \qquad \text{or} \qquad (\mathbf{e}_n)_i = R_{ij}(\mathbf{e}'_n)_j \tag{1.1}$$

where R is the rotation which connects the S and S' axes. I could write R in terms of Euler Angles, so that might be the connection.

Question: Why did I choose R in this manner? It seems to conflict with tensor doc where I would say something like $\mathbf{V}' = R\mathbf{V}$ for a vector. In tensor doc I say

$$\mathbf{e}_n \equiv S\mathbf{e}'_n \ . \tag{3.2.4}$$

which says $\mathbf{e}'_n = R\,\mathbf{e}_n$ which does conflict with (1.1) above. Ouch!

But there is this notion floating around somewhere that rotating the basis vectors one way is like rotating the vectors the other way. For "active" I rotate a vector **V** one way, and this is equivalent to "passive" where I instead rotate the basis vectors the other way. I do show in above (1.9) in frames doc that these are compatible

$$\mathbf{e}_n = R\mathbf{e}'_n \qquad \text{and} \qquad \mathbf{a}' = R\mathbf{a}$$

This is a *very annoying* topic and I need to get it 100% clarified.

In tensor doc the $\mathbf{e}'_n$ are axis-aligned basis vectors in x'-space.

The $\mathbf{e}_n$ are the tangent base vectors in x-space, and they are NOT aligned with x-space axes. The x-space aligned axes are called $\mathbf{u}_i$ in tensor doc. What then is the connection between the $\mathbf{e}'_n$ axis-aligned vectors in x'-space and the $\mathbf{u}_n$ axis-aligned vectors in x-space?

STOP. This confusion needs its own definitive separate document so I can nail it down hard.