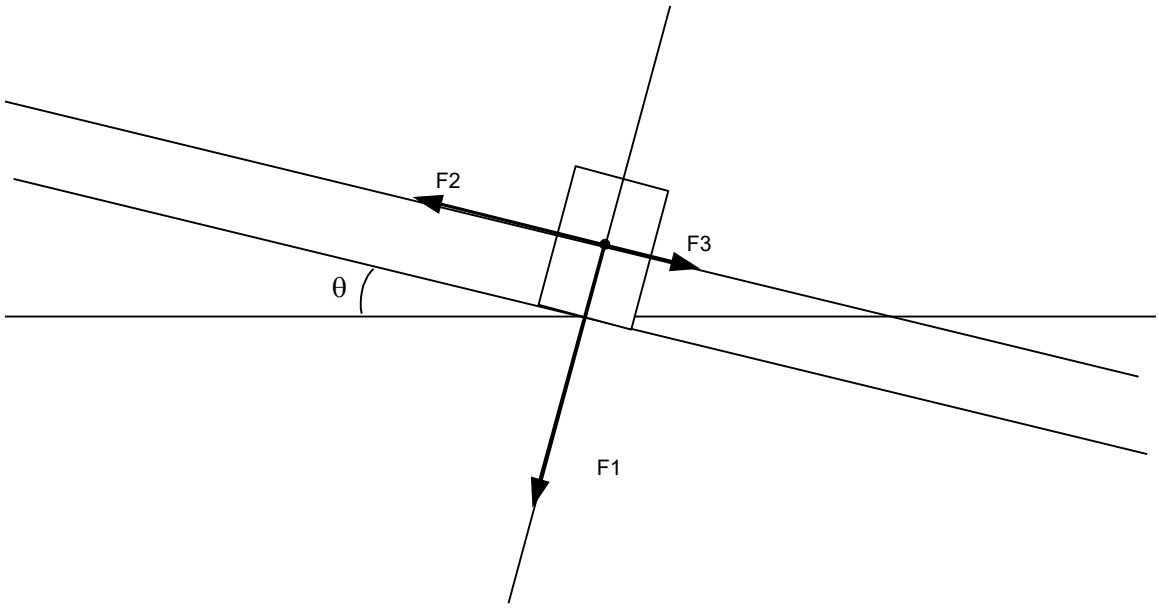

The key to this problem, like so many problems, it drawing a clear picture first. The above is one possibility for such a picture.

I think the "full" answer is that the car just starts slipping when

$| F_2 - F_3 | = c\, F_1$ $\qquad\qquad$ $c = .55$

The problem is figuring out what these three forces are.

F_1 has a positive contribution from gravity and centrifugal force:

$F_1 = mg \cos\theta + (mv^2/r) \sin\theta$ $\qquad\qquad$ // draw more detailed picture so see why, triangle deal

I guess I could have written these as two separate arrows. This F_1 is the force between the car and road perp to the road, so it is the one that friction applies to. Then we have the other forces as I drew them:

$F_2 = (mv^2/r) \cos\theta$ $\qquad\qquad$ // component of centrifugal force parallel to tilted road (to the left)

Then component of gravity force pushing car to the right parallel to the tilted road (to the right)

$F_3 = mg \sin\theta$

So this says (abs value needed since sign of $F_2 - F_3$ can be either way depending on the tilt)

$| F_2 - F_3 | = c\, F_1 \Rightarrow$

$| (mv^2/r) \cos\theta - mg \sin\theta | = c\, [mg \cos\theta + (mv^2/r) \sin\theta]$

which looks like a mess to solve for θ using standard algebra tricks (see below).

If we are on perfect black ice so c = 0, (which is the question asked in effect), then the solution is simpler,

$F_2 = F_3$

or

$(mv^2/r)\cos\theta = mg\sin\theta$

or

$\tan\theta = v^2/(rg) \sim (21)^2 / (80*9.8) = .5625$ $\theta = 29.4$ degrees

********** continued full solution, not required for problem asked ********

Mass m cancels so

$|\ (v^2/r)\cos\theta - g\sin\theta\ | = c\ [\ g\cos\theta + (v^2/r)\sin\theta]$

Result is dimensionally correct. I can test the result for the two extreme cases.

$\theta = 0$ $(v^2/r) = c\ g$ $v = \sqrt{crg}$ // this was your 21 m/sec answer

$\theta = 90$ $g = c\ (v^2/r)$

Here the "sideways" force on the car parallel to the road is the full mg gravity force.
The car stays on this vertically tilted road as long as $.55\ (mv^2/r) \geq (mg)$ which is the last $\theta = 90$ result.
Since equation is correct at the two extremes, it has a reasonable chance of being correct in between.

If I write the above equation as $A\cos\theta = B\sin\theta$ I can solve for $\tan\theta = A/B$. But have to do that in two separate "regimes";

If $(v^2/r)\cos\theta - g\sin\theta \geq 0$, which means if $\tan\theta \leq v^2/(rg)$, then I get this answer:

$\tan\theta = [\ (v^2/r) - cg\ ] / [\ c(v^2/r) + g\ ]$

Dimensionally correct. At $\theta = 0$ get the $v = \sqrt{crg}$ result as above.

If $(v^2/r)\cos\theta - g\sin\theta \leq 0$, which means if $\tan\theta \geq v^2/(rg)$, then I get this answer:

$\tan\theta = [\ (v^2/r) + cg\ ] / [\ - c(v^2/r) + g\ ]$

Dimensionally correct. At $\theta = 90$ get the $g = c\,(v^2/r)$ result as above.

The absolute value adds a definite complication to this problem. I have not checked my answers.
This is a pretty fancy problem. I have seen the circus motorcycle demo where they guy is inside a screen cage and does the $\theta = 90$ case. And I have seen tilted roadbeds. A very practical problem.
I guess the ideal would be to set the tilt so you get zero sideways force on the car at the posted speed limit. Then as rain changes c, it does not matter.