Flywheels PhL 10.22.04

The formula of interest is $E = 1/2\ I\omega^2$ where I is the moment of inertia

$I = \int m(r)r^2\, dV = MR^2$ for a rim , can write this as $I = \rho\ V\ R^2$

where ρ is the density of the flywheel material and V is its volume.

The "tensile stress" is just the force on a particle in the rim, centrifugal force. A particle at radius R would have an acceleration toward the center of $a = v\ d\theta/dt = v\omega = v^2/R = R\omega^2$. And force $ma = mv^2/R = m\omega^2 R$.

Tensile Force. You have a flat annular ring of inner ratios a and outer radius R . Consider the volume $dV = t\ r d\theta\ dr$ where r is between a and R somewhere. The force we have to supply toward the center to keep this element in rotation is

$dF = \rho dV * \omega^2 * r = \rho\ t\ r\ d\theta\ dr\ \omega^2\ r$

where t is our ring thickness in the z direction. This force acts on our little boundary area which is $t\ r\ d\theta$, so the force per unit area which is sort of a pressure (but a negative pressure, which is stress) would be

$dT = \rho\ \omega^2\ r\ dr$

This would be the tensile stress due just to our little mass dV tugging on our area dA. But all masses out to R are tugging together, so have to add them all up to get:

$$T = \int_a^R \rho\ \omega^2\ r\ dr = \rho\ \omega^2 \int_r^R r\ dr = (\ \rho\ \omega^2\ /2\ )\ [\ R^2 - r^2]$$

So stuff outside our location at r is pulling with this tensile stress. The maximal stress is at the smallest r, so we have

$T_{max} = (\ \rho\ \omega^2\ /2\ )\ [\ R^2 - a^2]$

For a flywheel rotor, there has to be some point where things attach to the central axis. It is a spoke in effect, and the worse case stress is at the center where a = 0 and the above formula gives

$T_{max} = (1/2)\ \rho\ \omega^2\ R^2$

Half way out the spoke you have less since a = R/2, then $[\ R^2 - a^2] = 3/4\ R^2$.

Now, we can write

$E = 1/2\ I\ \omega^2$ with $I \sim \rho V R^2$ so $E = 1/2\ \rho V R^2\ \omega^2 = V\ T_{max}$

where we imagine a design where almost all the mass is out at R, but of course we still need our spokes. Now we can fudge this with some fraction since all mass won't really be at R to get:

$E = f\ V\ T$ maybe $f = .75$

What does this say? The stored energy in a flywheel is proportional to the volume of the material times the tensile stress. The maximum you can store then is

$E_{max} = f\ V\ T_{max}$

because if you run it faster, it rips apart. This result is interesting. What you need is a large volume, and a large tensile force. The mass is not the important factor. This is why, for a given volume, you prefer a strong lightweight fiber rather than steel! Fibers have good T_{max} but have much less weight, so your flywheel can be light (and high RPM). I think fancy fibers can actually have 5x the T_{max} of steel.

A Pascal is a nt/m^2 , just the mks pressure unit. Here are some tensile strengths: (giga pascals)

Material	Tensile Strength (GPa)
cast iron	0.17
steel alloy	0.76
stainless steel	0.86
titanium	0.90
SiC	3.5
Kevlar	3-4

The claim is that a C-C bond is 200 GPa. Spider silk is 1.3 GPa ! 63 GPa has been measured for Bucky nanotubes.

Car comments. For a car, you want lightweight flywheel, so fiber very attractive.

But what about for a cheap wind-power storage unit in Torrey. Who cares how much it weighs? What we care about is how much it costs per unit if energy stored.

Note that 1 Gpa = 145,038 PSI.

Back to Energy Stored

$E = 1/2\ I\omega^2$ $I = f\ \rho\ V\ R^2 = f\ MR^2$ $T = T_{max} = (1/2)\ \rho\ \omega^2\ R^2$

$E = f\ MR^2\ \omega^2 / 2$

$E(joules) = f\ M(kg) R(m)^2\ \omega(sec^{-1})^2 / 2$

$M(kg) = M(lb)/2.2$

R(m) = R(ft)/3.28 1 foot = 12 in = 12*2.54 = 30.48 cm = .3048 m 1 m = 3.28 ft

E(joules) = f M(lb)/2.2 R(ft) ^{2} (1/3.28)^2 ω(sec^{-1}) ^{2} / 2 = .042 f M(lb)R(ft) ^{2} ω(sec^{-1}) ^{2} / 2

Compute either way:

m = 1000 lb
R = 10 feet
ω = 1 RPS = 6.28 radians/sec

E(joules) ~ 42,000

Now, kWh = 1000 watts* 3600 sec = 3.6 x 10^6 joules

42,000 J = .012 kWh = 12 watt-hours

So our huge wheel is useless. You have to have high RPM to make it go. The wheel could drive a 12 watt light bulb for an hour, not too bad, or a 100 watt bulb for .12 hours = 7 minutes.

OK, back to T_{max} = (1/2) ρ ω^2 R^2 Suppose made of water-cement, then ρ = 1 gm/cm^3 = 1,000 kg/m^3 . So then at this speed we have T_{maz} = 1/2 * 1000 * 40 * 3^2 = 200,000 Pascals = 0.2 MPa. I found a site that says concrete is about 4 MPa on tensile strength, so we could spin up to about 4x faster and increase storage by 16X then you have 200 watt-hours and you are doing 4 RPS. I suppose you could build a steel frame to hold your concrete with an outer metal rim, then you could go up to maybe .5 GPa or 500 MPa which is 500/.2 = 2500 x more, so you can then have 2500 * more energy than the 12 watt-hours or 30 kWh. Now you have something! But this thing is screaming around now at 50 X more ROM, so you are now doing 50 rotations per second or 3000 RPM. Water pumping seems so much safer!

What can you store in a reservoir of 100 m^3 that is 10 m up in the air? E = mhg = ρVgh = 1000*100*10*10 = 10^7 J = 3 kWh.

Comment: a nice feature of the high-ω low mass unit is that the bearing is not so much a gravitational problem! Now that 1 ft x 1 ft unit the site talks about doing 3 kWh looks pretty good! forget all that water and cement, just roll in t his little storage unit!

Conclusions of this Section

1. If you try to make a huge and cheap flywheel out of cement or some such, your problem is this: energy storage is only linear in M, so you need lots of Mass M, as in thousands of pounds. But that makes the bearing problem very unpleasant -- you have a Merry Go Round really. You can try and drive your mass out to a large radius, but then machine is too bulky. You just don't get big storage numbers with low RPM. You have to build a steel frame to hold it together.

2. The benefit of the light fiber high-RPM approach is this: (1) E is quadratic in ω, so you get your numbers up; (2) the mass stays low, so the bearing is not such a big problem. (3) the size of the whole unit stays small.

3. Maybe you could just rotate some large molecules instead at very high RPM. Or optically charge a small cubic crystal.

4. Makes the fuel cell seem interesting again. Chemical storage is good because the bond is so strong. Why not store energy as starch. A biological approach.