Active vs Passive in my docs PhL 11.23.16

This one page note sets out to review all my rotation-related docs, but does not get far! But I do realize a key thing here which is the strong association between tensor doc's way of doing things and covariant equations. I also get a hint that maybe I can use $u'_n = R^{-1}u_n$ as an equation to construct a version of the active/passive story which relates to tensor doc, but I don't actually do it here.

1. Tensor Doc

It includes the following equations, for example

$$\mathbf{e}'_n = R(\mathbf{x})\,\mathbf{e}_n \ . \qquad (3.3.2)$$

$$\mathbf{v}' = R(\mathbf{x})\,\mathbf{v} \ . \qquad (2.3.2)$$

$$\{\ \mathbf{V}',\ \mathbf{V}\ \} \qquad \mathbf{V}' \equiv R(\mathbf{x})\mathbf{V} \qquad \text{contravariant vector} \ . \qquad (2.9.2)$$

I do mention "frames of reference" in several places. A main idea is this one

> exact same form in different frames of reference which are related by a transformation. For example, one might have **F** = m**a** in frame S, and **F**' = m'**a**' in frame S', where these frames are related by a static rotation. **F** and **a** are tensorial vectors with respect to this rotation, and m and m' are tensorial scalars, and

The idea is that in Frame S' a covariant equation "has the same form but everything is primed". In the above example I would write **F**' = R**F** and **a**' = R**a** and m' = m, but I don't interpret this beyond implying that the Frame S' person would measure primed quantities. Frame appears a lot in App K about deformation tensors. But below K.5.26 I say this

Interpretation of the adjusted stress tensors discussed above.

In Chapter 2 we discuss the notion of the transformation of a contravariant vector $\mathbf{V}' = R\mathbf{V}$ in developmental notation. In x'-space, the vector components are $V'_i = R_{ij}V_i$ where V_i are the components in x-space. If R is a rotation matrix, then the unit basis vectors in the two spaces can be taken as Cartesian, call them $\mathbf{u}'_n$ in x'-space and $\mathbf{u}_n$ in x-space. We have these two expansions of $\mathbf{V}$:

$$\mathbf{V} = \Sigma_n \; V_n \mathbf{u}_n = \Sigma_n \; V'_n \mathbf{u}'_n \qquad \text{where} \qquad V_n = \mathbf{V} \bullet \mathbf{u}_n \qquad V'_n = \mathbf{V} \bullet \mathbf{u}'_n \; . \tag{K.5.25}$$

In the "active view" of things, we can think of $\mathbf{V}' = R\mathbf{V}$ as creating a new vector $\mathbf{V}'$ in x-space from the old vector $\mathbf{V}$ by rotating the vector $\mathbf{V}$ by R. In the "passive view", we think of the V'_i as the components of the original vector $\mathbf{V}$ projected onto the backwards-rotated basis vectors $\mathbf{u}'_n = R^{-1} \mathbf{u}_n$. To verify this relation between the basis vectors, we can write

$$V'_n = \mathbf{V} \bullet \mathbf{u}'_n \; = \mathbf{V} \bullet R^{-1}\mathbf{u}_n = R\mathbf{V} \bullet RR^{-1}\mathbf{u}_n = R\mathbf{V} \bullet \mathbf{u}_n = \mathbf{V}' \bullet \mathbf{u}_n = V'_n \; . \tag{K.5.26}$$

So one can think *either* of $\mathbf{V}$ being rotated forward in x-space into $\mathbf{V}'$ where $\mathbf{V}'$ has x-space components V'_n, *or* one can think of the V'_n as the components of $\mathbf{V}$ one measures in frame that is backwards rotated by R^{-1}, that is, $\mathbf{u}'_n = R^{-1} \mathbf{u}_n$.

This is the only place that the word "active" appears in tensor doc, and this is my *usual* active vs passive discussion.