The Distinction between the Inverse Problem and the Swap Notation PhL 9.10.12

Before I introduced the "swap" and "non-swap" notations into frames doc, I was relying on the inverse equations as being my forward swap equations. But you need to take ω → -ω and b → -b to make this work, and that is hard to keep track of. So I included both swap and non-swap results in both my forward and inverse problem summaries.

In frames doc the inverse problem is, I think, clearly stated. The "original problem" has frames S and S' and we compute Frame S things in terms of Frame S' things. In the inverse problem, we compute Frame S' things in terms of Frame S things. The activities in the two frames don't change. The physical situation does not change, we are just doing the inverse computations.

I show that the inverse equations can be obtained from the forward equations by a set of swap rules stated in (13.6), and one of these rules is ω ↔ -ω.

This all seems reasonable, but here is where I get confused. Must we consider Frame S' to be the non-inertial frame? Can't we choose either frame to be non-inertial? Do we make a decision on this somewhere? In my Section 3, I do say that Frame S' is non-inertial in my general description of the actions of the Observer there. And in my Fig 4.1 picture, I do imply that Frame S' is non-inertial. I don't require in that picture that Frame S be inertial, but I do imply that Frame S' is "rotating". I browse now to the start of Section 6 after the goal is stated in Section 5.

Go back to Section 2 and the G Rule. In that section, Frame S' is "rotating" relative to Frame S. Is the G Rule only valid if Frame S' is non-inertial and Frame S is inertial? I don't think so. I think this just has to do with rotation of one frame relative to another. I don't think "inertial" enters the picture until you actually talk about F = ma.

If that is correct, then we don't necessarily assume either S or S' is inertial or non-inertial until we get way down to Section 8 on the fictitious forces. Sections 6 and 7 regarding velocities and accelerations just use the G Rule, and I have just argued that it does not require either frame to be inertial. Of course if one frame IS inertial, then the other must be NON inertial. But they could both be non-inertial, or either one could be inertial.

So let's now browse into Section 8. Right off the bat we assume Frame S is inertial, maybe I should be stating this more clearly. Then we have true Newton's Law $\mathbf{F} = m\mathbf{a}$ in (8.1). And then we end up with the usual fictitious force stuff and Coriolis force is $-2m\,\boldsymbol{\omega} \times \mathbf{v}'$.

But suppose we assume Frame S' is inertial right at this point? The problem with assuming that is only that we have equations like this

$$\mathbf{a} = \mathbf{a}' + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + \ddot{\mathbf{b}}_S \qquad (7.6a)$$

S S' Euler Coriolis centripetal frame

which contain primed objects on the right, and this makes the bogus Newton's law work correctly since everything there is primed. For example,

$$m\mathbf{a}' = \mathbf{F}'_{\mathtt{eff}} = \mathbf{F} - m\ddot{\mathbf{b}}_S - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') - 2m\,\boldsymbol{\omega} \times \mathbf{v}' - m\dot{\boldsymbol{\omega}} \times \mathbf{r}' \qquad (8.4a)$$

$$m\mathbf{a}' = \mathbf{F}'_{\mathtt{eff}} = m\mathbf{a} - m\ddot{\mathbf{b}}_S - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') - 2m\,\boldsymbol{\omega} \times \mathbf{v}' - m\dot{\boldsymbol{\omega}} \times \mathbf{r}' \qquad (8.4b)$$

So this is not the right place to assume that Frame S' is inertial. The right place really would seem to be where I have done it, in that inverse problem section.

So OK, in that section I obtain this result in (13.3c).

$$\mathbf{a}' = \mathbf{a} - \dot{\boldsymbol{\omega}} \times \mathbf{r} - 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - \ddot{\mathbf{b}}_{S'} \qquad (13.3c)$$

Notice the opposite sign on the Coriolis term, just what you would expect from the brute force inversion process which is carried out right near (13.3a).

Then we come to section 13 (c) and fictitious forces. Yes, here I assume that Frame S' is inertial and Frame S is non-inertial. I get the true Newton as (13.8) with its **a**' acceleration. Hence (13.9). Then I write the bogus Newton in 13.10. This then leads to a + sign in the Coriolis, there is no way out of that fact.

Now in that section near 13.10, what is the meaning of the vector **ω** ? I suspect that is the big issue here. It is the same **ω** that we had in the original problem physical situation. Remember, the physical situation has not changed. So **ω** in this section near 13.10 refers to Fig 4.1 or Fig 4.2, it has not changed. It is the way Frame S' rotates relative to Frame S. That means that Frame S rotates at **-ω** relative to Frame S', and if Frame S' is inertial. But when you "set up" a problem as I have in the Foucault section figure D.1, my setup has Frame S rotating relative to Frame S' at **ω**, and that is then wrong for my equations!!!

So my "inverse equations" are correct if I am working on the inverse problem. They are not correct for the Fig D.1 setup, since I need to negate ω in that case.

So how should I deal with this issue in frames doc? By the way, this is why there is no **ω** ↔ **-ω** in the Marion swap rules. We are just swapping the labeling of things.

I want to give the reader a set of equations he/she can use if he/she wants to work in Marion notation. Maybe I can add a section f there?

OK, this led to a rewrite of sections 12 and 13 which are in a separate doc. I think I have now completely resolved this confusion and made frames doc more useful at the same time.
