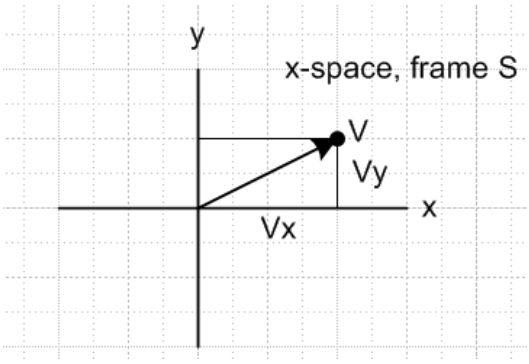
Passive View of V with Translation and Translation+Rotation PhL 7.1.12

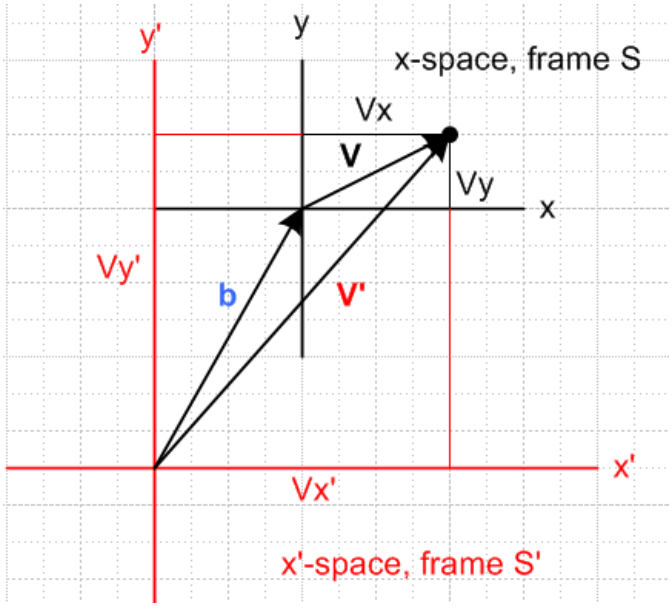
This doc provides a *precursor* to the equation **r'** = **r** + **b** used in frames doc. In both main pictures below, **V'** = **V** + **b** . I ended up never drawing or plotting velocity vectors in frames doc, maybe I should have.! This is a very early doc in this whole effort!

Part 1: Translation Only

1. We start with a vector sitting in x-space like so



2. We now imagine some other frame S' related to S by a simple 2D translation:



- The origin **x**=0 of S is located at position **x'** = **b** in S'.

- The basis unit vectors in either system are the same: $\hat{\mathbf{x}} = \hat{\mathbf{x}}'$ and $\hat{\mathbf{y}} = \hat{\mathbf{y}}'$

- We have these two ways to look at vector **V**

$\mathbf{V} = V_{\mathbf{x}}\hat{\mathbf{x}} + V_{\mathbf{y}}\,\hat{\mathbf{y}} = (V_{\mathbf{x}},V_{\mathbf{y}})$ = the vector **V** as seen in no-prime space

$\mathbf{V}' = V'_{\mathbf{x}}\hat{\mathbf{x}}' + V'_{\mathbf{y}}\hat{\mathbf{y}}' = (V'_{\mathbf{x}},V'_{\mathbf{y}})'$ = the same vector **V** as seen in primed space

Notice that we do not have **V** = **V'** . So *the prime is needed* to tell which frame we are in! The vector **V'** exists in x'-space whereas **V** exists in x-space, so it makes no sense to say **V** = **V'**.

- It certainly seems that this is true

$\mathbf{V}' = \mathbf{V} + \mathbf{b}$ $\qquad V'_{\mathbf{x}} = V_{\mathbf{x}} + b_{\mathbf{x}}$ and similarly for y

- Suppose **V** is the vector **x**. Then we have

$\mathbf{x}' = \mathbf{x} + \mathbf{b}$

- If **b** is changing in time, we then have

$d\mathbf{x}'/dt = d\mathbf{x}/dt + d\mathbf{b}/dt$

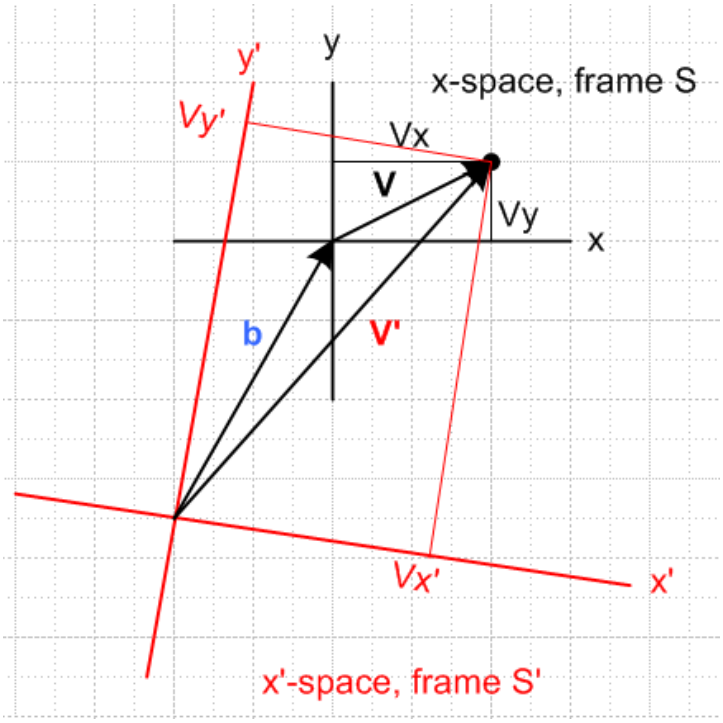
or

$\mathbf{v}' = \mathbf{v} + \mathbf{v}_{\mathbf{b}}$

Here **v** is a vector in x-space, **v'** is the corresponding vector in x'-space, and $\mathbf{v}_{\mathbf{b}}$ is in neither space.

Part 2: Translation and Rotation

Replace the earlier two-frame picture now with this picture



• The origin **x**=0 of S is located at position **x'** = **b** in S'. (same)

• The basis unit vectors in either system are different: $\hat{\mathbf{x}} = R\hat{\mathbf{x}}'$ and $\hat{\mathbf{y}} = R\hat{\mathbf{y}}'$ (different)

• We have these two ways to look at vector **V** (same)

$\mathbf{V} = V_x\hat{\mathbf{x}} + V_y\,\hat{\mathbf{y}} = (V_x, V_y)$ = the vector **V** as seen in no-prime space

$\mathbf{V}' = V'_x\hat{\mathbf{x}}' + V'_y\hat{\mathbf{y}}' = (V'_x, V'_y)'$ = the same vector **V** as seen in primed space

Notice that we do not have **V** = **V'** . So *the prime is needed* to tell which frame we are in! The vector **V'** exists in x'-space whereas **V** exists in x-space, so it makes no sense to say **V** = **V'**.

• It certainly seems that this is true

$\mathbf{V}' = R(\mathbf{V} + \mathbf{b})$

or

$V'_x\hat{\mathbf{x}}' + V'_y\hat{\mathbf{y}}' = R(V_x\hat{\mathbf{x}} + V_y\,\hat{\mathbf{y}} + \mathbf{b})$

or

$V'_x\hat{\mathbf{x}}' + V'_y\hat{\mathbf{y}}' = R(V_x\hat{\mathbf{x}} + V_y\,\hat{\mathbf{y}} + \mathbf{b})$