This is true support material do not delete. It derives equation 15.30 by the brute force method. I later learned you can get the same results by a trick, but I want the brute force stuff kept. All the sinsin+coscos stuff is here. -- 8.18.12

Note that page numbering is turned on in this template.

$$\mathbf{r}'(t) = x'\,\hat{\mathbf{x}}' + y'\,\hat{\mathbf{y}}'$$

where (15.29)

$$x' = (r'_0 - Vt)\cos(\Omega t)$$
$$y' = (r'_0 - Vt)\sin(\Omega t)$$

$$\hat{\mathbf{x}}' = \cos\varphi\,\hat{\mathbf{x}} + \sin\varphi\,\hat{\mathbf{y}}$$
$$\hat{\mathbf{y}}' = -\sin\varphi\,\hat{\mathbf{x}} + \cos\varphi\,\hat{\mathbf{y}} \qquad (15.6)$$

$$\mathbf{r}'(t) = x'\,[\cos\varphi\,\hat{\mathbf{x}} + \sin\varphi\,\hat{\mathbf{y}}] + y'\,[-\sin\varphi\,\hat{\mathbf{x}} + \cos\varphi\,\hat{\mathbf{y}}]$$

$$= (x'\cos\varphi - y'\sin\varphi)\,\hat{\mathbf{x}} + (x'\sin\varphi + y'\cos\varphi)\,\hat{\mathbf{y}}$$

$$= (r'_0 - Vt)[\,(\cos(\Omega t)\cos\varphi - \sin(\Omega t)\sin\varphi)\,\hat{\mathbf{x}} + (\cos(\Omega t)\sin\varphi + \sin(\Omega t)\cos\varphi)\,\hat{\mathbf{y}}\,]$$

$$= (r'_0 - Vt)[\,(\cos(\Omega t)\cos\varphi - \sin(\Omega t)\sin\varphi)\,\hat{\mathbf{x}} + (\sin\varphi\cos(\Omega t) + \cos\varphi\sin(\Omega t))\,\hat{\mathbf{y}}\,]$$

$$= (r'_0 - Vt)[\,(\cos(\varphi + \Omega t)\,\hat{\mathbf{x}} + (\sin(\varphi + \Omega t)\,\hat{\mathbf{y}}\,]$$

$$\mathbf{v}' = (v')'_{\mathbf{x}}\,\hat{\mathbf{x}}' + (v')'_{\mathbf{y}}\,\hat{\mathbf{y}}'$$

where (15.30)

$$(v')'_{\mathbf{x}} = -V\cos(\Omega t) - \Omega\,(r'_0 - Vt)\sin(\Omega t)$$
$$(v')'_{\mathbf{y}} = -V\sin(\Omega t) + \Omega\,(r'_0 - Vt)\cos(\Omega t)$$

$$\mathbf{v}' = (v'_{\mathbf{x}}\cos\varphi - v'_{\mathbf{y}}\sin\varphi)\,\hat{\mathbf{x}} + (v'_{\mathbf{x}}\sin\varphi + v'_{\mathbf{y}}\cos\varphi)\,\hat{\mathbf{y}}$$

$$(v'_{\mathbf{x}}\cos\varphi - v'_{\mathbf{y}}\sin\varphi) =$$

$$[-V\cos(\Omega t) - \Omega\,(r'_0 - Vt)\sin(\Omega t)]\cos\varphi$$

$$- [-V\sin(\Omega t) + \Omega\,(r'_0 - Vt)\cos(\Omega t)]\sin\varphi$$

$$= -V\cos\varphi\cos(\Omega t) - \Omega\,(r'_0 - Vt)\cos\varphi\sin(\Omega t)$$

$+V \sin\varphi \sin(\Omega t) - \Omega (r'_0 - Vt) \sin\varphi \cos(\Omega t)]$

$= -V [\cos\varphi \cos(\Omega t) - \sin\varphi \sin(\Omega t)] - \Omega (r'_0 - Vt)[\sin\varphi \cos(\Omega t) + \cos\varphi \sin(\Omega t)]$

$= -V \cos(\varphi+\Omega t) - \Omega (r'_0 - Vt) \sin(\varphi+\Omega t)$

To get the other result, take $\cos\varphi \rightarrow \sin\varphi$ and $\sin\varphi \rightarrow -\cos\varphi$. Do this in the second last line above

$(v'_x \sin\varphi + v'_y \cos\varphi)$

$= -V [\sin\varphi \cos(\Omega t) + \cos\varphi \sin(\Omega t)] - \Omega (r'_0 - Vt)[-\cos\varphi \cos(\Omega t) + \sin\varphi \sin(\Omega t)]$

$= -V [\sin(\varphi+\Omega t)] - \Omega (r'_0 - Vt)[-\cos\varphi \cos(\Omega t) + \sin\varphi \sin(\Omega t)]$

$= -V [\sin(\varphi+\Omega t)] + \Omega (r'_0 - Vt)[\cos\varphi \cos(\Omega t) - \sin\varphi \sin(\Omega t)]$

$= -V [\sin(\varphi+\Omega t)] + \Omega (r'_0 - Vt)[\cos(\varphi+\Omega t)]$

$= -V \sin(\varphi+\Omega t) + \Omega (r'_0 - Vt)\cos(\varphi+\Omega t)$

So we can then write

$$\mathbf{v}' = (v'_x \cos\varphi - v'_y \sin\varphi)\,\hat{\mathbf{x}} + (v'_x \sin\varphi + v'_y \cos\varphi)\,\hat{\mathbf{y}}$$

$$= (v')_x\,\hat{\mathbf{x}} + (v')_y\hat{\mathbf{y}}$$

where

$(v')_x = -V \cos(\varphi+\Omega t) - \Omega (r'_0 - Vt) \sin(\varphi+\Omega t)$
$(v')_y = -V \sin(\varphi+\Omega t) + \Omega (r'_0 - Vt)\cos(\varphi+\Omega t)$

$(v')_x = (v'_x \cos\varphi - v'_y \sin\varphi) = -V \cos(\varphi+\Omega t) - \Omega (r'_0 - Vt) \sin(\varphi+\Omega t)$

$$\mathbf{a}' = (a')'_x\,\hat{\mathbf{x}}' + (a')'_y\,\hat{\mathbf{y}}'$$

where (15.31)

$(a')'_x = 2\Omega V\sin(\Omega t) - \Omega^2 x'$
$(a')'_y = -2\Omega V\cos(\Omega t) - \Omega^2 y'$

$$\mathbf{a}' = ((a')'_x \cos\varphi - (a')'_y \sin\varphi)\,\hat{\mathbf{x}} + ((a')'_x \sin\varphi + (a')'_y \cos\varphi)\,\hat{\mathbf{y}}$$

$$= (a')_x\hat{\mathbf{x}} + (a')_y\,\hat{\mathbf{y}}$$

where

$$(a')_x = ((a')'_x \cos\varphi - (a')'_y \sin\varphi)$$

$$= [2\Omega V\sin(\Omega t) - \Omega^2 x']\cos\varphi$$

$$- [-2\Omega V\cos(\Omega t) - \Omega^2 y']\sin\varphi$$

$$= [2\Omega V\sin(\Omega t) - \Omega^2 (r'_0 - Vt)\cos(\Omega t)]\cos\varphi$$

$$- [-2\Omega V\cos(\Omega t) - \Omega^2 (r'_0 - Vt)\sin(\Omega t)]\sin\varphi$$

$$= [2\Omega V\sin(\Omega t) - \Omega^2 (r'_0 - Vt)\cos(\Omega t)]\cos\varphi$$

$$+ [2\Omega V\cos(\Omega t) + \Omega^2 (r'_0 - Vt)\sin(\Omega t)]\sin\varphi$$

$$= 2\Omega V \{ \sin(\Omega t)\cos\varphi + \cos(\Omega t)\sin\varphi \} + \Omega^2 (r'_0 - Vt) \{ \sin(\Omega t)\sin\varphi - \cos(\Omega t)\cos\varphi \}$$

$$= 2\Omega V \{ \sin(\varphi + \Omega t) \} + \Omega^2 (r'_0 - Vt) \{ \sin(\Omega t)\sin\varphi - \cos(\Omega t)\cos\varphi \}$$

$$= 2\Omega V \{ \sin(\varphi + \Omega t) \} - \Omega^2 (r'_0 - Vt) \{ -\sin(\Omega t)\sin\varphi + \cos(\Omega t)\cos\varphi \}$$

$$= 2\Omega V \{ \sin(\varphi + \Omega t) \} - \Omega^2 (r'_0 - Vt) \{ \cos(\varphi + \Omega t) \}$$

$$= 2\Omega V \sin(\varphi + \Omega t) - \Omega^2 (r'_0 - Vt)\cos(\varphi + \Omega t)$$

To get the other result, take $\cos\varphi \to \sin\varphi$ and $\sin\varphi \to -\cos\varphi$. Do this in the 5th last line above

$$(a')_y = 2\Omega V \{ \sin(\Omega t)\sin\varphi - \cos(\Omega t)\cos\varphi \} + \Omega^2 (r'_0 - Vt) \{ -\sin(\Omega t)\cos - \cos(\Omega t)\sin\varphi \}$$

$$= -2\Omega V \{ -\sin(\Omega t)\sin\varphi + \cos(\Omega t)\cos\varphi \} - \Omega^2 (r'_0 - Vt) \{ \sin(\Omega t)\cos\varphi + \cos(\Omega t)\sin\varphi \}$$

$$= -2\Omega V \{ \cos(\varphi + \Omega t) \} - \Omega^2 (r'_0 - Vt) \{ \sin(\varphi + \Omega t) \}$$

$$= -2\Omega V \cos(\varphi + \Omega t) \} - \Omega^2 (r'_0 - Vt) \sin(\varphi + \Omega t)$$

Therefore we find that

$$\mathbf{a'} = (a')_x \hat{\mathbf{x}} + (a')_y \hat{\mathbf{y}}$$

where

$$(a')_x = 2\Omega V \sin(\varphi + \Omega t) - \Omega^2 (r'_0 - Vt)\cos(\varphi + \Omega t)$$

$$(a')_y = -2\Omega V \cos(\varphi + \Omega t) \} - \Omega^2 (r'_0 - Vt) \sin(\varphi + \Omega t)$$

$$a_x = 2\Omega V\sin(\varphi + \Omega t) - \Omega^2(r'_0 - Vt)\cos(\varphi + \Omega t)$$
$$- \dot{\omega}[-b\cos\varphi + (r'_0 - Vt)\sin(\varphi + \Omega t)] - \omega^2[b\sin\varphi + (r'_0 - Vt)\cos(\varphi + \Omega t)]$$
$$- 2\omega[-V\sin(\varphi + \Omega t) + \Omega (r'_0 - Vt)\cos(\varphi + \Omega t)]$$

$$= [2\Omega V - \dot{\omega} (r'_0 - Vt) + 2\omega V] \sin(\varphi + \Omega t)$$

$$+ [-\Omega^2(r'_0 - Vt) - \omega^2 (r'_0 - Vt) - 2\omega \Omega (r'_0 - Vt)]\cos(\varphi + \Omega t)$$

$$+ \dot{\omega} b\cos\varphi - \omega^2 b\sin\varphi$$

$$= [2(\omega+\Omega)V - \dot{\omega} (r'_0 - Vt)] \sin(\varphi + \Omega t)$$

$$+ [-(\omega^2 + \Omega^2)r'_0 - 2\omega \Omega (r'_0 - Vt)]\cos(\varphi + \Omega t)$$

$$+ \dot{\omega} b\cos\varphi - \omega^2 b\sin\varphi$$

$$= [2(\omega+\Omega)V - \dot{\omega} (r'_0 - Vt)] \sin(\varphi + \Omega t)$$

$$+ [-(\omega - \Omega)^2 r'_0 + 2\omega \Omega Vt)]\cos(\varphi + \Omega t)$$

$$+ \dot{\omega} b\cos\varphi - \omega^2 b\sin\varphi$$