How does Gold get 4-105? PhL 7.24.12

[Here I realize that Goldstein is using one of the "cross velocities" in his discussion, and this is now all explained in frames doc.]

First, how does he get (4-104) ? In my notation, here is what he does

$$(d\mathbf{r}'/dt)_S = (d\mathbf{r}'/dt)_{S'} + \boldsymbol{\omega} \times \mathbf{r}'$$

or

$$\mathbf{v}'_S = \mathbf{v}'_{S'} + \boldsymbol{\omega} \times \mathbf{r}'$$

which with the S↔S' rule would translate into

$$(d\mathbf{r}/dt)_s = (d\mathbf{r}/dt)_r + \boldsymbol{\omega} \times \mathbf{r}$$

or

$$\mathbf{v}_s = \mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r}$$

Therefore we see that he is using

$$\mathbf{v}_s = \mathbf{v}'_S \qquad \mathbf{v}_r = \mathbf{v}'_{S'}$$

Therefore, we find that these are the translation rules (including S↔S')

$$\mathbf{r}' \rightarrow \mathbf{r} \qquad \mathbf{v}'_S \rightarrow \mathbf{v}_s \qquad \mathbf{v}'_{S'} \rightarrow \mathbf{v}_r$$
$$\mathbf{a}'_S \rightarrow \mathbf{a}_s \qquad \mathbf{a}'_{S'} \rightarrow \mathbf{a}_r$$

Start with (4-104)

$$\mathbf{v}_s = \mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r} \qquad \text{r = terrestrial}$$

Apply the G rule to $\mathbf{v}_s$

$$(d\mathbf{v}_s/dt)_s = (d\mathbf{v}_s/dt)_r + \boldsymbol{\omega} \times \mathbf{v}_s \qquad \text{// agrees with first line of 4-105}$$

Now apply $(d/dt)_r$ to 4-104

$$(d\mathbf{v}_s/dt)_r = (d\mathbf{v}_r/dt)_r + \boldsymbol{\omega} \times (d\mathbf{r}/dt)_r = \mathbf{a}_r + \omega \times \mathbf{v}_r \qquad \mathbf{a}_r = (d\mathbf{v}_r/dt)_r$$

Install this into the previous line to get

$$(d\mathbf{v}_s/dt)_s = \mathbf{a}_r + \omega \times \mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{v}_s$$

Now use 4-104 in here to get

$(d\mathbf{v}_s/dt)_s = \mathbf{a}_r + \omega \times \mathbf{v}_r + \boldsymbol{\omega} \times [\mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r}]$ $\qquad\qquad$ $\mathbf{a}_s = (d\mathbf{v}_s/dt)_s$

$\mathbf{a}_s = \mathbf{a}_r + 2\,\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ $\qquad\qquad$ // agrees with second line of 4-105

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How do I translate this all to my notation? I will try my initial idea 4-104 says for me

$\mathbf{v}_r \rightarrow (d\mathbf{r}'/dt)_{S'} = \mathbf{v}'_{S'}$

$\mathbf{v}_s \rightarrow (d\mathbf{r}'/dt)_S = \mathbf{v}'_S$

The if this is correct, the translation of (4-104) is this

$\mathbf{v}'_S = \mathbf{v}'_{S'} + \boldsymbol{\omega} \times \mathbf{r}'$ $\qquad\qquad$ r' = terrestrial = frame S'

This is one of my supposedly valid equations which leads me to the idea shown for v_r and v_s.

Now apply the G rule to $\mathbf{v}'_S = v_s$

$(d\mathbf{v}'_S/dt)_S = (d\mathbf{v}'_S/dt)_{S'} + \boldsymbol{\omega} \times \mathbf{v}'_S$ $\qquad\qquad$ // my version of first line of 4-105

Now apply $(d/dt)_{S'}$ to 4-104 assuming ω = constant

$\mathbf{v}'_S = \mathbf{v}'_{S'} + \boldsymbol{\omega} \times \mathbf{r}'$

$(d\mathbf{v}'_S/dt)_{S'} = (d\mathbf{v}'_{S'}/dt)_{S'} + \boldsymbol{\omega} \times (d\mathbf{r}'/dt)_{S'} = \mathbf{a}'_{S'} + \boldsymbol{\omega} \times \mathbf{v}'_{S'}$

Install this into two lines back to get

$(d\mathbf{v}'_S/dt)_S = \mathbf{a}'_{S'} + \boldsymbol{\omega} \times \mathbf{v}'_{S'} + \boldsymbol{\omega} \times \mathbf{v}'_S$

Now in the last term use $\mathbf{v}'_S = \mathbf{v}'_{S'} + \boldsymbol{\omega} \times \mathbf{r}'$ and we then get

$(d\mathbf{v}'_S/dt)_S = \mathbf{a}'_{S'} + \boldsymbol{\omega} \times \mathbf{v}'_{S'} + \boldsymbol{\omega} \times \mathbf{v}'_{S'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')$

which says

$\mathbf{a}'_S = \mathbf{a}'_{S'} + 2\,\boldsymbol{\omega} \times \mathbf{v}'_{S'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')$

and all is well.

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The only problem is that I think I have showed in Gold Rule version 8 that

$$\mathbf{a'_S} = \mathbf{a'_{S'}} + \boldsymbol{\omega} \times \mathbf{v'_S} \qquad (7.2)$$

and then this becomes, using 6 lines above,

$$\mathbf{a'_S} = \mathbf{a'_{S'}} + \boldsymbol{\omega} \times \mathbf{v'_{S'}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'})$$

and this just does not look the same! as 3 lines above!!! I am then missing the factor of 2 !