Frames doc edit log 8.3.12

This file used to be called proofing frames.doc, and it has notes on many proofings that took place in 2012. Today is 11.18.16 and I am reading the doc as part of my Goldstein Ch 4 review.

8.3.12.

I just finished updating Problem 3, and will now proof Problems 1 and 2.

8.4.12.

Had a big Paradox regarding $v_r = v'_r$ and resolved it. Then did a rewrite of Section 14 on curvilinear coordinates and just got that done.

I think the idea of extending things to special relativity should wait until I run into that subject somewhere, I don't want to tackle it on my own right now. Same for general relativity. So there are really no new sections to add except an introduction.

8.5.12

Did a very careful proofing of all of **Section 1**. This stuff I think is "very good". I have never seen these topics discussed this carefully in any book. I hope others will agree. Have not done "we removal", just doing the logic, checking equation references, etc.

Did careful proofing of **Section 2**. Added Hunt and Crowe references, so just starting to build the list of references. Section 2 is long winded but I think the extra words and steps are worth it since the G Rule is so fundamentally important.

Did careful proofing of **Section 3**, think this detail of what the Observer in Frame S' can see and do is worth while, section is still pretty short. Has no equations of its own.

Section 4 proofing complete, I added a picture of the Rotor I rode on with Karen. Not many equations.

Section 5 proofing is done, it iss very short, but I like it as a separate section to clarify the forest for the trees. No equations.

Section 6 I just did a little cleanup, proofing complete.

Section 7 went OK, lots of fine detail here, I doubt any other source has done anything like this. I am showing how to use the G Rule in very many examples here. Equation numbers all OK.

Section 8 proofing is done. I added the numbers for the earth on the frame part that is often neglected.

Signing off for this day, will continue proofing tomorrow. Need References and Introduction too.

8.6.12

Section 9 on Marion. Had to adjust a few things. This is a mere 1 page section, but it took a lot of work as I recall. I did not recheck the document references.

Section 10 proofing done. It is rather amazing that he is so different from Marion, but I think I have it nailed. Some day I might send a note to the GPS authors.

Section 11 proofing done, section is very short, it is needed to "fulfill the goal of Section 5". Note that I am checking all equation references in all my proofing.

Section 12 proofing done. Found a few bad equ nos and did some tunings. This is the summary section.

Section 13 proofing is underway. Am ready to start (e). Done now, this is a difficult section. I altered the ending a bit, adding the word symmetry. I don't care if my argument sells because the brute force method shows it is right.

Section 14 done. I decided to make Appendix A at the end to have more modularity and more reasonable equation numbers. It is much better that way.

Section 15 is now underway. (a) Somewhat painful, at least partitioned a bit into small pieces. Proofing complete. // All done. I added a piece on the high school frames of reference movies, very good. This is a tough section and I am sure it would be faster to use the polar coordinates in the right way, but I will leave it as is. These are examples of applying the equations I show. I did add an Exercise from my Paradox thing.

Big Error just discovered at 9:30 PM. Equation (15.32) and all that follows it is wrong because I have omitted the primes on x and y hat. I added them already to the previous 3 equations. So my 6 trajectories are going to be wrong as well.

8.7.12

Since I am now redoing Problem 2, and since it is based on the outputs of Problem 1, I am now carefully reviewing Problem 1 to avoid a huge error propagation.

Problem 2: // I have now heavily edited Ant's Motion in Frame S' opening section of Problem 2, and right there I convert to the $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ expansions. That in itself was pretty interesting and took lots of work but the results are easy to interpret. // Trajectory edited, now starting the velocity section. // Velocity done, starting accel. // OK, at 2 PM I have all new plots, and I think Problem 2 proofing is complete. It was a long haul.

Problem 3: OK, it is proofed! Added bogus M&T plots.

8.8.12

Reference and intro are now complete. Now doing More proofing. **Section 1** in progress // and is now done. This really is an excellent section IMHO. It addresses topics I think people have always wondered about but have never read about.

Section 2 "read" easily and quickly, I think it is fine. **Section 3** is brief and I think makes the points I want it to make.

Section 4 underway. Part (c) is painful, but I don't really want to delete it. It is true that $\dot{\mathbf{b}}_S$, is most easily understood in the special cases. It never hurts I think to pick some object and try to interpret it. I admit this is a weak section and produces not much of use. The special cases are important to mention. But overall, I pronounce Section 4 as readable, not too long, and it maintains interest with some photos and the three scenarios, relieving the boredom induced by section (c).

Section 5 is instant! Since the reader is about to dive into the trees of the forest with many applications of the G Rule and many mysterious equations, I want to get the goal solidified beforehand.

Section 6 I reread fast, it is pretty well organized, the special cases are considered, it is OK.

Section 7 is pure workman (accels), but does comment on Coriolis 2 and how it arose.

OK, I have completed my pass through the entire document. I think.

Now is the time to look harder at "other peoples' documents" on the subject of rotating frames. // I browsed for an hour or so, finding many offerings but none really like mine. Only I would spend 85 pages exhuming all the fine formal details of this subject. Most sources get the acceleration equation, talk fictional forces, then do Coriolis and Foucault.

8.9.12

I reviewed Lai up to the point where he mentions the G Rule, then I started looking at my original Goldstein notes. Then I found the **Taylor** book and I see that he has quite a long chapter on my subject,. longest I have seen, and I am reading through it. Nice section on the tides. He uses I instead of we, something I have not seen before. He assumes a common origin for these two frames (I do not). Frames are called S (rotating frame) and S_0 (fixed frame), script font. He uses that Goldstein phrase that his S_0 frame is "fixed in the earth" (queasy feeling). Common origin is center of the earth. Rotating frame has basis vectors $\mathbf{e}_i$ and $\mathbf{Q} = Q_i\mathbf{e}_i$ expansion. His entire section 9.4 is to derive the G Rule! His name for it is "an important identity", p 342. My $\boldsymbol{\omega}$ is his $\boldsymbol{\Omega}$ and he assumes it is constant. He then gets the fictional forces equation. It is a good presentation, glad to have the book.

Did Coriolis use the G Rule in 1835?

""Sur les équations du mouvement relatif des systèmes de corps"

Read more of Crowe, altered my history note on the G Rule.

Got rid of my lousy section 4(c) about interpreting $\dot{\mathbf{b}}_S$. My mess really just shows that you cannot interpret a cross velocity very well. I replaced it with some brief comments.

Did pagination. As usual, each section starts on a new page, and I made each of the three problems start on a new page as well. Pages are cheap. Did a full spell check, this doc is not too long for that.

I am finally ready to try to make a PDF. Doc is about 100 pages.

Pictures look crappy as usual unless blown up in Acrobat 4.

Overdots are not high enough in these equations:

(1.30), (1.32), (1.33) certain ones, (2.9), start of Section 8 too low on $\dot{\omega}$, all the $\dot{\omega}$ need raising.

Try the other viewer. Graphics much better, I should comment on that right in the pdf.

How do I globally raise dots? I want to open all field codes as usual (an option), then have to search, that was easy, about 120 replaced. Let's now do the same for single r dots, be careful!

Next, I want to unbold any bolded dots. It found 145, that went well. Found only two doubles bold.

PDF shows good dots, added link to the viewer. Now check paging. OK. Added comment on graphics right at the start. Thing is ready to fly.

Problem found: in Section 1 (a) heading, subscripts are bad. Since first heading, fix it! In fact I see this in many headings, so do that task tomorrow. Get it right please! Could it be since I edited them in the contents? Maybe make a test document for this.

August 16, 2012.

Many changes and additions have been made. Today I proofed/changed the **Overview** and the **Summary**. Then I did **Section 2** on the G Rule. Then short **Section 3**. Then **Section 4**. Then **Section 5**. **Section 6** is extremely "dry", but I checked everything on this pass. It is OK. **Section 7** is equally dry, I added a summary comment to this effect! Am checking every single thing in all these proofings. **Section 8** is very long. a,b,c done. OK, after a long time, Section 8 complete, now 7 PM. **Section 9** done, I added a little sentence on Taylor's notation. Done. **Section 10** done. **Section 11** done. **Section 12** done. **Section 13** done, found some horrible errors in the inverse equations, all fixed. **Section 14** is done. Section 15 which is very very long is done. **App A** done. **App B** done.

I have to completely repaginate everything.

Aug 18, 2012.

Adding Tensor G Rule Appendix C has caused revision of Section 1 (i), Section 2, so these need to be reproofed. I also need to get derivations of Section 1 (i) equations into repository.

Section 1 (i) proofed. **Section 2** proofed. **Appendix C** proofed, it is very good, a fitting finish for this long paper. Since App B got messed with recently, I will reread it now. **Appendix B** proofed.

Now another shot on the opening comments. **Overview** and **Summary** proofed!

Contents -- 2 pages
Overview -- 1 page
Summary -- 2 pages

Section 1 -- last page is 18
Section 2 -- 4 pages, fine.
Section 3 -- only 1 1/2 pages, fine.
Section 4 -- last page is 31
Section 5 -- 1/3 page!
Section 6 -- 3 pages exactly, good fit
Section 7 -- 4 pages, did one break adjust
Section 8 -- a monster section, last page is 64, all OK
Section 9 -- 1 1/3 pages, easy
Section 10 -- 2 1/2 pages, easy
Section 11 -- 1/2 page, easy
Section 12 -- summary is just one page, done.
Section 13 -- last page is 79. all OK, made pictures smaller for swap thing.
Section 14 -- 2 1/3 pages, easy.
Section 15 -- This is so long, I give and subsection its own page break start top reduce interactions for change I might make. DONE. I just added the coupled equations solution which I had wrong.
Appendix A -- short and done.
Appendix B -- 3 pages, good.
Appendix C -- done.

Pagination is now complete and it is 4 PM. Did a PDF, and I will now just visually scan for problems.

Errata:
p 103 an x hat missing prime
p 111 below 15.49 parens are screwed up
p 119 underbar too far application 3
p 119 space before In this (no extra space)

I will not fix these and make a new PDF at once. Oops

remove blue from the TOC. Must do this after every TOC update.

5:15 PM it is out there! I pushed both Bing and Google as well, see doc in web folder area.

Sept 11, 2012.

The new App D is installed, I just finished setting a good pagination. And then I did a full read-through with a few small edits, so this Appendix is ready to roll.

Overview: good.
Summary: good, tiny things fixed. Ends nicely on a page boundary.
Section 12 OK (swap and no swap stuff)
Section 13: needed pagination work, is now OK.
Section 1 review: something is wrong with (1.38). Equation 1 and 8 have the same LHS.
Section 2 review: all OK
Section 4 OK, skipped 3.

OK, enough. Tomorrow I will scan pagination only and then republish so I can move on.

Sept 12, 2012.

I did more pagination, added some stuff to the Marion Section comparing it to my swap notation which I think is helpful. Goldstein is so messy that I just skipped it. Did a final TOC and changed blue to black there and am now ready for a PDF with today's release date.

Oct 21, 2012

Today I will add section headers now that I know how to do it! DONE.
Did one errata.
Update the front matter.
Remake TOC and tune it.
Checked all bookmarks present in previous release.
Go for a cycle!

Oct 23-24. Decided to redo the L section, decoupling the two reference points. This causes a lot to change, but makes the comparison with Lai work right. ON Oct 24 using red I carefully verified the key results now called 11.3 and 11.4, then back to black. I have proofed sections (a), (b) and (c) with full accuracy checks. Decided to separate the fictitious force fluid dynamics into a new subsection.

Now I have to carefully update all my summary sections for my new L expressions.

• the **forward non-swap** equations will now be these:

$$\mathbf{F'}_{\mathbf{fict}} = \underset{\text{frame}}{-m\ddot{\mathbf{b}}_S} \underset{\text{centrifugal}}{- m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'})} \underset{\text{Coriolis}}{- 2m\,\boldsymbol{\omega} \times \mathbf{v'}} \underset{\text{Euler}}{- m\dot{\boldsymbol{\omega}} \times \mathbf{r'}} \qquad (8.6)$$

For Special Case # 1 problems (**ω** axis passes through Frame S origin), we have

$$\mathbf{F'}_{\mathbf{fict}} = \underset{\text{centrifugal}}{-m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})} \underset{\text{Coriolis}}{- 2m\,\boldsymbol{\omega} \times \mathbf{v'}} \underset{\text{Euler}}{- m\dot{\boldsymbol{\omega}} \times \mathbf{r}} \qquad \text{Special Case \#1} \qquad (8.12)$$

$$\mathbf{L}^{(c)} = \mathbf{L'}^{(c')} + (\mathbf{r'} - \mathbf{c} + \mathbf{b})\,(\boldsymbol{\omega} \times \mathbf{r'} + \dot{\mathbf{b}}_S) + (\mathbf{c'} - \mathbf{c} + \mathbf{b}) \times \mathbf{v'} \qquad (11.3)$$

$$\dot{\mathbf{L}}^{(c)} = \dot{\mathbf{L}}'^{(c')} + (\mathbf{c'} - \mathbf{c} + \mathbf{b}) \times \mathbf{a'} - \dot{\mathbf{c}}_S \times [\mathbf{v'} + \boldsymbol{\omega} \times \mathbf{r'} + \dot{\mathbf{b}}_S] + \dot{\mathbf{c}}'_{S'} \times \mathbf{v'}$$
$$+ (\mathbf{r'} - \mathbf{c} + \mathbf{b}) \times [\,\dot{\boldsymbol{\omega}} \times \mathbf{r'} + 2\,\boldsymbol{\omega} \times \mathbf{v'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) + \ddot{\mathbf{b}}_S] \qquad (11.4)$$

and the fictitious forces will be these

$$\mathbf{N'}^{(c')}{}_{\mathbf{fict}} = -(\mathbf{c'} - \mathbf{c} + \mathbf{b}) \times m\mathbf{a'} + \dot{\mathbf{c}}_S \times [m\mathbf{v'} + m\boldsymbol{\omega} \times \mathbf{r'} + m\dot{\mathbf{b}}_S] - \dot{\mathbf{c}}'_{S'} \times m\mathbf{v'}$$
$$- (\mathbf{r'} - \mathbf{c} + \mathbf{b}) \times [\,m\dot{\boldsymbol{\omega}} \times \mathbf{r'} + 2m\boldsymbol{\omega} \times \mathbf{v'} + m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) + m\ddot{\mathbf{b}}_S] \qquad (11.11)$$

where the second line is just $(\mathbf{r'} - \mathbf{c} + \mathbf{b}) \times \mathbf{F'}_{\mathtt{fict}}$ from (8.6).

• To get the forward *swap* equations, leave **b** and **ω** fixed, buy swap all primed with non-primed other things: (including c↔c')

$$\mathbf{F}_{\mathtt{fict}} = -m\ddot{\mathbf{b}}_{S'} - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2m\,\boldsymbol{\omega} \times \mathbf{v} - m\dot{\boldsymbol{\omega}} \times \mathbf{r} \qquad (8.6)_{\mathtt{s}}$$

frame centrifugal Coriolis Euler

For Special Case # 1 problems (**ω** axis passes through Frame S' origin), we have

$$\mathbf{F}_{\mathtt{fict}} = -m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) - 2m\,\boldsymbol{\omega} \times \mathbf{v} - m\dot{\boldsymbol{\omega}} \times \mathbf{r'} \qquad \text{Special Case \#1} \qquad (8.12)_{\mathtt{s}}$$

centrifugal Coriolis Euler

$$\mathbf{L'}^{(c')} = \mathbf{L}^{(c)} + (\mathbf{r} - \mathbf{c'} + \mathbf{b})\,(\boldsymbol{\omega} \times \mathbf{r} + \dot{\mathbf{b}}_{S'}) + (\mathbf{c} - \mathbf{c'} + \mathbf{b}) \times \mathbf{v} \qquad (11.3)_{\mathtt{s}}$$

$$\dot{\mathbf{L}}'^{(c')} = \dot{\mathbf{L}}^{(c)} + (\mathbf{c} - \mathbf{c'} + \mathbf{b}) \times \mathbf{a} - \dot{\mathbf{c}}'_{S'} \times [\mathbf{v} + \boldsymbol{\omega} \times \mathbf{r} + \dot{\mathbf{b}}_{S'}] + \dot{\mathbf{c}}_S \times \mathbf{v}$$
$$+ (\mathbf{r} - \mathbf{c'} + \mathbf{b}) \times [\,\dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \ddot{\mathbf{b}}_{S'}] \qquad (11.4)_{\mathtt{s}}$$

$$\mathbf{N}^{(c)}{}_{\mathtt{fict}} = -(\mathbf{c} - \mathbf{c'} + \mathbf{b}) \times m\mathbf{a} + \dot{\mathbf{c}}'_{S'} \times [m\mathbf{v} + m\boldsymbol{\omega} \times \mathbf{r} + m\dot{\mathbf{b}}_{S'}] - \dot{\mathbf{c}}_S \times m\mathbf{v}$$
$$-(\mathbf{r} - \mathbf{c'} + \mathbf{b}) \times [\,m\dot{\boldsymbol{\omega}} \times \mathbf{r} + 2m\boldsymbol{\omega} \times \mathbf{v} + m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + m\ddot{\mathbf{b}}_{S'}] \qquad (11.11)$$

where the second line is just $(\mathbf{r} - \mathbf{c'} + \mathbf{b}) \times \mathbf{F}_{\mathtt{fict}}$ from $(8.6)_{\mathtt{s}}$.

I have done them carefully in red, then back to black. Now we have the inverse problem summary to worry about:

The Inverse Problem. Here we don't show any fictional forces or torques because the frame on the left is inertial! We are just trying to get primed objects in terms of unprimed ones and we do this with these rules:

inverse non-swap = forward swap + negate b and ω

So here I will fix up the new L equations:

inverse non-swap L equations:

$$\mathbf{L'}^{(c')} = \mathbf{L}^{(c)} + (\mathbf{r} - \mathbf{c'} - \mathbf{b})\,(-\boldsymbol{\omega} \times \mathbf{r} - \dot{\mathbf{b}}_{S'}) + (\mathbf{c} - \mathbf{c'} - \mathbf{b}) \times \mathbf{v} \qquad (11.3)_{\mathtt{s}}$$

$$\dot{\mathbf{L}}'^{(c')} = \dot{\mathbf{L}}^{(c)} + (\mathbf{c} - \mathbf{c'} - \mathbf{b}) \times \mathbf{a} - \dot{\mathbf{c}}'_{S'} \times [\mathbf{v} - \boldsymbol{\omega} \times \mathbf{r} - \dot{\mathbf{b}}_{S'}] + \dot{\mathbf{c}}_S \times \mathbf{v}$$
$$+ (\mathbf{r} - \mathbf{c'} - \mathbf{b}) \times [\,-\dot{\boldsymbol{\omega}} \times \mathbf{r} - 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - \ddot{\mathbf{b}}_{S'}] \qquad (11.4)_{\mathtt{s}}$$

inverse swap L equations:

$$\mathbf{L}^{(c)} = \mathbf{L}'^{(c')} + (\mathbf{r}' - \mathbf{c} - \mathbf{b})\,(-\boldsymbol{\omega} \times \mathbf{r}' - \dot{\mathbf{b}}_S) + (\mathbf{c}' - \mathbf{c} - \mathbf{b}) \times \mathbf{v}' \qquad (11.3)$$

$$\dot{\mathbf{L}}^{(c)} = \dot{\mathbf{L}}'^{(c')} + (\mathbf{c}' - \mathbf{c} - \mathbf{b}) \times \mathbf{a}' - \dot{\mathbf{c}}_S \times [\mathbf{v}' - \boldsymbol{\omega} \times \mathbf{r}' - \dot{\mathbf{b}}_S] + \dot{\mathbf{c}}'_S \times \mathbf{v}'$$
$$+ (\mathbf{r}' - \mathbf{c} - \mathbf{b}) \times [-\dot{\boldsymbol{\omega}} \times \mathbf{r}' - 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') - \ddot{\mathbf{b}}_S] \qquad (11.4)$$

Made the red with edit, now back to black. Now install: DONE.

Preparation for Oct 24 release.

(1) remake TOC DONE
(2) check pagination of sections 1 starting with (h), Section 11, Section 12, Section 13 after (c), Refs. DONE
(3) remake TOC again, DONE
(4) clean up equation number positions using format copy where new equations done. DONE
(5) fix Lai reference, DONE
(6) update the Overview on Section 11, mention fluid and Reynolds.
(7) manual tune of the TOC DONE
(8) create a PDF
(9) quick scan of PDF looks OK.

Update 3/5/16.

Some guy from Poland put in a Maple request on this doc, so thought this would be a good time to do a review. I guess this "proofing frames" doc is what I now call an edit log.doc, so I will put notes here. I am reading the PDF and will add errata as they are encountered to the errata file. // Took a lot of hours.

Review

Interesting that the opening of Ch1 deals exactly with my issue in wedge doc. In the review below I am NOT checking all equations and equation refs. It is just a review to see what is going on in this doc, and whether I should release it to Researchgate.

Overview OK

Chapter 1 p 7
(a) had doubts about 1.2 right side, but it is correct.
(b) OK
(c) OK
(d) OK, meaning of two vectors being equal
(e) OK, so far I like my doc
(f) OK
(g) OK, I am still liking it
(h) OK

(i) OK
(j) OK

This is a pretty complicated collection of "facts", but we see the 8 accelerations.

Chapter 2 p 22

only one section, subject is "the G rule: $\partial_S \mathbf{a} = \partial_{S'} \mathbf{a} + \boldsymbol{\omega} \times \mathbf{a}$

I wonder if my presentation is the wordiest in the world?

Chapter 3 p 26

only one section, talks about Observer in Frame S' measuring various things he can measure.

Chapter 4 p 28

(a) OK, description of Fig 1.4 in some detail
(b) OK, Fig 4.1 is viewed from a different eye position
(c) seems OK, about vector **b**
(d) OK, special case where rotation axis contains Frame S origin
(e) OK, this time ω axis passes through Frame S' origin. I am liking it all so far.
(f) OK, I love it: picture of turntable and of the Rotor I rode with Karen in days of olde.
(g) OK, the earth and turntable are both Special Case #1 of earlier.
(h) OK, the flying camera platform as inverse problem

Chapter 5 p 35

very short, just states the general problem we are trying to solve.

Chapter 6 p 36

opening text, OK
(a) through (e), OK, just stating lots of equations
(f) Comment 1: be careful with prime location!
2: $\mathbf{r}' \neq \mathrm{R}\mathbf{r}$ $\mathbf{v}' \neq \mathrm{R}\mathbf{v}$ since non inertial frames.
3: **b** does not need concern about prime location

Chapter 7 p 39

(a) OK, relation between $\mathbf{a}_{S}$ and **a**' and the "Coriolis factor of 2"

$$\mathbf{a}'_S = \mathbf{a}' + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') \quad . \tag{7.4}$$

(b) OK, relates **a** to **a**'

$$\underset{S}{\mathbf{a}} = \underset{S'}{\mathbf{a}'} + \underset{\text{Euler}}{\dot{\boldsymbol{\omega}} \times \mathbf{r}'} + \underset{\text{Coriolis}}{2\,\boldsymbol{\omega} \times \mathbf{v}'} + \underset{\text{centripetal}}{\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')} + \underset{\text{frame}}{\ddot{\mathbf{b}}_S} \tag{7.6a}$$

and a longer version is stated as well.

(c) OK. another relation of a's
(f) OK, summary of the previous 3 sections

(d) Acceleration Summary

$$\mathbf{a} = \mathbf{a}' + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + \ddot{\mathbf{b}}_S \qquad (7.6a)$$

$$\mathbf{a}'_S = \mathbf{a}' + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') \qquad (7.4)$$

$$\mathbf{a}_{S'} = \mathbf{a}' + \ddot{\mathbf{b}}_{S'} \qquad (7.7)$$

(e) OK, similar results for vector b

$$\ddot{\mathbf{b}}_S = \ddot{\mathbf{b}}_{S'} + \dot{\boldsymbol{\omega}} \times \mathbf{b} + 2\boldsymbol{\omega} \times \dot{\mathbf{b}}_{S'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{b}) \; . \qquad (7.11)$$

Maybe this Chapter 7 is the one I called "dry" above. Yes, both Ch 6 and Ch 7 are dry as dust. Chapter are called Sections in this paper, by the way.

Note Added. Calculation regarding $\dot{\omega}$ on page 51 (now 52) bottom is wrong. Also I give no reference for the seasonal variation in ω for the earth. I found some day here

https://books.google.com/books?id=_6My7JR6dRgC&pg=PA133&lpg=PA133&dq=earth+rotation+%22seasonal+variations%22&source=bl&ots=NJhhvbNrkm&sig=bUMTZjRr3_LW5MJcqgmHzKrTo3Y&hl=en&sa=X&ved=0ahUKEwjqwI6rgqrLAhXktYMKHebrAl4Q6AEIVTAJ#v=onepage&q=seasonal&f=false

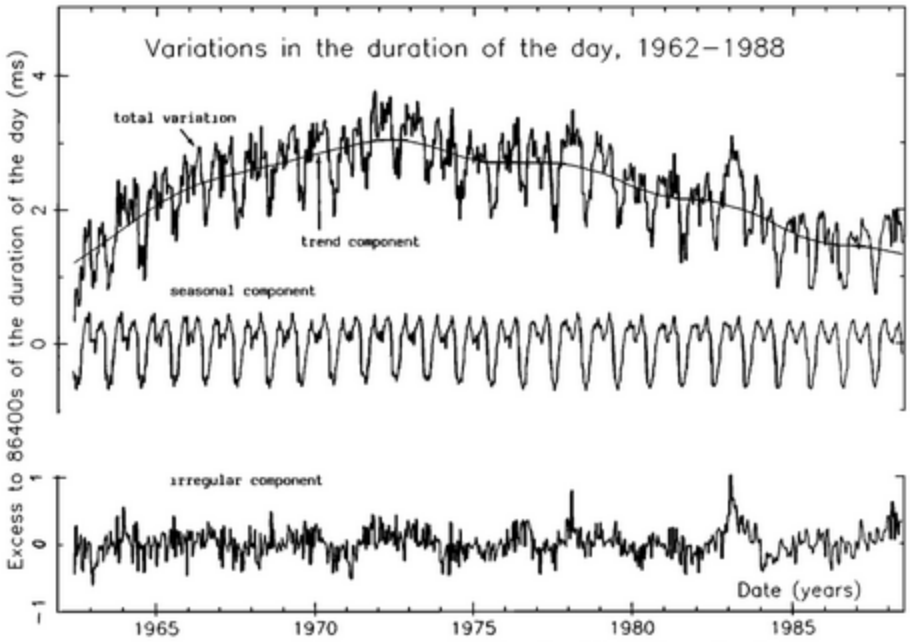


Fig. 1. Duration of the day, corrected for the effect of zonal tides. Total variation and decomposition in trend, seasonal and irregular components.

and the regular wave is the "seasonal component" which is indeed about 1 msec/day peak to peak and maybe then 0.5 msec RMS, so I am in the right ballpark with my claim.

Here is the repaired calculation:

$dT/T_{day} \sim 0.5 \text{ msec}/T_{day}$

$\omega = 2\pi/T$ $\qquad \dot{\omega} = d\omega/dt = -2\pi T^{-2} dT/dt = 2\pi(dT/dt)/T^2 \ (\text{sec}^{-1})$

Then $\qquad \dot{\omega} = 2\pi * (0.5 \text{ msec}/T_{day}) * (1/T_{day})^2 = \pi \ (1 \text{ msec})/T_{day}^3 = \text{sec}^{-2}$

$= \pi\,(.001)/(86400)^3 \sim 3 \times 10^{-3} / [10^5]^3 = 3 \times 10^{-18}$\

```
day := 24*60*60;
```

$day := 86400$

```
evalf(Pi* 0.001 / day^3);
```

$.4870893628\ 10^{-17}$

So the correct result is $\dot{\omega} \sim 10^{-17}$ sec^{-2} and not the 10^{-8} sec^{-2} which I quote. Another error is that it is the long vector r that is involved, not the short vector r'. Here then is my replacement text:

Let T = 24*60*60 = 86400 ~ 10^5 sec be the nominal period of a day. The day has a seasonal variation in its duration of roughly (see e.g. https://www.iers.org/IERS/EN/Science/EarthRotation/LODplot.html)

$dT/dt \sim 0.5$ msec/day ~ 10^{-8} // rms

with smaller short term variations. From this we compute a rough value for $\dot{\omega}$,

$|\dot{\omega}| = |d(2\pi/T)/dt| = 2\pi T^{-2} (dT/dt) \sim 6 * 10^{-10} * 10^{-8} \sim 10^{-17}$ sec^{-2} .

For activities on the surface of the earth, $r \approx R_E \sim 10^7$ m so

$\dot{\omega} \times r \sim 10^{-17} * 10^7 \sim 10^{-10} \sim 10^{-9}$ g

Thus in (8.12) we neglect the Euler term to get

$$\mathbf{F'_{eff}} = (m\mathbf{g_0} + \text{possible other real forces}) - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2m\,\boldsymbol{\omega} \times \mathbf{v}' \quad . \qquad (8.13)$$

Comment: As shown later in Section 8 (g), the tidal force is ~ 10^{-7} g.

Bug? In Section 8 g I claim that Fig 8.11 is Special Case #2 with ω = 0 and two sets of axes always aligned. I don't understand that. Well due to the gimbals, frame S' is fixed wrt the stars in orientation. In figure 4.5 I show rotation axis ω passing through the frame S' origin. I guess i am allowed to put that axis anywhere. In the earth case, then yes, ω = 0 since Frame S' is not rotating. The main thing is Ω .

This is a long chapter with lots of meat in it

Chapter 8 p 43

(a) OK, find expression for fictitious forces F_{fict}, do special case (not good for Earth)
(b) OK, I "interpret" the various fictitious forces, this is a difficult section to read today
(c)
OK: Arm-waving Coriolis interpretation, not bad.
OK: superposition of Coriolis and Centrifugal force. Only the first "drifts right".
if you ignore centrifugal, Coriolis would give circular motion!
(d) OK Special Case #1 simplify, then applies to earth surface problems.
(e) OK (found numerical error, but it does not affect conclusions, fix in errata doc and above)

(f) OK, tension in a tethered satellite pair, is tidal force etc
(g)
OK to end of digression on page 61
OK to end of page 64, nice picture and discussion of tides IMHO
OK to end of section (g) on page 68

Chapter 9 p 69
OK, compares notation to Marion's books and Taylor.

Chapter 10 p 71 comparing to Goldstein and GPS
(a) OK
(b) OK
(c) OK on a hidden Gold approximation.

Chapter 11 p 74
(a) OK, force torque and angular momentum stuff
(b) OK, shows L difference when referenced to points c and c'.
(c) OK, analogous discussion of "fictitious torques" in a rotating frame situation.
(d) OK, very fancy integral versions of things. (fluids etc)
(e) OK, fluid integral version of fictitious forces
all stuff verified in Lai equations thank goodness!
(f) OK, I give derivations of various Reynold's transport stuff.

Chapter 12 Forward Problem p 86
(a) OK, summarize "forward problem" equations in "non-swap notation"
(b) OK, summarize "forward problem" equations in "swap notation"

Chapter 13 Inverse Problem p 90
(a) OK, summarize "inverse problem" equations in "non-swap notation" : brute force
(b) OK, summarize "inverse problem" equations in "non-swap notation" : Swap Rules Method
(c) OK, summarize "inverse problem" equations in "non-swap notation" , more stuff than (a)
(d) OK, summarize "inverse problem" equations in "swap notation"
(e) OK, why the Swap Rules Method works

Chapter 14 Curvilinear versions of S and S' p 98
OK, we provide curvilinear coordinates ξ_i and ξ'_i for frames S and S', so four systems.
You might have ξ_i = sphericals and ξ'_i = toroidals
I then show how a key equations appears in all four systems. Fancy stuff!
A good topic for me to address.

Chapter 15 Ant on Turntable Problems p 101

(a) OK, basis ant kinematics in S and S' frames, details about various basis vectors,
This section has all setup work needed for the three ant problems.
(b) OK, ant crawls to origin of S'
(c) OK, ant spirals while crawling to origin in S'

(d) OK, the and flyover problem, but plots don't agree with Marion's numbers
(e) OK, the 4-projectiles problem with more plots

Appendix A: Rotation matrices p 134
OK

Appendix B: tensor doc notation comparison p 138
OK

Appendix C: G Rule for tensors p 141
OK, I like it, I hope it is right!

Appendix D: Foucault p 147
OK, opening material finds the small angle assumption solution
(a) setting up the spherical coordinate systems (kinematics)
(b) OK on qualitative solution.
(c) OK on three scalar equations to describe the spherical pendulum
(d) the full problem for a non-rotating earth
(e) the true Foucault Pendulum on the rotating earth.

References.
OK

So I have reread this entire paper today. I had forgotten how chock full of details and examples it is. Only 160 pages, one of my shorter docs. No one has ever emailed me until today, so that is why I took a pass and collected errata for a new release before Researchgate.

The guy from Poland asks for my Maple code, so I will next want to create a Maple index.

Update 11.17.16

I am reading this paper again as part of my Goldstein Ch 4 review. I think it is a good paper, and lots of errata are piling up, so I will probably do a cycle, but for now I am just reading

Overview	OK
Summary	skipped
Chapter 1	OK
Chapter 2	OK

Update 11.18.16

Continuing my reading (no equation checks on this pass).

Chapter 3	very short and clear, shows how rotating S' frame observer measures things.

Chapter 4

(a) OK detailed explanation of fig 4.1
(b) OK explanation of fig 4.2 where green circle is in plane of paper
(c) OK, seems mostly preparatory for later
(d) Special Case #1: rotation axis through Frame S origin, OK
(e) Special Case #2: rotation axis through Frame S' origin, OK
(f) turntable kinematics, excellent and OK (special case #1 example)
(g) the earth, another Special Case 1 example, just fine. OK
(h) the camera platform as Special Case 2 example

Chapter 5
OK, one paragraph stating our Problem.

Chapter 6 Determination of velocities
opening text OK
(a) OK on first of the four velocities.
(b) OK on the second
(c) OK on third // I don't do the 4th velocity for some reason
(d) velocity summary OK
(e) velocities for the special cases
(f) comments
1. OK
2. OK
3. OK

Chapter 7 Determination of accelerations
(a) OK, getting one of the 8 accelerations
(b) OK, lots of tedious algebra!!! Getting a second acceleration
(c) OK, getting a third acceleration
(d) OK, summary of these three accelerations
(e) OK, getting results for b double dot

Chapter 8. Fictitious Forces
(a) OK, development of these fictitious forces
(b) OK, a tough section, interpret Euler and Cor forces
(c) OK, another tough section. Maybe summaries at start will help.
(d) OK, fict forces in Special Case 1
(e) surface of the earth problems, g_0 vs g, seems OK
(f) tethered satellites discussion, OK
(g) tides on the earth a very LONG section, it does get support from other books OK

Pause for doing some of the Summary

1 - 10: OK

Chapter 9. Compare to Marion and Taylor OK

Chapter 10: Compare to Goldstein Very painful but probably OK.

Chapter 11

(a) introduction, seems OK, I talk about a bar and two chucks
(b) lots of messy algebra and a typo which is now on the errata list, OK
(c) if have no verification that what I say here is valid, but will call it OK
(d) fictitious torques in the Lai world, very messy so good to add, guess OK
(e) fictitious forces in the Lai world, and a translation table OK
(f) another Lai contact section, Reynolds Transport. Just trying to "tie in" to Lai. OK
end of Chapter 11

Comment: This is a brutal monograph! Hard to imagine anyone could read it through. It is a collection of what I have learned about these subjects. The subject is intrinsically difficult, I just don't think there is a simple simon way to do all this stuff. I have lots of examples,

Chapter 12. Forward summary OK

Chapter 13: The inverse problem and Swap Rules OK

Chapter 14: Curvilinear coordinates. OK, at least I face up to this question. And
I give a reasonably systematic answer with some examples.

Near (14.10), why are some of the ω symbols italicized, seems wrong and confusing? But ω is a vector like any other, and it has Cartesian components $(\omega)_i$ and curvilinear ones $(\omega)_i$ just the way I write things for the vector V, so I think this is all correct.

The ant problems are amazingly complicated for being such simple problems. The point is to exercise my general formulae given in Sections 12 and 13 which would apply to ANY problem.

Chapter 15. This is ALL the ant problems. I think they are all very good, I hope there are not too many algebra errors. Someone ELSE really has to check this stuff. So Ch 15 OK.

Appendices. Just reading through them. I really wonder if the G rule for tensors is backed up by any other author?

OK at 7:45 I finally finished this very long review. I think I will hold off on doing a Frames update until I am further along in my Goldstein review. The main thing is the G rule.

Update 11.26.16

By this time I have reviewed active/passive/covariance and A,B,C and Frames and all that stuff, and I am ready to start into updating frames doc.

I just rewrote Appendix C from scratch, it makes no mention of tensor doc or Appendix B, and I think it is very much better and clearer than the old Appendix C. I have installed the new one into frames v1 doc.

Now what to do with Appendix B? Should I just get rid of it? I really see no reason to keep it.

(1) I am trying to reduce connections to tensor doc which is not really "passive" oriented
(2) nobody cares about A/B notation.

OK, out it goes. I always have old copies in the earlier releases. So

Step 1: Delete the appendix B from v1 frames doc. DONE

Step 2: Rename all App C to be App B. DONE

Step 3: Rename all App D stuff to be App C. DONE

Step 4: Adjust the heading names for the new App B and App C.

I will leave tensor analysis in as a reference even though I might not refer to it any more.

Update 11.27.16

I went through a very long list of errata and made the fixes in v1. This included a long pass to add periods after equations that need periods (maybe 200). There are some errata left still that require more work and I will go after them next. I gathered them all together in the errata list at the bottom.

As of 3 PM I have worked through some of the more moderate errata, but some big ones are left. I think I am going to have to do another reading pass to get back into the details after my long digression on my "confusions with rotations" which lasted several days.

Another Frame Doc Proofing and Review

Overview -- good

Summary -- good, long, a few edits were done.

1(a) I just rewrote part of this section to make it clearer.

1(b) OK, did many fine edits

1(c) Much improvement, and added Active/Passive section. Did a lot of work here.

1(d) I maintain Example numbers always increasing.

1(e) very good, I think reader OK on matrix expo.

1 (f) very good, I checked the cone solution, I keep adding edits.

1(g) again, very clear

1(h) things are OK down to the start of angular momentum. And that is OK too, very good
no mysteries here.

1(i) OK, d/dt on scalars.

1(j) I better derive these 8 equations, at least on scratch. // Ouch, I have run aground regarding a claim about rotation matrices that I don't know how to prove! But then I realizes this whole section does not really do what I want. I want to show non-commute in all cases but the natural cases.

Update 11.28.16

I rewrote Section 1 (j) and it is immensely better now. I am finally done with Section 1. There is a huge amount of meat and potatoes in this section, I think it is excellent and sets the stage quite well. Added more facts to Section 1 (j) and eq nums are changed in that section. Proved the comm theorem in full.

So I now continue with my review

2. Added Marion books and Taylor and fully checked all references! I completely redid the derivation here of the G Rule, and removed some useless stuff, and it is much better.

3. Describes the observer at rest in Frame S', not much content here, a rest for the reader. All OK. This Section 3 introduces no new equation numbers.

4. This is a 7 page chapter, so I need to take is slowly, doing a close reading of everything and full eq num checks and so on.

4(a). OK, this is just a description emphasizing the generality of Figure 4.1.

4(b). OK, draws and describes Fig 4.2 where circular motion is in the plane of paper.

4(c) OK, just a few comments about motions of the b vector.

4(d) This is Special Case #1 where **b** does cone motion in Frame S.

4(e) This is special case #2, everything seems OK to me.

4(f) Excellent setup for the turntable with mention of Karen's Rotor.

4(g) I just capitalized Earth, Moon and Sun everywhere in this doc. This section is OK and clear.

4 (h) Very short, it is the camera platform as a special case 2 example.

So much for Chapter 4. I think it is ready for Prime Time.

5. Very short, states the Goal of the Next Two Sections.

6 opening	OK	Velocities
6 (a)	OK, not clear why the result is useful	
b (b)	OK, again still hazy where this is headed.	
6 (c)	OK, fixed a reference, still hazy	
6 (d)	OK summary. I have checked every detail and EN ref so far!	
6 (e)	the special cases, added new eq nums here, so we careful with later refs!	
6 (f)	cleaned up the 3rd comment, end on nice page boundary.	
7 opening	OK, cleaned up, parallel to 6	Accelerations
7(a)	OK, did every detail, it is good on showing Coriolis factor of 2	
7(b)	OK, full check on all	
7(c)	OK, very short	
7(d)	summary, gave it a new number	
7(e)	OK, new eq nums, back repair same.	
8(a)	OK, no changes , all checked	Fictitious Forces
8(b)	I don't like this section :	

The comments of Section 8 for intuition always go back to **r'** being the "long vector" on a merry go round or other. I think I veer off course in this section 8 (b).

I have rewritten Section 8 (b) in a separate file. I need to fix up words like "centrifugal" to get it cleaner, but then I will just install this replacement section into frames doc v1. I am going to dump the weird tilted cone picture which is impossible to understand and adds nothing. Figures in Section 8 are then going to get renumbered unless I can come up with a Figure to add. Enough for today at 7:30 PM. Progress was good, but many miles still to go.

Update 11.29.16

I have installed my new Section 8 (b) and I think it is much better. So I can now continue the review from yesterday with Section 8 (c)

8(c) This section needs no major fixes, I centered the pictures
arm waving section OK
comments OK
superposition idea OK

8(d) seems OK, point is to derive (8.12) for Special Case #1 problems (I guess).

8(e) Euler small for earth; g and g_0 in non-swap then swap notation. All OK

8(f) Bogged down right at the start, I don't know what "center of gravity" means for N objects each having some different gravity. This ate the rest of the day.

Update 11.30.16

At 2 PM I am done with "center of gravity" and ready to resume on Section 8 (f).

8(f) tether is done. I had to add equations here, so Section 8(g) will have to have its eq nums all altered and then I have to back-check xrefs. I have much more confidence now in the tether application of the rotating frames equations. I think the factor of 3 is correct.

I keep re-reading 8f on the tether, making small edits on each pass.

8(g) STOP.

What now does center of gravity say about the earth-moon drawing Fig 8.10? I would say that each has a certain center of gravity relative to the other (with the other treated as a point mass at its center).

8(g) RESUME, I will add a comment that R_{cog} exists but we are not using it.

The basic picture	OK
Frame S and Frame S'	OK
How the tidal model fits in	OK
One fictitious force in Frame S'	OK
Digression: The relation between r12 and Ω	OK

STOP. Sign error found below (8.31?) It causes me to disagree with Taylor, need to study.
Stopping at 8 PM, enough for today.

Update 12.6.16

I have now finished a first draft of 18-page Appendix D on the subject of the center of gravity and the tether. Not surprisingly, trying to answer questions raises more and new questions. So right now here are some of those:

1. I argue that $\mathbf{N}^{(rcog)} = 0$ and $\mathbf{N}^{(0)} = \mathbf{r}_{cog} \times \mathbf{F}$ as part of the definition of $\mathbf{r}_{cog}$. The other part is the distance rule which sets magnitude r_{cog}. Question: does this apply only in an inertial frame, or does it apply also in a rotating frame?

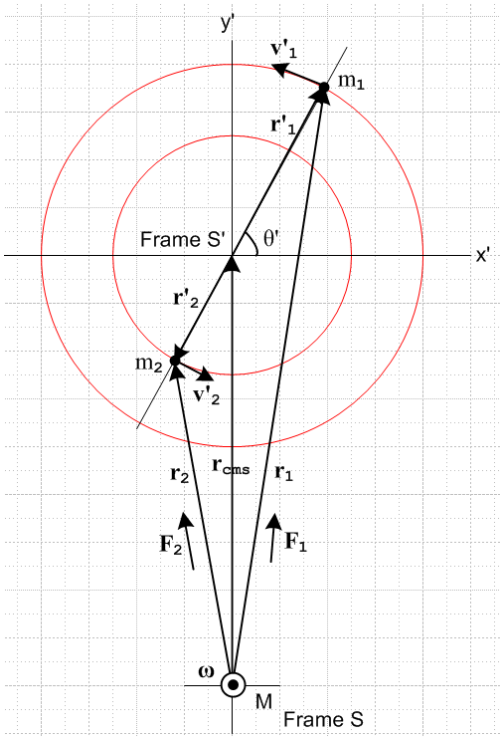
2. I compute the torque about tether center point ("origin") and I show "restoring force". What does this torque have to do with the torques of item 1 above? And what about the fact that this "origin" is not the center of gravity or of mass?

3. Does it make sense to say anything about a freeze-frame of the tether since you know it must be in motion? Am I apply statics to a case where there is motion and so statics is wrong?

I will go off in a new doc " App D questions" and try to resolve these questions and see what yet new questions arise from the study of these questions. Sisyphus lives!

Concession: It is Dec 6, I leave town Dec 20, there is no way I can get a new frames doc release done in that time and have it be good. So I hereby give up on this goal and will pursue multiple interests once again.

OK, did other things but then had a session here. I am trying to apply my completely general frames of reference equations to the tether problem even though I probably did that already in the main text. I then want to get an equation of motion for the two masses at some arbitrary angle. It now seems to me much better to put the CMS at the center of your circles picture to allow different masses. I convinced myself that the cog moves only very slightly relative to the cms and adds a motion which does not matter much in solving problems. So I drew a whole new CMS based tether picture.,



Since this whole thing is in a non-inertial frame (it orbits the earth), I have to use the full frames doc equations including fictitious forces and fictitious torques. I tried a long shot on that but made mistakes which I can try to fix tomorrow. Notice the simple gravity torques are:

$$\mathbf{N}_1^{(0)} = \mathbf{r}_1 \times \mathbf{F}_1 = 0$$
$$\mathbf{N}_2^{(0)} = \mathbf{r}_2 \times \mathbf{F}_2 = 0$$

whereas these to not vanish,

$$\mathbf{N}_1^{(0')} = \mathbf{r'}_1 \times \mathbf{F}_1$$
$$\mathbf{N}_2^{(0')} = \mathbf{r'}_2 \times \mathbf{F}_2$$

Update 12.8.16 through 12.10.16

I have decided to back up and update all eq nums to my standard format before trying to carry out an update of App D. The platform needs to be more solid than it is, with pieces sliding around. This will happen in the v2 doc I just made, in case I decide to go back.

1. Notation, important role of the Prime Symbol, and other Preliminaries

1.1 The basis vectors e_n and e'_n and two ways in which they are related a

old	new	
1.1	1.1.1	
1.2	1.1.2	
1.3	1.1.3	
1.4	1.1.4	done

1.2 Expansions of a vector and use of primes and parentheses b

old	new	
1.5	1.2.1	
1.6	1.2.2	
1.7	1.2.3	
1.8	1.2.4	done

1.3 Special case where a'_i is unambiguous c

old	new	
1.9	1.3.1	
1.10	1.3.2	
1.11	1.3.3	
1.12	1.3.4	
1.13	1.3.5	
1.14	1.3.6	done

1.4 When are Two Vectors Equal? d

Fig 1.1	1.4.1	done

1.5 The Small Rotation of a vector about an axis e

old	new	
1.15	1.5.1	
1.16	1.5.2	
1.17	1.5.3	
1.18	1.5.4	
1.19	1.5.5	
1.20	1.5.6	
1.21	1.5.7	
1.22	1.5.8	
Fig 1.2	1.5.9	
	1.5.10 new	done

1.6 The time rate of change of a rotating vector f

old	new	
1.23	1.6.1	
Fig 1.3	1.6.2	
1.24	1.6.3	done

1.7 Rate of change of the basis vectors g

old	new	
1.25	1.7.1	
1.26	1.7.2	
1.27	1.7.3	
1.28	1.7.4	done

1.8 Notations for the many time derivatives of vectors r, r', b and L h

old		
1.29	1.8.1	
1.30	1.8.2	
1.31	1.8.3	
1.32	1.8.4	
none	1.8.5	
1.33	1.8.6	
none	1.8.7	
none	1.8.8	
1.34	1.8.9	
none	1.8.10	
Fig 1.4	1.8.11	
none	1.8.12	
1.35	1.8.13	
none	1.8.14	
none	1.8.15	
none	1.8.16	
1.36	1.8.17	done

1.9 Notations for the many time derivatives of vectors r, r', b and L i

old	new	
1.37	1.9.1	done

1.10 Notations for the many time derivatives of vectors r, r', b and L

old	new
1.38	1.10.1

1.39 1.10.2
1.40 1.10.3
1.41 1.10.4
1.42 1.10.5 done

2. The G Rule for arbitrary vector a and its derivation

No need to renumber this chapter since only one section!
But have to fix all refs. done

3. The Apparatus and its Observer at Rest in Frame S'

This Section has no numbered equations, but I fixed the refs.

4. The Relationship between the Two Frames S and S'

4.1 Explanation of Fig 4.1: Frame S in the plane of paper a

old	new	
Fig 4.1	(4.1.1)	done

4.2 Explanation of Fig 4.2: Vector ω pointing directly out of paper b

old	new	
Fig 4.2	(4.2.1)	done

4.3 Comments on $\dot{\mathbf{b}}_S$ and $\dot{\mathbf{b}}_{S'}$ c

old	new	
(4.1)	(4.3.1)	done

4.4 Special Case #1 : ω axis through Frame S origin d

old	new	
Fig 4.3	4.4.1	
4.2	4.4.2	
4.3	4.4.3	
Fig 4.4	4.4.4	done

4.5 Special Case #2 : ω axis through Frame S' origin e

old	new	
Fig 4.5	4.5.1	
4.4	4.5.2	done

4.6 The Turntable **f**

old	new	
Fig 4.6	4.6.1	
Fig 4.7	4.6.2	
Fig 4.8	4.6.3	done

4.7 The Earth **g**

old	new	
Fig 4.9	(4.7.1)	done

4.8 The Flying Camera Platform
done

5. The Goal of the next two sections
done

6. Determination of velocities

For this Section I leave equation numbers as there were since so few of them.

6.1 Velocity $v_{s'}$	**a**
6.2 Velocity $v \equiv v_s$	**b**
6.3 Velocity v'_s	**c**
6.4 Velocity Summary	**d**
6.5 Velocities for Special Cases	**e**
6.6 Comments	**f**
But go through on xrefs!	done

7. Determination of accelerations

Same idea, Leave eq nums as is, but replace letters with numbers in titles

7.1 Acceleration a'_s	**a**	**done**
7.2 Acceleration $a \equiv a_s$	**b**	**done**
7.3 Acceleration $a_{s'}$	**c**	**done**
7.4 Acceleration Summary	**d**	**done**
7.5 Relation between $\ddot{b}_s$ and $\ddot{b}_{s'}$	**e**	**done**

These last two sections are have heavy internal xref and I would not want to renumber them. There are only about 13 equations in each Section.

8. The Fictitious Forces

8.1 Development of the Fictitious Forces a

old	new	
none	8.1.1	
8.1	8.1.2	
8.2	8.1.3	
8.3	8.1.4	
8.4a	8.1.5	
8.4b	8.1.6	
8.5	8.1.7	
8.6	8.1.8	
8.2 app	8.1.9	
8.4a app	8.1.10	
8.4b app	8.1.11	
none	8.1.12	
8.6 app	8.1.13	done

8.2 Interpretation of the Centrifugal and Euler Fictitious Forces b

old	new	
Fig 8.1	8.2.1	
8.7	8.2.2	
8.8	8.2.3	
Fig 8.2	8.2.4	
8.9	8.2.5	
8.10	8.2.6	
8.11	8.2.7	
Fig 8.3	8.2.8	done

8.3 Interpretations of the Coriolis Fictitious Force c

Fig 8.5	8.3.1	
none	8.3.2	
Fig 8.6	8.3.3	
none	8.3.4	
none code	8.3.5	done

8.4 Special Case #1 Problems d

8.12	8.4.1	done

8.5 Problems on the surface of the Earth e

none	8.5.1

none	8.5.2	
none	8.5.3	
none	8.5.4	
8.13	8.5.5	
8.14	8.5.6	
8.15	8.5.7	
8.16	8.5.8	
none	8.5.9	
8.13S	8.5.5$_{\mathbf{s}}$	
8.14S	8.5.6$_{\mathbf{s}}$	
8.15S	8.5.7$_{\mathbf{s}}$	
8.16S	8.5.8$_{\mathbf{s}}$	done

8.6 Tethered satellites and Tidal Forces

8.17	8.6.1	
8.18	8.6.2	
8.19	8.6.3	
8.20	8.6.4	
8.21	8.6.5	
Fig 8.7	8.6.6	
8.22	8.6.7	
8.23	none	
8.24	8.6.8	
none	8.6.9	
8.25	8.6.10	
Fig 8.8	8.6.11	
none	8.6.12	
8.26	8.6.13	
8.27	8.6.14	done

8.7 Tides on the Earth **g**

none	8.7.1	
Fig 8.10	8.7.2	
8.24	8.7.3	
8.25	8.7.4	
8.26	8.7.5	
Fig 8.11	8.7.6	
8.27	8.7.7	
none	8.7.8	
none	8.7.9	
8.28	8.7.10	
8.29	8.7.11	
none	8.7.12	
8.30	8.7.13	

Fig 8.12	8.7.14
8.31	8.7.15
8.32	8.7.16
8.33	8.7.17
8.34	8.7.18
8.35	8.7.19
8.36	8.7.20
8.37	8.7.21
8.38	8.7.22
8.39	8.7.23
8.40	8.7.24
Fig 8.13	8.7.25
none code	8.7.26
8.41	8.7.27
8.42	8.7.28
8.43	8.7.29
Fig 8.14	8.7.30
Fig none	8.7.31
8.44	8.7.32
8.45	8.7.33
8.46	8.7.34
8.47	8.7.35
8.48	8.7.36
none	8.7.37
none	8.7.38 done!

9. Comparison with Marion (1970), Thornton & Marion (2003) and Taylor (2005)

No eq num changes here but do all xref! done

10. Comparison with Goldstein (1950) and Goldstein, Poole and Safko (2001)

No eq num changes here but do all xref! done

At this point I just pasted in replacements for sections 11.2 and 11.3 from "section 11b rewrite v1". Now I continue:

11. Angular Momentum and Fictitious Torques; the Reynolds Transport Theorem

11.1 Introduction a

none	11.1.1
none	11.1.2
none	11.1.3
Fig 11.1	11.1.4
none	11.1.5

none	11.1.6	
Fig 11.2	11.1.7	done

11.2 Expression of $L^{(c)}$ and $\dot{L}^{(c)}$ in terms of Frame S' objects b

This section was a complete rewrite in new eq nums, so I have no translation table.
Here is a partial translation table

But do the xrefs: done

11.3 Fictitious Torques and Newton's Rotational Law in a non-inertial frame c

This section was a complete rewrite in new eq nums, so I have no translation table.
But do the xrefs: done

old	new
11.2	11.2.14
11.3	11.2.14
11.4	11.2.15
11.5	11.2.16
11.6	11.3.2
11.8	11.3.5, 11.3.4
11.11	11.3.8
11.12	

11.4 Application: Fictitious Torques in Fluid Dynamics d

11.16	11.4.1	
11.17	11.4.2	
11.18	11.4.3	
11.19	11.4.4	
11.20	11.4.5	
11.21	11.4.6	
none	11.4.7	done

11.5 Application: Fictitious Forces in Fluid Dynamics e

11.22	11.5.1	
11.23	11.5.2	
11.24	11.5.3	
11.25	11.5.4	
none	11.5.5	done

11.6 Comments on the Reynolds Transport Theorem f

none	11.6.1

11.26	11.6.2	
11.27	11.6.3	
11.28	11.6.4	
11.29	11.6.5	
11.30	11.6.6	
none	11.6.7	
11.31	11.6.8	
11.32	11.6.9	
none	none	done

12. Summary of the Forward Problem Solution

old	new	
Fig12.1	12.1	
Fig12.2	12.1s	done

13. The Inverse Problem

13.1 Brute Force Method

13.1	13.1.1	
13.2a	13.1.2	
13.2b	13.1.3	
13.3a	13.1.4	
13.3b	13.1.5	
13.3c	13.1.6	done

13.2 Swap Rules Method

13.6	13.2.1	
none	13.2.2	done

13.3 Summary of the Inverse Problem Equations (non-swap notation)

Fig 13.1	13.3.1
13.7	13.3.2

Pause: Why is my inverse problem incomplete compared to the forward problem equations?

For example, in Section 12.1 I give Forward Problem results with three distinct sections.

Definitions and Equations
Fictitious Forces (Section 8)
Fictitious Torques (Section 11)

All this stuff is repeated in Section 12.2 in swap notation.

In Section 13.1 I define the inverse problem, then in 13.2 I demonstrate the brute force method, and 13.2 I show how you get the same results from using Swap Rules.

Then in Section 13.3 I summarize the Inverse Problem equations, but I only have the first of those three sections!

Pause Pause: Stack Push! I no longer buy my argument as to why the Swap Rules which applied to a set of valid equations produces a new set of valid equations. I tried to show this graphically in Section 13, but I just don't believe my own argument. Although things end up generally the same in the final picture as in the initial picture, distances are different, things are not completely the same.

OK, I rewrote this section and I guess I am more or less convinced that the argument is OK. I installed the rewrite, and now Stack Pop.

Restabilize the Summary Sections. They have spun out of control with stale data.

Restabilize Section 12.1 : redid again, adding new eq nums!

Restabilize Section 12.2 : obtain this directly from 12.1 results
first part now OK, prior to fict forces
fict force part OK
torque OK
Done with 12.2, added new eq numbs overall.

Restabilize 13.3. OK. Now have (a) through (g)

13.3 has destabilized again. Stop answering the phone !!! 6 calls from MRL each one breaks my intense concentration in getting this fine detail right.

high noon on 12.10.16. I think the four summaries are now stable.

<u>So let's now back up and try to continue our eq num alterations.</u>

14. Rotating Frames in Curvilinear Coordinates

14.3 14.4
etc

Check xrefs. done

15. Ant on Turntable Problems

Kinematics common to all Ant Problems a //decided to have no number

Fig 15.1 15.1
15.1 15.2
15.2 15.3
15.3 15.4

15.4	15.5
15.2a	15.6
15.4a	15.7
15.5	15.8
15.6	15.9
15.7	15.10
15.8	15.11
15.9	15.12

15.1 Problem 1: Ant crawls at constant speed V to the Origin of Frame S' b

Fig 15.2	15.1.1
15.10	15.1.2
15.11	15.1.3
15.12	15.1.4
15.13	15.1.5
15.14	15.1.6
15.15	15.1.7
15.16	15.1.8
15.17	15.1.9
15.18	15.1.10
15.19	15.1.11
15.20	15.1.12
15.21	15.1.13
15.22	15.1.14
15.23	15.1.15
15.24	15.1.16
15.25	15.1.17
15.26	15.1.18
15.27	15.1.19
Fig 15.3	15.1.20
Fig 15.4	15.1.21
Fig 15.5	15.1.22
Fig 15.6	15.1.23

Now do all the references. done

15.2 Problem 2: Ant spirals in at constant V and Ω to the Origin of Frame S' c

15.29a	15.2.1
15.30a	15.2.2
15.31a	15.2.3
15.29b	15.2.4
15.30b	15.2.5
15.31b	15.2.6
15.32	15.2.7

15.33	15.2.8
15.34	15.2.9
15.34a	15.2.10
15.35	15.2.11
15.36	15.2.12
Fig 15.7	15.2.13
Fig 15.8	15.2.14
Fig 15.9	15.2.15

Now do refs: done

15.3 Problem 3: Inverse Problem: Ant flies in Frame S at constant velocity V d

15.37	15.3.1
15.38	15.3.2
15.39	15.3.3
15.40	15.3.4
15.41	15.3.5
15.42	15.3.6
15.43	15.3.7
15.44	15.3.8
Fig 15.10	15.3.9
Fig 15.11	15.3.10
Fig 15.12	15.3.11
Fig 15.13	15.3.12
Fig 15.14	15.3.13
Fig 15.15	15.3.14
Fig 15.16	15.3.15
Fig 15.17	15.3.16
Fig 15.18	15.3.17

refs: done

15.4 Problem 4: The Projectile Problem of Section 8.3

15.45	15.4.1
15.46	15.4.2
15.47	15.4.3
15.48	15.4.4
15.49	15.4.5
none	15.4.6
none	15.4.7
none	15.4.8
15.50	15.4.9
15.51	15.4.10
none	15.4.11

none 15.4.12
none 15.4.13

refs: done!

Appendix A: Derivation of R(ξ) for Spherical Coordinates

Making this change

A.1 A.1
none A.2
A.2 A.3
A,n A.n+1

Scan for refs: done

Appendix B: The G Rule for a Tensor of Rank n

Scan for refs: done

Appendix C: The Foucault Pendulum

none C.1
none C.2

C.1 Drawings, Notation, and Coordinates a

Fig C.1 C.1.1
C.1 C.1.2
Fig C.2 C.1.3
C.2 C.1.4

refs: done

C.2 Qualitative Solution b

Fig C.3 C.2.1

scan ref: done

C.3 The Equations of Motion for a Foucault Pendulum c

C.3 C.2.1
C.4 C.2.2
C.5 C.2.3
C.6 C.2.4

C.7	C.2.5
C.8	C.2.6
C.9	C.2.7
C.10	C.2.8
C.11	C.2.9

refs: done

C.4 The Spherical Pendulum d

C.12	C.4.1
C.13	C.4.2
C.14	C.4.3
C.15	C.4.4
C.16	C.4.5
C.17	C.4.6
C.18	C.4.7
C.19	C.4.8
C.20	C.4.9
C.21	C.4.10
none	C.4.11
none	C.4.12
C.22	C.4.13
C.23	C.4.14
Fig C.4	C.4.15
C.24	C.4.16
C.25	C.4.17

refs: done

C.5 The Foucault Pendulum e

C.26	C.5.1
none	C.5.2
none	C.5.3
C.27	C.5.4
C.28	C.5.5
C.29	C.5.6
C.30	C.5.7
none	C.5.8
none	C.5.9

refs: done

This concludes my incredibly painful and lengthy equation number update of frames doc. So now it is more maintainable.