If we apply the G Rule (2.1) to vector **b**, we find that

$$(d\mathbf{b}/dt)_S = (d\mathbf{b}/dt)_{S'} + \boldsymbol{\omega} \times \mathbf{b} \tag{4.1}$$

or

$$(d[b'_i \mathbf{e}'_i]/dt)_S = (db'_i/dt)\,\mathbf{e}'_i + b'_i(d\mathbf{e}'_i/dt)_S \ . \tag{4.2}$$

We want to clearly understand these two terms which contribute to $(d\mathbf{b}/dt)_S$ :

$$\text{1st term} = (d\mathbf{b}/dt)_{S'} = (db'_i/dt)\,\mathbf{e}'_i = [(d\mathbf{b}/dt)_S]|^{\mathbf{e}'i \text{ fixed}} \tag{4.3}$$

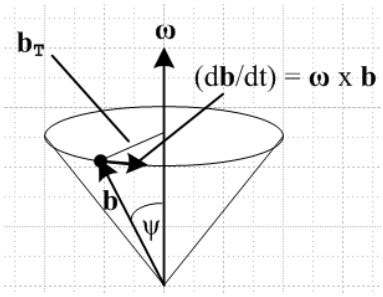
$$\text{2nd term} = \boldsymbol{\omega} \times \mathbf{b} = b'_i(d\mathbf{e}'_i/dt)_S = [(d\mathbf{b}/dt)_S]|^{b'i \text{ fixed}} \tag{4.4}$$

The 2nd term is entirely caused by the fact that the $\mathbf{e}'_i$ are rotating in frame S, which of course is due to the rotation of Frame S' about the $\boldsymbol{\omega}$ axis.

*Why* is this 2nd term contribution to $(d\mathbf{b}/dt)_S$ in the direction $\boldsymbol{\omega} \times \mathbf{b}$ ? It is because $(d\mathbf{e}'_i/dt)_S = \boldsymbol{\omega} \times \mathbf{e}'_i$:

$$\text{2nd term} = b'_i(d\mathbf{e}'_i/dt)_S = b'_i\,[\,\boldsymbol{\omega} \times \mathbf{e}'_i\,] = \boldsymbol{\omega} \times (b'_i\,\mathbf{e}'_i) = \boldsymbol{\omega} \times \mathbf{b}.$$

If our 1st term were zero, we would have $(d\mathbf{b}/dt)_S = \boldsymbol{\omega} \times \mathbf{b}$ which we know indicates that **b** is doing a certain circular motion in Frame S,




So we may regard the 1st term as measuring the *deviation* from this circular motion. This 2nd term is unaffected by any ongoing *change* in the vector $\boldsymbol{\omega}$ or by any ongoing *change* in the location of the rotation axis which contains vector $\boldsymbol{\omega}$.

The 1st term then represents the deviation from the circular motion noted above. There is such a deviation even when $\boldsymbol{\omega}$ = constant and the ω axis does not move ($\mathbf{v}_{\omega par} = 0$) and radius $a_\omega$ = constant. This 1st term deviation exists in this case because $\boldsymbol{\omega} \times \mathbf{b}$ is not tangent to the green circle in Fig 2, but the actual motion of **b** *is* tangent to the circle.

When $\boldsymbol{\omega} = \boldsymbol{\omega}(t)$, there are other sources of this 1st term deviation as well. For example, if $\boldsymbol{\omega}$ is in the process of changing direction, the green circle is in the process of tipping out of the plane of paper, and this might contribute to the motion of the S' origin (tip of vector **b**) a component perpendicular to the plane of paper. Or, it might be that the rotation axis indicated by vector $\boldsymbol{\omega}$ is translating sideways in the above picture at some instantaneous velocity $\mathbf{v}_{\omega par}$, and this would contribute to the motion of the S' origin (tip of vector **b**) another amount in the plane of paper.

In order to write down a precise expression for the 1st term $(d\mathbf{b}/dt)_{S'}$, we need to know many details of how the two frames are arranged. Basically, we need to know about $\boldsymbol{\omega}(t)$ (three parameters) and we need to know the instantaneous radius of the green circle $r_{\omega}(t)$ (a fourth parameter). Then we might compute the "tipping green circle effect" just mentioned. Similarly, we would need to know about the instantaneous parallel velocity of the $\boldsymbol{\omega}$ rotation axis $\mathbf{v}_{\omega par}(t)$ (another two parameters). We are thinking here of these parameters just mentioned as being the "drivers" of Fig 2, and then the vector **b** comes out however it does. (In Special Case #2 below we will instead think of $\boldsymbol{\omega}$ and **b** as the 6 independent parameters. )

Rather than attempt such a calculation for the general case, we shall just *leave* this first term $(d\mathbf{b}/dt)_{S'}$ sitting in all our equations and defer the task of computing $(d\mathbf{b}/dt)_{S'}$ to a specific application. As noted in (1.34), the shortened symbol for $(d\mathbf{b}/dt)_{S'}$ is $\dot{\mathbf{b}}_{S'}$ ,

$$\dot{\mathbf{b}}_{S'} \equiv (d\mathbf{b}/dt)_{S'} = \text{1st term} = (db'_i/dt)\,\mathbf{e}'_i = [(d\mathbf{b}/dt)_S]|^{\mathbf{e}'_i \text{ fixed}} \tag{4.5}$$

In this shortened notation the G Rule (4.1) says

$$\dot{\mathbf{b}}_S = \dot{\mathbf{b}}_{S'} + \boldsymbol{\omega} \times \mathbf{b} \tag{4.6}$$

Two special cases are of particular interest:

---

The quantity $\dot{\mathbf{b}}_S = (d\mathbf{b}/dt)_S$ describes the velocity of the Frame S' origin relative to that of Frame S. When $\omega = 0$, Frame S' merely translates relative to Frame S with velocity $\dot{\mathbf{b}}_S$ and acceleration $\ddot{\mathbf{b}}_S$ . When $\omega \neq 0$, however, for a general placement of the rotation axis one must regard **b**, $\dot{\mathbf{b}}_S$ and $\ddot{\mathbf{b}}_S$ as quantities derived from the location and movement of that axis and from the value of $\boldsymbol{\omega}$, all of which one imagines are controlled by some mechanical outside agency. An exception is Special Case #2 below where the rotation axis passes through the Frame S' origin.

The quantity $\dot{\mathbf{b}}_{S'} = (d\mathbf{b}/dt)_{S'}$ is a "cross velocity" in the sense of Section 1 (h) and is hard to interpret. In Special Case #1 below, however, the vector b is effectively glued to the Frame S' axes and $\dot{\mathbf{b}}_{S'} = 0$, causing simplification of many of the equations which will be obtained below.

---

$$\dot{\mathbf{b}}_S = \dot{\mathbf{b}}_{S'} + \boldsymbol{\omega} \times \mathbf{b} \tag{2.10}$$