old earth problems section

(d) Earth problems

The arrangement of frames was shown in the Figure in Section 4 (g). Recall that

$$\mathbf{F'_{eff}} = \mathbf{F} - m\ddot{\mathbf{b}}_S - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) - 2m\,\boldsymbol{\omega} \times \mathbf{v'} - m\dot{\boldsymbol{\omega}} \times \mathbf{r'} \qquad (8.4a)$$

frame centrifugal Coriolis Euler

For problems involving motions of objects near the surface of the Earth, one has

$$\mathbf{F} = m\mathbf{g} + \text{possible other forces} \qquad (8.12)$$

where $\mathbf{g} = -g\,\hat{\mathbf{r}}$. Possible other forces might include air friction, wind, the action of magnetic fields on charged particles, etc. For the earth $\dot{\boldsymbol{\omega}} \sim 0.5$ msec/day $\sim 10^{-8}$ sec^{-2} for seasonal variations and much less for short term variations. Even for r' ~ 1 km, we can neglect the Euler term $\dot{\boldsymbol{\omega}} \times \mathbf{r'} \sim 10^{-5} \sim 10^{-6}$ g, so we are left with

$$\mathbf{F'_{eff}} = (m\mathbf{g} + \text{possible other forces}) - m\ddot{\mathbf{b}}_S - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) - 2m\,\boldsymbol{\omega} \times \mathbf{v'} \quad . \qquad (8.13)$$

Carrying out a static experiment ($\mathbf{v'} = 0$) to measure the $\mathbf{g}$ vector at some location on the Earth (no other forces in this experiment), one finds

$$\mathbf{F'_{eff}} = m\mathbf{g} - m\ddot{\mathbf{b}}_S - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) = m[\mathbf{g} - \ddot{\mathbf{b}}_S - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'})] = m\mathbf{g_{local}}$$

where

$$\mathbf{g_{local}} \equiv \mathbf{g} - m\ddot{\mathbf{b}}_S - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}). \qquad (8.14a)$$

In this special case #1 problem we have from (7.12), neglecting the $\dot{\boldsymbol{\omega}} \times \mathbf{b}$ term,

$$\ddot{\mathbf{b}}_S = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{b}) \quad . \qquad (7.12)$$

Then since (6.1) $\mathbf{r} = \mathbf{r'} + \mathbf{b}$, we can write $\mathbf{g_{local}}$ in the more intuitive form

$$\mathbf{g_{local}} \equiv \mathbf{g} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad . \qquad (8.14b)$$

Note that $\mathbf{g_{local}}$ is not proportional to $\hat{\mathbf{r}}$. In practice, the locally flat Earth is perpendicular to $\mathbf{g_{local}}$ and that is the "g" one would be using in any local calculations. Then for an object in motion near the surface of the Earth, one has

$$\mathbf{F'_{eff}} = m\mathbf{g_{local}} - 2m\,\boldsymbol{\omega} \times \mathbf{v'} + \text{possible other forces}$$

and so only the Coriolis force of our earlier discussion is of concern. No terms are neglected in this equation, there is no approximation other than $\dot{\boldsymbol{\omega}} = 0$. The frame and centrifugal effects are both included in $m\mathbf{g}_{local}$.

We can ask how much $\mathbf{g}$ and $\mathbf{g}_{local}$ differ. For the Earth $\omega = 7.27 \times 10^{-5}$ sec^{-1}, so if $|\mathbf{r}'|$ is say 1 km,

$\omega^2 r' = (7.27)^2\ 10^{-10}\ 10^3 \sim 5 \times 10^{-6}$ m/sec^2 $\sim 10^{-6}$ g // $\boldsymbol{\omega}$ x ($\boldsymbol{\omega}$ x $\mathbf{r}'$) term very small

On the other hand, $\ddot{\mathbf{b}}_S = \boldsymbol{\omega}$ x ($\boldsymbol{\omega}$ x $\mathbf{b}$) $\sim \omega^2 b$. Since $b \approx 6.37 \times 10^6$ m we have

$\omega^2 b = (7.27)^2\ 10^{-10}\ 6.37 \times 10^6 = 344 \times 10^{-4} \sim 3 \times 10^{-2}$ m/sec^2 $\sim 3 \times 10^{-3}$ g $\approx \omega^2 r$

The conclusion is that $\mathbf{g}$ and $\mathbf{g}_{local}$ differ by up to 1/3 of 1% with direction as in (8.14b).

In some problems this $\ddot{\mathbf{b}}_S \sim \omega^2 b \sim 3 \times 10^{-2}$ may in fact be negligible, justifying the list of approximate equations given below (8.6). But when using $m\mathbf{g}_{local}$, the $\ddot{\mathbf{b}}_S$ term vanishes altogether from the problem along with the centripetal term.