Note that page numbering is turned on in this template.

For the $(d\mathbf{L}/dt)_S$ equation, we start with its alternate form (9.10)

$$(d\mathbf{L}/dt)_S = \boldsymbol{\omega} \times \mathbf{L} + \boldsymbol{\omega} \times (\mathbf{r} \times \mathbf{v}') + \mathbf{r} \times (\dot{\boldsymbol{\omega}} \times \mathbf{r}) + \mathbf{b} \times \mathbf{a}' + (d\mathbf{L}'/dt)_{S'}$$

to find that

$$(d\mathbf{L}'/dt)_{S'} = (d\mathbf{L}/dt)_S - \boldsymbol{\omega} \times \mathbf{L} - \boldsymbol{\omega} \times (\mathbf{r} \times \mathbf{v}') - \mathbf{r} \times (\dot{\boldsymbol{\omega}} \times \mathbf{r}) - \mathbf{b} \times \mathbf{a}' \qquad (10.5)$$

which we leave "as is". One can insert (10.2) and (10.3) for **v**' and **a**', but the result is unpleasant.

We may now summarize the solution of the Inverse Problem:

$$\mathbf{r}' = \mathbf{r} - \mathbf{b} \qquad (10.1)$$

$$\mathbf{v}' = \mathbf{v} - \boldsymbol{\omega} \times \mathbf{r} \qquad (10.2)$$

$$\mathbf{a}' = \mathbf{a} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - \dot{\boldsymbol{\omega}} \times \mathbf{r} - 2\,\boldsymbol{\omega} \times \mathbf{v} \qquad (10.3)$$

$$\mathbf{L}' = \mathbf{L} - \mathbf{r} \times \mathbf{b} - \mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad (10.4)$$

$$(d\mathbf{L}'/dt)_{S'} = (d\mathbf{L}/dt)_S - \boldsymbol{\omega} \times \mathbf{L} - \boldsymbol{\omega} \times (\mathbf{r} \times \mathbf{v}') - \mathbf{r} \times (\dot{\boldsymbol{\omega}} \times \mathbf{r}) - \mathbf{b} \times \mathbf{a}' \qquad (10.5)$$

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We now have to replace both **v**' and **a**'. First **v**' :

$$-\boldsymbol{\omega} \times (\mathbf{r} \times \mathbf{v}') = -\boldsymbol{\omega} \times (\mathbf{r} \times [\mathbf{v} - \boldsymbol{\omega} \times \mathbf{r}]) = -\boldsymbol{\omega} \times (\mathbf{r} \times \mathbf{v}) + \boldsymbol{\omega} \times (\mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r})$$

From (9.13) with $\mathbf{r} \leftrightarrow \boldsymbol{\omega}$ we know that

$$\boldsymbol{\omega} \times (\mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r}) = -(\boldsymbol{\omega} \bullet \mathbf{r})(\boldsymbol{\omega} \times \mathbf{r}) = (\mathbf{r} \bullet \boldsymbol{\omega})(\mathbf{r} \times \boldsymbol{\omega})$$

so that

$$-\boldsymbol{\omega} \times (\mathbf{r} \times \mathbf{v}') = -\boldsymbol{\omega} \times (\mathbf{r} \times \mathbf{v}) + (\mathbf{r} \bullet \boldsymbol{\omega})(\mathbf{r} \times \boldsymbol{\omega})$$

As for the **a**' term, we have

$$-\mathbf{b} \times \mathbf{a}' = -\mathbf{b} \times [\mathbf{a} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - \dot{\boldsymbol{\omega}} \times \mathbf{r} - 2\,\boldsymbol{\omega} \times \mathbf{v}]$$

$$= \mathbf{-\,b} \times \mathbf{a} \mathbf{-\,b} \times (\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})) + \mathbf{b} \times (\dot{\boldsymbol{\omega}} \times \mathbf{r}) + 2\,\mathbf{b} \times (\boldsymbol{\omega} \times \mathbf{v})$$

Inserting these two results gives

$$(d\mathbf{L}'/dt)_{S'} = (d\mathbf{L}/dt)_S - \boldsymbol{\omega} \times \mathbf{L} - \boldsymbol{\omega} \times (\mathbf{r} \times \mathbf{v}') - \mathbf{r} \times (\dot{\boldsymbol{\omega}} \times \mathbf{r}) - \mathbf{b} \times \mathbf{a}'$$

$$= (d\mathbf{L}/dt)_S - \boldsymbol{\omega} \times \mathbf{L} - \boldsymbol{\omega} \times (\mathbf{r} \times \mathbf{v}) + (\mathbf{r} \bullet \boldsymbol{\omega})(\mathbf{r} \times \boldsymbol{\omega}) - \mathbf{r} \times (\dot{\boldsymbol{\omega}} \times \mathbf{r})$$

$$\mathbf{-\,b} \times \mathbf{a} \mathbf{-\,b} \times (\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})) + \mathbf{b} \times (\dot{\boldsymbol{\omega}} \times \mathbf{r}) + 2\,\mathbf{b} \times (\boldsymbol{\omega} \times \mathbf{v})$$

or

$$(\mathbf{L}'/dt)_{S'} = (d\mathbf{L}/dt)_S - 2\,\boldsymbol{\omega} \times \mathbf{L} + (\mathbf{r} \bullet \boldsymbol{\omega})(\mathbf{r} \times \boldsymbol{\omega}) - \mathbf{r} \times (\dot{\boldsymbol{\omega}} \times \mathbf{r})$$
$$\mathbf{-\,b} \times \mathbf{a} \mathbf{-\,b} \times (\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})) + \mathbf{b} \times (\dot{\boldsymbol{\omega}} \times \mathbf{r}) + 2\,\mathbf{b} \times (\boldsymbol{\omega} \times \mathbf{v})$$

There must be something simpler! The G rule for L' says

$$(d\mathbf{L}'/dt)_S = \boldsymbol{\omega} \times \mathbf{L}' + (d\mathbf{L}'/dt)_{S'}$$

which we c