Note that page numbering is turned on in this template.

where are the right we just drop one of the frame labels for a more compact notation.

We c

$$\begin{array}{llll} \dot{\mathbf{a}}_S & \dot{\mathbf{a}}_{S'} & \dot{\mathbf{a}}'_S & \dot{\mathbf{a}}'_{S'} \\ \mathbf{c}_S & \mathbf{c}_{S'} & \mathbf{c}'_S & \mathbf{c}'_{S'} \end{array} \qquad \text{where } \mathbf{c} = \dot{\mathbf{a}} \qquad (1.29)$$

To make equations more compact, we shall often use the compressed notation on lines (1.30). Here of course **a** and **c** = $\dot{\mathbf{a}}$ are just generic names, but we might have **a** = position and **c** = velocity. Some other notations for these four derivatives are these

$$\begin{array}{llll} (d\mathbf{a}/dt)_S & (d\mathbf{a}/dt)_{S'} & (d\mathbf{a}'/dt)_S & (d\mathbf{a}'/dt)_{S'} \\ (d/dt)_S\mathbf{a} & (d/dt)_{S'}\mathbf{a} & (d/dt)_S\mathbf{a}' & (d/dt)_{S'}\mathbf{a}' \\ \partial_S\mathbf{a} & \partial_{S'}\mathbf{a} & \partial_S\mathbf{a}' & \partial_{S'}\mathbf{a}' \end{array} \qquad (1.29)$$

Here, we use $\partial_S \equiv (d/dt)_S$ as a shorthand for this operator, and this ∂ symbol has nothing to do with partial differentiation.

In a purely schematic sense, we can put the four derivatives in a matrix like this,

$$\begin{pmatrix} (d\mathbf{a}/dt)_S & (d\mathbf{a}/dt)_{S'} \\ (d\mathbf{a}'/dt)_S & (d\mathbf{a}'/dt)_{S'} \end{pmatrix} = \begin{pmatrix} (d/dt)_S\mathbf{a} & (d/dt)_{S'}\mathbf{a} \\ (d/dt)_S\mathbf{a}' & (d/dt)_{S'}\mathbf{a}' \end{pmatrix} = \begin{pmatrix} \partial_S\mathbf{a} & \partial_{S'}\mathbf{a} \\ \partial_S\mathbf{a}' & \partial_{S'}\mathbf{a}' \end{pmatrix} = \begin{pmatrix} \dot{\mathbf{a}}_S & \dot{\mathbf{a}}_{S'} \\ \dot{\mathbf{a}}'_S & \dot{\mathbf{a}}'_{S'} \end{pmatrix} = \begin{pmatrix} \mathbf{c}_S & \mathbf{c}_{S'} \\ \mathbf{c}'_S & \mathbf{c}'_{S'} \end{pmatrix}$$

The first column involves $(d/dt)_S$ derivatives, while the second $(d/dt)_{S'}$. The first row involves the vector **a** while the second involves some vector **a'** which is related to **a** "in some way". Here are some examples of vectors related in some way:

$\mathbf{r} = \mathbf{r}' + \mathbf{b}$ (**r**,**r'**) related in some way *(1.24)*

$\mathbf{v}_S = \mathbf{v}'_S + \dot{\mathbf{b}}_S$ ($\mathbf{v}_S$, $\mathbf{v}'_S$) related in some way *(6.4)*

The second equation results from applying $(d/dt)_S$ to the first.

Given the 2x2 matrix construct of the four derivatives, we can think of two of these derivatives as being "diagonal derivatives" and the other two as being "off diagonal" or "cross derivatives". In the diagonal cases, we have matching of the primes between the $(d/dt)_X$ operator and the vector acted upon, whereas in the cross-derivatives there is a mismatch.

What about *second* derivatives?

$$\partial_S \partial_S \mathbf{a} = \partial_S \mathring{\mathbf{a}}_S = \ddot{a}_{SS}$$
$$\partial_S \partial_{S'} \mathbf{a} = \partial_S \mathring{\mathbf{a}}_{S'} = \ddot{a}_{SS'}$$
$$\partial_{S'} \partial_S \mathbf{a} = \partial_{S'} \mathring{\mathbf{a}}_S = \ddot{a}_{S'S}$$
$$\partial_{S'} \partial_{S'} \mathbf{a} = \partial_{S'} \mathring{\mathbf{a}}_{S'} = \ddot{a}_{S'S'}$$

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Suppose we let symbols X and Y represent two frames of reference. Then

$$\partial_X \partial_Y \mathbf{a} = (d/dt)_X (d/dt)_Y \mathbf{a} = (d/dt)_X (d\mathbf{a}/dt)_Y = (d[(d\mathbf{a}/dt)_Y]/dt)_X$$

Then we have

$$\partial_X \partial_Y \mathbf{a} = (d[(d\mathbf{a}/dt)_Y]/dt)_X$$
$$\partial_Y \partial_X \mathbf{a} = (d[(d\mathbf{a}/dt)_X]/dt)_Y$$
$$\partial_X \partial_X \mathbf{a} = (d[(d\mathbf{a}/dt)_X]/dt)_X = (d^2\mathbf{a}/dt^2)_X = \partial_X{}^2 \mathbf{a}$$
$$\partial_Y \partial_Y \mathbf{a} = (d[(d\mathbf{a}/dt)_Y]/dt)_Y = (d^2\mathbf{a}/dt^2)_Y = \partial_Y{}^2 \mathbf{a}$$

$$\partial_S \partial_S \mathbf{a} = \partial_S (da/dt)_S =$$

$$(d/dt)_S (d/dt)_S \mathbf{a} = (d/dt)_S \mathbf{c}_S = (d\mathbf{a}_S/dt)_S$$
$$(d/dt)_S (d/dt)_S \mathbf{a'} = (d/dt)_S \mathbf{a'}_S = (d\mathbf{a'}_S/dt)_S$$

$$(d/dt)_{S'} (d/dt)_{S'} \mathbf{a} = (d/dt)_{S'} \mathbf{a}_{S'} = (d\mathbf{a}_{S'}/dt)_{S'}$$
$$(d/dt)_{S'} (d/dt)_{S'} \mathbf{a'} = (d/dt)_{S'} \mathbf{a'}_{S'} = (d\mathbf{a'}_S/dt)_{S'}$$

$$(d/dt)_S (d/dt)_{S'} \mathbf{a} = (d/dt)_S \mathbf{a}_{S'} = (d\mathbf{a}_{S'}/dt)_S$$
$$(d/dt)_S (d/dt)_{S'} \mathbf{a'} = (d/dt)_S \mathbf{a'}_{S'} = (d\mathbf{a'}_{S'}/dt)_S$$

$$(d/dt)_{S'} (d/dt)_S \mathbf{a} = (d/dt)_{S'} \mathbf{a}_S = (d\mathbf{a}_S/dt)_{S'}$$
$$(d/dt)_{S'} (d/dt)_S \mathbf{a'} = (d/dt)_{S'} \mathbf{a'}_S = (d\mathbf{a'}_S/dt)_{S'}$$

$$\begin{pmatrix} (d\mathbf{a}/dt)_S & (d\mathbf{a}/dt)_{S'} \\ (d\mathbf{a'}/dt)_S & (d\mathbf{a'}/dt)_{S'} \end{pmatrix} = \begin{pmatrix} (d/dt)_S \mathbf{a} & (d/dt)_{S'} \mathbf{a} \\ (d/dt)_S \mathbf{a'} & (d/dt)_{S'} \mathbf{a'} \end{pmatrix} = \begin{pmatrix} \mathring{\mathbf{a}}_S & \mathring{\mathbf{a}}_{S'} \\ \mathring{\mathbf{a}}'_S & \mathring{\mathbf{a}}'_{S'} \end{pmatrix}$$