Tides References PhL 8.13.12

The time has come to find some real references on this subject. Surely some modern author has computed the tidal force on the earth from the moon, and has computed the displacement that it creates.

I will start with wiki page on tides

The tides page is general but does have a Physics section. Newton Principia had a version. Our first reference is this:

J Wahr (1995). *Earth Tides in "Global Earth Physics", American Geophysical Union Reference Shelf #1,*. pp. 40–46.

"Maclaurin was the first to write about the Earth's rotational effects on motion. Euler realized that the tidal force's *horizontal* component (more than the vertical) drives the tide."

Laplace Tidal Equations are still in use today, it says.

Arthur Thomas Doodson developed and published in 1921[24] the first modern development of the tide-generating potential in harmonic form: Doodson distinguished 388 tidal frequencies.[25] Some of his methods remain in use.[26]

^ A T Doodson (December, 1921). "The Harmonic Development of the Tide-Generating Potential". *Proceedings of the Royal Society of London. Series A* **100** (704): 305–329. Bibcode 1921RSPSA.100..305D.

Current procedure for analysing tides follows the method of harmonic analysis introduced in the 1860s by William Thomson (our friend Kelvin)

Page says that usual analysis uses Fourier analysis, something far from what I have been doing.

So this is a good page, lots of references, lots of data, just over 25 minutes is mentioned. I can now go down the list of links and see if something good pops up.

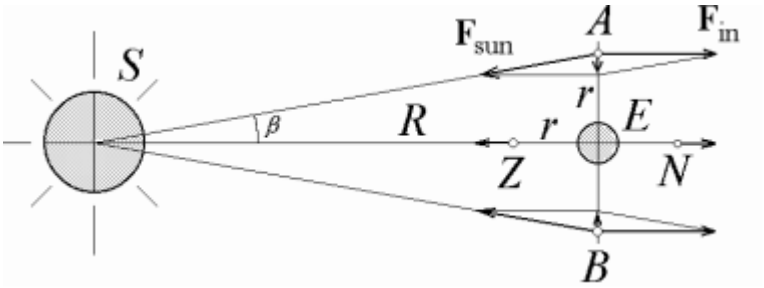
A dynamical picture of the oceanic tides

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This guy takes note of many bogus treatments in his opening paragraph.

He then gets right into the issue of the earth rotating while maintaining its orientation, my gimbals frame, his frying pan idea. This is a translating frame, not a rotating frame. Later he will do a rotating frame analysis I think. Yes. Points in the frame accelerate to the other object, yes. He seems to open with the sun-earth situation rather than the sun moon case. So far so good. In this frame we have a pseudo force $-m\mathbf{a}_0$. So I will think of object 1 left = sun, object 2 right = earth. He hammers on the point that the pseudo force is the same for all points in or on the earth, which may be my issue. Of course the grav force will be different at different points. I will ponder this later. He says that the pseudo force exactly balances the grav force at the center of the earth. Elsewhere, the difference is the tidal force.

He then produces figure 1 which is like my figure.



Point A is occupied by a satellite say. Vector F_{in} points exactly to the right, a key idea. This is the pseudo force vector for everything in the non-rotating gimbals earth frame of reference. The center of the earth is not at point A, only the satellite is at point A. The vector shown the tidal forces both pushing inward toward the earth. The lengths of the F_{sun} and F_{in} vectors are essentially the same if r is small. Geometry then shows that $F_{tid} = m(M_1G/R_1^2)\sin\beta = m\,(M_1G/R_1^2)(r/R)$, I agree.

He then considers points Z and N, and here the usual approximation is made, and we learn that a factor of 2 arises relative to the points A and B.

$$F_Z = G\frac{mM_{\text{sun}}}{(R-r)^2} - ma_0 = ma_0\left[\frac{R^2}{(R-r)^2} - 1\right] \approx ma_0\frac{2r}{R} = G\frac{mM_{\text{sun}}}{R^2}\frac{2r}{R}, \qquad (1)$$

Comment: this author is excellent. He is saying all the words I have been wanting to hear said, whereas Taylor just wings it. He then says the moon-earth situation is essentially the same, just change the numbers. Says the moon is 2.2 larger effect.

I stop at this point and will pick up later. He says basically Taylor is right and I am wrong. So that gimbals thing makes all the difference.

Then in section VI he starts throwing in the effect of the earth's rotation. Now centrifugal forces come into play, and they are much larger than tidal forces. He notes that the earth is oblate due to this force. He then says that since this centrifugal acceleration due to earth rotation is the same all around the earth, it does not affect the tides.

OK, so I have done enough reading for the moment. It is my task now to figure out WHY my analysis gives a different result. I am making some wrong assumption somehow. My approach is not the gimbals frame but the rotating frame where the earth rotates exactly one time per orbit around the moon. It seems

such a small difference. Author above argues that rotation should have no effect at all, yet I get an effect which is 41 times larger than his tidal force!

I think my "center cavity" business is correct.

Idea #1: In my picture my earth rotation is not about the center of the earth as author above considers, my rotation is about the center of mass which is NOT the center of the earth, and that is why I am picking up "differential" centrifugal effects!! The error of my ways is that the earth does not DO this kind of rotation! That is the issue, that is the discrepancy!!! My analysis would apply for the moon which DOES do this kind of rotation.