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Finally, the first equation in *** provides the string tension T(t).

As we shall see in Section C.8, if the spherical pendulum is started in some narrow elliptical orbit, that orbit precesses clockwise roughly in proportion to the area of that ellipse. This "intrinsic apsidal precession" occurs even if the Earth is not rotating. In order to use a spherical pendulum as a "Foucault" pendulum (to be discussed below), one must launch the spherical pendulum with a sufficiently small elliptical area (ideally that area is zero) so the Foucault precession is not masked by the intrinsic precession. Zero area corresponds to h = 0 and $L_z = 0$ so $\dot{\varphi} = 0$ in (C.5.16).

The Foucault/Intrinsic signal ratio is improved by making the pendulum very long (67 m in Paris). In theory, perhaps by using the "burned string" launch method, one can have h = 0 so $\dot{\varphi} = 0$ exactly and then the spherical pendulum becomes a plane pendulum moving in the plane $\varphi = \varphi_0$. Schumacher and Tarbet (2009) discuss this subject and propose an active electronic device to neutralize the intrinsic precession so that one can have a much shorter length Foucault pendulum.

(f) The spherical pendulum for small θ is still complicated unless h is very small

Setting sinθ = θ and cosθ = 1, the equations of motion (C.5.1) become

$$\ddot{\theta}^2 + \theta^2 \dot{\varphi}^2 = -(g/\ell) + T/(m\ell) \qquad \hat{\mathbf{r}}$$

$$\ddot{\theta} - \theta \dot{\varphi}^2 = -(g/\ell)\,\theta \qquad \hat{\boldsymbol{\theta}}$$

$$2\dot{\theta}\dot{\varphi} + \theta\ddot{\varphi} = 0 \quad . \qquad \hat{\boldsymbol{\varphi}} \qquad \text{(C.5.17)}$$

Since in general $\dot{\varphi}^2$ is not small relative to (g/ℓ), one cannot trivially solve the second equation to find simple harmonic motion for θ. If one assumes that $\dot{\varphi}^2 << (g/\ell)$ (meaning very small h) and then neglects the $\dot{\varphi}^2$ term in the second equation, one obtains $\theta(t) \approx \theta_0 \cos(\Omega t)$ where $\Omega = \sqrt{g/\ell}$ which is the usual angular frequency for a small-angle plane pendulum.