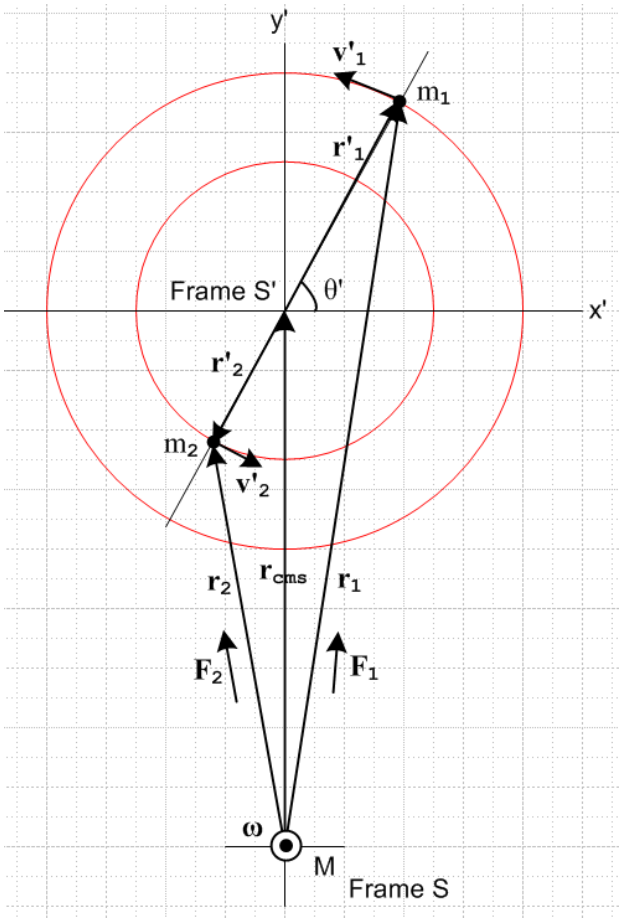
Appendix D questions PhL 12.6.16

Previous doc has good results, but here I am going to just start over on the equations of motion problem for the tether in motion. Lagrangian approach would be best probably, but I want to do it high school style. The first aid is the new picture



Frame S is inertial, Frame S' is non-inertial for two reasons. (1) satellite is moving around the earth (2) the cms is moving relative to the cog. I am hoping to ignore item (2) completely. This problem serves as a check on my fancy frames doc equations involving torque which I claim are exact.

From my summary of the forward problem, I know that for forces,

$\mathbf{F} = m\mathbf{a}$ // true Newton's Law in inertial Frame S *(8.1)*

$\mathbf{F'}_{\mathtt{eff}} = m\mathbf{a'}$ // fake Newton's Law in rotating Frame S' *(8.3)*

$$\mathbf{F'}_{\mathtt{eff}} = \mathbf{F} + \mathbf{F'}_{\mathtt{fict}} \ . \qquad (8.5)$$

$$\mathbf{F'}_{\mathtt{fict}} = -m\ddot{\mathbf{b}}_{\mathtt{S}} - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) - 2m\,\boldsymbol{\omega} \times \mathbf{v'} - m\dot{\boldsymbol{\omega}} \times \mathbf{r'} \ . \qquad (8.6)$$

frame centrifugal Coriolis Euler

and for torques

$$\mathbf{N}^{(c)} = \dot{\mathbf{L}}^{(c)} \qquad \text{// true Newton's Rot Law in Frame S} \qquad (11.6)$$

$$\mathbf{N'}^{(c')}_{\mathbf{eff}} = \mathbf{N}^{(c)} + \mathbf{N'}^{(c')}_{\mathbf{fict}} \qquad \text{// } \textit{defines } \mathbf{N'}^{(c')}_{\mathbf{fict}} \qquad (11.8)$$

$$\mathbf{N}^{(c)} + \mathbf{N'}^{(c')}_{\mathbf{fict}} = \dot{\mathbf{L}}^{(c')} \quad . \qquad (11.9)$$

$$\mathbf{N'}^{(c')}_{\mathbf{fict}} = -(\mathbf{c'} - \mathbf{c} + \mathbf{b}) \times m\mathbf{a'} + \dot{\mathbf{c}}_S \times [m\mathbf{v'} + m\boldsymbol{\omega} \times \mathbf{r'} + m\dot{\mathbf{b}}_S] - \dot{\mathbf{c}}'_{S'} \times m\mathbf{v'}$$
$$-(\mathbf{r'} - \mathbf{c} + \mathbf{b}) \times [\, m\dot{\boldsymbol{\omega}} \times \mathbf{r'} + 2m\boldsymbol{\omega} \times \mathbf{v'} + m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) + m\ddot{\mathbf{b}}_S] \qquad (11.11)$$

My presentation needs work, it is not parallel and the summary is incomplete. I don't say $F = \dot{p}$ for the linear case. So maybe

$$\mathbf{L}^{(c)} = (\mathbf{r}-\mathbf{c}) \times m\mathbf{v}$$

$$\dot{\mathbf{L}}^{(c)} = m(\mathbf{r}-\mathbf{c}) \times \mathbf{a} - \dot{\mathbf{c}}_S \times m\mathbf{v} = (\mathbf{r}-\mathbf{c}) \times \mathbf{F} - \dot{\mathbf{c}}_S \times m\mathbf{v}$$

The symbol ω is already used up. I want to say $\mathbf{L}^{(c)} = I^{(c)}\boldsymbol{\omega}$ in parallel with $\mathbf{p} = m\mathbf{v}$ but that implies I think that ω here is about point **c**. If I were to pick **c** = the ω axis point in Fig 11.3, then I could use ω the same variable, but that is not very general-case. So I have to stick with the above. So I could restate like so

$\mathbf{F} = m\mathbf{a} = \dot{\mathbf{p}}$	$\mathbf{N}^{(c)} = \dot{\mathbf{L}}^{(c)}$	Newton
$\mathbf{F'}_{\mathbf{eff}} = m\mathbf{a'} = \dot{\mathbf{p}}'$	$\mathbf{N'}^{(c')}_{\mathbf{eff}} = \dot{\mathbf{L}}^{(c')}$	effective Newton
$\mathbf{F'}_{\mathbf{eff}} = \mathbf{F} + \mathbf{F'}_{\mathbf{fict}}$	$\mathbf{N'}^{(c')}_{\mathbf{eff}} = \mathbf{N}^{(c)} + \mathbf{N'}^{(c')}_{\mathbf{fict}}$	
$\mathbf{F'}_{\mathbf{fict}} = \mathbf{F'}_{\mathbf{eff}} - \mathbf{F} = \dot{\mathbf{p}}' - \dot{\mathbf{p}}$	$\mathbf{N'}^{(c')}_{\mathbf{fict}} = \mathbf{N'}^{(c')}_{\mathbf{eff}} - \mathbf{N}^{(c)} = \dot{\mathbf{L}}^{(c')} - \dot{\mathbf{L}}^{(c)}$	

$$\mathbf{F'}_{\mathbf{fict}} = -m\ddot{\mathbf{b}}_S - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) - 2m\,\boldsymbol{\omega} \times \mathbf{v'} - m\dot{\boldsymbol{\omega}} \times \mathbf{r'} \quad . \qquad \textit{(8.6)}$$
frame centrifugal Coriolis Euler

$$\mathbf{N'}^{(c')}_{\mathbf{fict}} = -(\mathbf{c'} - \mathbf{c} + \mathbf{b}) \times m\mathbf{a'} + \dot{\mathbf{c}}_S \times [m\mathbf{v'} + m\boldsymbol{\omega} \times \mathbf{r'} + m\dot{\mathbf{b}}_S] - \dot{\mathbf{c}}'_{S'} \times m\mathbf{v'}$$
$$-(\mathbf{r'} - \mathbf{c} + \mathbf{b}) \times [\, m\dot{\boldsymbol{\omega}} \times \mathbf{r'} + 2m\boldsymbol{\omega} \times \mathbf{v'} + m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) + m\ddot{\mathbf{b}}_S] \qquad (11.11)$$

Is this the best I can do? Has several confusions I think.

Item 1: Look at equation $\mathbf{N'^{(c')}_{eff}} = \mathbf{N^{(c)}} + \mathbf{N'^{(c')}_{fict}}$. Normally you add two torques with respect to the same reference point! Does this make sense with arbitrary c and c' vectors as shown in Fig 11.3 ?? Forces don't have reverence points, so this issue does not arise.

Suppose we start with Frames S and S' with different origins and S' is rotating and we have this

$$\mathbf{N'^{(c')}_{eff}} = \mathbf{N^{(c)}} + \mathbf{N'^{(c')}_{fict}}$$

If we were to first turn off the rotation (set $\omega = 0$) and then align the axes, it seems we should have

$$\mathbf{N'^{(c')}_{eff}} \rightarrow \mathbf{N^{(c)}} \qquad \mathbf{r} - \mathbf{c} = \mathbf{r'} - \mathbf{c'} \qquad \mathbf{c} - \mathbf{c'} = \mathbf{b}$$

But in this case you need to have $\mathbf{c'} \rightarrow \mathbf{c}$ or it does not make sense. So I don't think I can set these two points arbitrarily as I have done, so lets now rewrite Section 11 (b) (for starters). There I will have c' and c be vectors to the same point in space!