This is the Title PhL 1.11.15

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$$I = \int_0^{\pi} d\theta \frac{\sin\theta\,(b-r\cos\theta)}{(r^2+b^2-2rb\cos\theta)^{3/2}} \qquad \text{den} = (r_1^2)^{3/2} = r_1^3$$

Let

$$r_1^2 = r^2 + b^2 - 2rb\cos\theta$$

$$2r_1 dr_1 = +2rb\sin\theta d\theta \qquad \Rightarrow \qquad \sin\theta d\theta = r_1 dr_1/(rb)$$

$$r\cos\theta = (r^2+b^2- r_1^2)/2b$$

$$b - r\cos\theta = b - (r^2+b^2- r_1^2)/2b = (2b^2 - r^2-b^2+r_1^2)/(2b) = (b^2-r^2+r_1^2)/2b$$

Then

$$I = \int (r_1 dr_1/rb) * (b^2-r^2+r_1^2)/2b * (1/r_1^3)$$

$$= (2rb^2)^{-1} \int (r_1 dr_1 * (b^2-r^2+r_1^2) * (1/r_1^3)$$

$$= (2rb^2)^{-1} \int dr_1 * (b^2-r^2+r_1^2) (1/r_1^2)$$

$$= (2rb^2)^{-1} \int dr_1 [(b^2-r^2) / r_1^2 + 1]$$

$$= (2rb^2)^{-1} [(b^2-r^2) \int dr_1/r_1^2 + \int dr_1\, 1]$$

What are the endpoints?

$$r_1^2 = r^2 + b^2 - 2rb\cos\theta$$

$$r_{1low}^2 = r^2 + b^2 - 2rb = (r-b)^2 \qquad r_{1lo} = |b-r| = \alpha$$

$$r_{1hi}^2 = r^2 + b^2 + 2rb = (r+b)^2 \qquad r_{1hi} = b+r = \beta$$

So Answer is

$$(2rb^2)^{-1}\,[\;\; (b^2-r^2)\int_{\alpha}^{\beta} dr_1/r_1^2 \;\; + \;\; \int_{\alpha}^{\beta} dr_1\; 1\;]$$

$$= \;\; (2rb^2)^{-1}[\;\; (b^2-r^2)\;\{-(1/r_1)\}_{\alpha}^{\beta} \; + (\beta-\alpha)\;\;]$$

$$= \;\; (2rb^2)^{-1}\,[\;\; (b^2-r^2)\;\{-(1/\beta)+(1/\alpha)\} \;\; + (\beta-\alpha)\;\;]$$

$$= \;\; (2rb^2)^{-1}\,[\;\; (b^2-r^2)\;\{-(1/(b+r))+(1/|b-r|)\} \;\; + (b+r - |b-r|\;)\;\;]$$

$$= \;\; (2rb^2)^{-1}\,[\;\; (b^2-r^2)\;\{\frac{1}{|b-r|} \; - \frac{1}{b+r}\} \;\; + b+r - \;|b-r|)\;\;]$$

$$= \;\; (2rb^2)^{-1}[\;\; (b^2-r^2)\;\{\frac{(b+r) - |b-r|}{(b+r)|b-r|}\} \;\; + b+r - \;|b-r|)\;)\;\;]$$

$$= \;\; (2rb^2)^{-1}[\;\; (b+r)(b-r)\;\{\frac{(b+r) - |b-r|}{(b+r)|b-r|}\} \;\; + b+r - \;|b-r|)\;)\;\;]$$

$$= \;\; (2rb^2)^{-1}[\;\; (b-r)\;\{\frac{(b+r) - |b-r|}{|b-r|}\} \;\; + \;\; b+r - \;|b-r|)\;)\;\;]$$

If b > r this says

$$= \;\; (2rb^2)^{-1}[\;\; (b-r)\;\{\frac{(b+r) - (b-r)}{(b-r)}\} \;\; + \;\; b+r - \;(b-r)\;)\;\;]$$

$$= \;\; (2rb^2)^{-1}[\;\; 2r \;\; + 2r \;\;] \; = 4r/(2rb^2) \; = 2/b^2$$

If b < r this says

$$= \;\; (2rb^2)^{-1}[\;\; (b-r)\;\{\frac{(b+r) + (b-r)}{-(b-r)}\} \;\; + \;\; b+r + (b-r)\;)\;\;]$$

$$= \;\; (2rb^2)^{-1}[\;-2b \;\; + 2b)\;\;]$$

$$= 0$$