Debugging Maple's Numeric Dsolve problem PhL 1.10.17

I have two sample files and they both show the same problem. One is a second order system, the other is a corresponding first order system. They both make the "cannot convert to first order" error. I did a web scan and various people get the same error, but none of their hints fixed my problem.

Here is the second order system showing the error:

$$y\ddot{x} - x\ddot{y} = -\omega^2 yx - 2\omega\dot{z}x$$

$$b\ddot{x} - x\ddot{z} = +\omega^2 bx + 2\omega x\dot{y}$$

$$x^2+y^2+z^2 = r_1^2$$

I will work in mws file called debug1.mws. I will start with the above mess and keep simplifying it. I suspect the last equation is the problem but want to confirm this feeling.

Step 1: Let's try this instead

$$y\ddot{x} - x\ddot{y} = yx$$

$$1000\ddot{x} - x\ddot{z} = 2x$$

$$x^2+y^2+z^2 = 100$$

Here is my equation entry

```
restart; with(DEtools):
eq1 := y(t)*diff(x(t),t,t) - x(t)*diff(y(t),t,t) = y(t)*x(t);
```

$$eq1 := y(t)\left(\frac{\partial^2}{\partial t^2}x(t)\right) - x(t)\left(\frac{\partial^2}{\partial t^2}y(t)\right) = y(t)\,x(t)$$

```
eq2 := 1000*diff(x(t),t,t)- x(t)*diff(z(t),t,t) = 2*x(t);
```

$$eq2 := 1000\left(\frac{\partial^2}{\partial t^2}x(t)\right) - x(t)\left(\frac{\partial^2}{\partial t^2}z(t)\right) = 2\,x(t)$$

```
eq3 := x(t)^2+y(t)^2+z(t)^2 = 100;
```

$$eq3 := x(t)^2 + y(t)^2 + z(t)^2 = 100$$

```
eqs := {eq1,eq2,eq3};
```

And I continue on to generate the error message

```
> eqs := {eq1,eq2,eq3}:
> funcs := {x(t),y(t),z(t)}:
> inits := {x(0)=1,y(0)=0,z(0)=0,D(x)(0)=0,D(y)(0)=0,D(z)(0)=0 }:
> convertsys(eqs,inits,funcs,t,Y,YP);
Error, (in convertsys) unable to convert to an explicit first-order system
```

Step 2. Simplify to

$\ddot{x} - \ddot{y} = yx$
$1000\ddot{x} - \ddot{z} = 2x$
$x^2+y^2+z^2 = 100$ Same error.

Step 3. Simplify to

$\ddot{x} = y$
$\ddot{x} + \ddot{z} = x$
$x^2+y^2+z^2 = 100$ Same error.

Step 4. Simplify to

$\ddot{x} = y$
$x^2+y^2 = 100$ Same error

Here is the code for the above:

```
restart; with(DEtools):
eq1 := diff(x(t),t,t) = y(t);
```

$$eq1 := \frac{\partial^2}{\partial t^2} x(t) = y(t)$$

```
eq2 := x(t)^2+y(t)^2 = 100;
```

$$eq2 := x(t)^2 + y(t)^2 = 100$$

```
eqs := {eq1,eq2}:
funcs := {x(t),y(t)}:
inits := {x(0)=1,y(0)=sqrt(100-1),D(x)(0)=0 }:
convertsys(eqs,inits,funcs,t,Y,YP);
ror, (in convertsys) unable to convert to an explicit first-order system
```

Maybe all equations have to be ODE's with at least a first derivative?

Step 5. Try this

$\ddot{x} = y$
$\dot{x}^2+y^2 = 100$ Same error

```
eq1 := diff(x(t),t,t) = y(t);
```

$$eq1 := \frac{\partial^2}{\partial t^2} x(t) = y(t)$$

```
eq2 := diff(x(t),t)^2+y(t)^2 = 100;
```

$$eq2 := \left(\frac{\partial}{\partial t} x(t)\right)^2 + y(t)^2 = 100$$

```
eqs := {eq1,eq2}:
funcs := {x(t),y(t)}:
inits := {x(0)=1,y(0)=0,D(x)(0)=0 }:
convertsys(eqs,inits,funcs,t,Y,YP);
ror, (in convertsys) unable to convert to an explicit first-order sys
```

Step 6. Try this

$\ddot{x} = y$

$\dot{x}+y = 100$ Same error

```
eq1 := diff(x(t),t,t) = y(t);
```

$$eq1 := \frac{\partial^2}{\partial t^2} x(t) = y(t)$$

```
eq2 := diff(x(t),t)+y(t) = 100;
```

$$eq2 := \left(\frac{\partial}{\partial t} x(t)\right) + y(t) = 100$$

```
eqs := {eq1,eq2}:
funcs := {x(t),y(t)}:
inits := {x(0)=1,y(0)=0,D(x)(0)=0 }:
convertsys(eqs,inits,funcs,t,Y,YP);
ror, (in convertsys) unable to convert to an explicit first-order syst
```

Step 6. Try this

$\dot{y} = x + 2$

$\dot{x}+y = 100$

The error goes away. This is already a first order system:

```
eq1 := diff(y(t),t) = x(t)+2;
```

$$eq1 := \frac{\partial}{\partial t} y(t) = x(t) + 2$$

```
eq2 := diff(x(t),t)+y(t) = 100;
```

$$eq2 := \left(\frac{\partial}{\partial t} x(t)\right) + y(t) = 100$$

```
eqs := {eq1,eq2}:
funcs := {x(t),y(t)}:
inits := {x(0)=1,y(0)=0}:
convertsys(eqs,inits,funcs,t,Y,YP);
```

$$[[YP_1 = -Y_2 + 100, YP_2 = Y_1 + 2], [Y_1 = x(t), Y_2 = y(t)], 0, [1, 0]]$$

Step 7. Try this

$\dot{y} = x + 2$

$\dot{x}+y^2 = 100$ no error

Step 8: Try this

$\dot{y} = x + 2$

$x+y = 100$ yes error

```
eq1 := diff(y(t),t) = x(t)+2;
```

$$eq1 := \frac{\partial}{\partial t} y(t) = x(t) + 2$$

```
eq2 := x(t)+y(t) = 100;
```

$$eq2 := x(t) + y(t) = 100$$

```
eqs := {eq1,eq2}:
funcs := {x(t),y(t)}:
inits := {y(0)=0}:
convertsys(eqs,inits,funcs,t,Y,YP);
ror, (in convertsys) unable to convert to an explicit first-order s
```

I think this confirms that each equation must have at least one derivative! I can reformulate the above Step 8 problem this way:

$\dot{y} = (100-y) + 2$

```
restart; with(DEtools):
eq1 := diff(y(t),t) = (100-y(t))+2;
```

$$eq1 := \frac{\partial}{\partial t} y(t) = 102 - y(t)$$

```
eqs := {eq1}:
funcs := {y(t)}:
inits := {y(0)=0}:
convertsys(eqs,inits,funcs,t,Y,YP);
```

$$[[YP_1 = 102 - Y_1], [Y_1 = y(t)], 0, [0]]$$

RULE #1: Every equation must have at least one derivative!

Step 9: Go back now to the original problem

$y\ddot{x} - x\ddot{y} = -\omega^2 yx - 2\omega\dot{z}x$

$b\ddot{x} - x\ddot{z} = +\omega^2 bx + 2\omega x\,\dot{y}$

$x^2+y^2+z^2 = r_1^2$

How can I restate this so all equations have derivatives? Let's quickly jump to a simple case which I think shows the same problem:

$\ddot{x} - \ddot{y} = x+2$

$\ddot{x} = +y$

$x^2+y^2 =25$ Same error

Make it even simpler

$\dot{x} - \dot{y} = x+2$

$\dot{x} = +y$

$x^2+y^2 =25$ Same error

```
eq1 := diff(x(t),t)-diff(y(t),t)  = x(t)+2;
```

$$eq1 := \left(\frac{\partial}{\partial t}x(t)\right) - \left(\frac{\partial}{\partial t}y(t)\right) = x(t) + 2$$

```
eq2 := diff(x(t),t) = y;
```

$$eq2 := \frac{\partial}{\partial t}x(t) = y$$

```
eq3 := x(t)^2+y(t)^2 = 25;
```

$$eq3 := x(t)^2 + y(t)^2 = 25$$

```
eqs := {eq1,eq2,eq3}:
funcs := {x(t),y(t)}:
inits := {x(0)=3,y(0)=4}:
convertsys(eqs,inits,funcs,t,Y,YP);
Error, (in convertsys) unable to convert to an explicit first-order system
```

RULE #2: Even if you system seems *obviously* first order, you can still get this error!

One reason here for failure is that in effect I have 2 equations for just one function x(t).

Now try to eliminate y from these three equations. Do this system

$\dot{x} - \dot{y} = x+2$

$y = \sqrt{25-x^2}$

Here is a path to success:

```
restart; with(DEtools):
y := (t) -> sqrt(25-x(t)^2);
```

$$y := t \to \sqrt{25 - x(t)^2}$$

```
yd := diff(y(t),t);
```

$$yd := -\frac{x(t)\left(\frac{\partial}{\partial t}x(t)\right)}{\sqrt{25 - x(t)^2}}$$

```
eq1 := diff(x,t) - yd = x(t)+2;
```

$$eq1 := \frac{x(t)\left(\frac{\partial}{\partial t}x(t)\right)}{\sqrt{25 - x(t)^2}} = x(t) + 2$$

```
eqs := {eq1}:
funcs := {x(t)}:
inits := {x(0)=1}:

convertsys(eqs,inits,funcs,t,Y,YP);
```

$$\left[\left[YP_1 = \frac{\sqrt{25 - Y_1{}^2}\,(Y_1 + 2)}{Y_1}\right], [Y_1 = x(t)], 0, [1]\right]$$

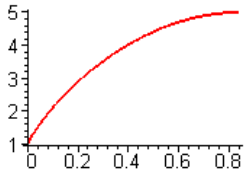
```
dsolve(eqs union inits,funcs,type=numeric,method=classical,output=listprocedure);
```

$$[t = (\mathbf{proc}(t) \ldots \mathbf{end}), x(t) = (\mathbf{proc}(t) \ldots \mathbf{end})]$$

```
f := dsolve(eqs union inits,funcs,type=numeric,method=classical,output=listprocedure);
```

$$f := [t = (\mathbf{proc}(t) \ldots \mathbf{end}), x(t) = (\mathbf{proc}(t) \ldots \mathbf{end})]$$

```
odeplot(f,[t,x(t)],0..3.5,numpoints=1500,color=red,thickness=2);
```



It is not complaining about the ambiguity of the square root! It just takes it positive I presume.

RULE #3: Elementary square roots are not a problem.

Step 10: Let's back up to Step 3 and try a subs with x,y,z, mimicking the above

$\ddot{x} = y$

$\ddot{x} + \ddot{z} = x$

$x^2+y^2+z^2 = 100$

I will eliminate z

$z = \sqrt{100-x^2-y^2}$

Success! Here it is. First part,

```
restart; with(DEtools):with(plots):
z := (t) -> sqrt(100-x(t)^2-y(t)^2);
```

$$z := t \to \sqrt{100 - x(t)^2 - y(t)^2}$$

```
zdd := diff(z(t),t,t);
```

$$zdd := -\frac{1}{4}\frac{\left(-2\,x(t)\left(\frac{\partial}{\partial t}x(t)\right) - 2\,y(t)\left(\frac{\partial}{\partial t}y(t)\right)\right)^2}{\left(100 - x(t)^2 - y(t)^2\right)^{\left(\frac{3}{2}\right)}} + \frac{1}{2}\frac{-2\left(\frac{\partial}{\partial t}x(t)\right)^2 - 2\,x(t)\left(\frac{\partial^2}{\partial t^2}x(t)\right) - 2\left(\frac{\partial}{\partial t}y(t)\right)^2 - 2\,y(t)\left(\frac{\partial^2}{\partial t^2}y(t)\right)}{\sqrt{100 - x(t)^2 - y(t)^2}}$$

```
eq1 := diff(x(t),t,t) = y(t);
```

$$eq1 := \frac{\partial^2}{\partial t^2}x(t) = y(t)$$

```
eq2 := diff(x(t),t,t) + zdd = x(t);
```

$$2 :=$$

$$\left(\frac{\partial^2}{\partial t^2}x(t)\right) - \frac{1}{4}\frac{\left(-2\,x(t)\left(\frac{\partial}{\partial t}x(t)\right) - 2\,y(t)\left(\frac{\partial}{\partial t}y(t)\right)\right)^2}{\left(100 - x(t)^2 - y(t)^2\right)^{\left(\frac{3}{2}\right)}} + \frac{1}{2}\frac{-2\left(\frac{\partial}{\partial t}x(t)\right)^2 - 2\,x(t)\left(\frac{\partial^2}{\partial t^2}x(t)\right) - 2\left(\frac{\partial}{\partial t}y(t)\right)^2 - 2\,y(t)\left(\frac{\partial^2}{\partial t^2}y(t)\right)}{\sqrt{100 - x(t)^2 - y(t)^2}} = x(t)$$

and now the second part

```
> eqs := {eq1,eq2}:
> funcs := {x(t),y(t)}:
> inits := {x(0)=1.6,y(0)=3.2,D(x)(0)=0,D(y)(0)=0}:
>
> convertsys(eqs,inits,funcs,t,Y,YP);
```

$$\left[YP_1 = Y_2,\ YP_2 = Y_3,\ YP_3 = Y_4,\ YP_4 = -\left(Y_3\left(100 - Y_1{}^2 - Y_3{}^2\right)^{\left(\frac{3}{2}\right)} - 2\,Y_1\,Y_2\,Y_3\,Y_4 - 100\,Y_2{}^2 + Y_2{}^2\,Y_3{}^2 - 100\,Y_1\,Y_3 + Y_1{}^3\,Y_3 + Y_1\,Y_3{}^3 - 100\,Y_4{}^2 + Y_4{}^2\,Y_1{}^2 - Y_1\left(100 - Y_1{}^2 - Y_3{}^2\right)^{\left(\frac{3}{2}\right)}\right) \Big/ \left(Y_3\left(-100 + Y_1{}^2 + Y_3{}^2\right)\right)\right], \left[Y_1 = x(t),\ Y_2 = \frac{\partial}{\partial t}x(t),\ Y_3 = y(t),\ Y_4 = \frac{\partial}{\partial t}y(t)\right], 0, [1.6, 0, 3.2, 0]\right]$$

```
> f := dsolve(eqs union inits,funcs,type=numeric,method=classical,output=listprocedure);
```

$$f := \left[t = (\mathbf{proc}(t)\ \ldots\ \mathbf{end}),\ x(t) = (\mathbf{proc}(t)\ \ldots\ \mathbf{end}),\ \frac{\partial}{\partial t}x(t) = (\mathbf{proc}(t)\ \ldots\ \mathbf{end}),\ y(t) = (\mathbf{proc}(t)\ \ldots\ \mathbf{end}),\ \frac{\partial}{\partial t}y(t) = (\mathbf{proc}(t)\ \ldots\ \mathbf{end})\right]$$

```
> f(.2);
```

$$t(.2) = .2,\ x(t)(.2) = 1.662573872926857,\ \left(\frac{\partial}{\partial t}x(t)\right)(.2) = .6437419077652143,\ y(t)(.2) = 3.258519710532320,$$

Once again, it does not mind the square roots.

Step 11: Now back to the original problem!

$$y\ddot{x} - x\ddot{y} = -\omega^2 yx - 2\omega\dot{z}x$$

$b\ddot{x} - x\ddot{z} = +\omega^2 bx + 2\omega x \dot{y}$

$x^2+y^2+z^2 = r_1^2$

I will try to eliminate z, mimicking the Step 10 case. I will do this in "satellite in Cartesians v2a.mws". It works! Here is the code:

First, here is the stuff used to eliminate z

```
z := (t) -> sqrt(r1^2-x(t)^2-y(t)^2);
```

$$z := t \to \sqrt{r1^2 - x(t)^2 - y(t)^2}$$

```
zd := diff(z(t),t);
```

$$zd := \frac{1}{2}\frac{-2\,x(t)\left(\frac{\partial}{\partial t}x(t)\right) - 2\,y(t)\left(\frac{\partial}{\partial t}y(t)\right)}{\sqrt{r1^2 - x(t)^2 - y(t)^2}}$$

```
zdd := diff(z(t),t,t);
```

$$zdd := -\frac{1}{4}\frac{\left(-2\,x(t)\left(\frac{\partial}{\partial t}x(t)\right) - 2\,y(t)\left(\frac{\partial}{\partial t}y(t)\right)\right)^2}{\left(r1^2 - x(t)^2 - y(t)^2\right)^{\left(\frac{3}{2}\right)}} + \frac{1}{2}\frac{-2\left(\frac{\partial}{\partial t}x(t)\right)^2 - 2\,x(t)\left(\frac{\partial^2}{\partial t^2}x(t)\right) - 2\left(\frac{\partial}{\partial t}y(t)\right)^2 - 2\,y(t)\left(\frac{\partial^2}{\partial t^2}y(t)\right)}{\sqrt{r1^2 - x(t)^2 - y(t)^2}}$$

Then here are the two equations in x(t) and y(t) only,

```
> eq1 := y(t)*diff(x(t),t,t)-x(t)*diff(y(t),t,t)=-w^2*y(t)*x(t)-2*w*zd*x(t);
```

$$eq1 := y(t)\left(\frac{\partial^2}{\partial t^2}x(t)\right) - x(t)\left(\frac{\partial^2}{\partial t^2}y(t)\right) = -w^2\,y(t)\,x(t) - \frac{w\left(-2\,x(t)\left(\frac{\partial}{\partial t}x(t)\right) - 2\,y(t)\left(\frac{\partial}{\partial t}y(t)\right)\right)x(t)}{\sqrt{r1^2 - x(t)^2 - y(t)^2}}$$

```
> eq2 := b*diff(x(t),t,t)- x(t)*zdd = w^2*b*x(t)+2*w*x(t)*diff(y(t),t);
```

$$eq2 := b\left(\frac{\partial^2}{\partial t^2}x(t)\right) - x(t)\left(-\frac{1}{4}\frac{\left(-2\,x(t)\left(\frac{\partial}{\partial t}x(t)\right) - 2\,y(t)\left(\frac{\partial}{\partial t}y(t)\right)\right)^2}{\left(r1^2 - x(t)^2 - y(t)^2\right)^{\left(\frac{3}{2}\right)}} + \frac{1}{2}\frac{-2\left(\frac{\partial}{\partial t}x(t)\right)^2 - 2\,x(t)\left(\frac{\partial^2}{\partial t^2}x(t)\right) - 2\left(\frac{\partial}{\partial t}y(t)\right)^2 - 2\,y(t)\left(\frac{\partial^2}{\partial t^2}y(t)\right)}{\sqrt{r1^2 - x(t)^2 - y(t)^2}}\right) = w^2\,b\,x(t) + 2\,w\,x(t)\left(\frac{\partial}{\partial t}y(t)\right)$$

And now we try to solve:

```
b := 1000;
```

$$b := 1000$$

```
r1 := 10;
```

$$r1 := 10$$

```
w := 1;
```

$$w := 1$$

```
eqs := {eq1,eq2}:
funcs := {x(t),y(t)}:
inits := {x(0)= 1,y(0) = 0, D(x)(0)=0,D(y)(0)=0};
```

$$inits := \{x(0) = 1, y(0) = 0, D(x)(0) = 0, D(y)(0) = 0\}$$

```
f := dsolve(eqs union inits,funcs,type=numeric,method=classical,output=listprocedure);
```

$$f := \left[t = (\mathbf{proc}(t) \ \ldots \ \mathbf{end}), x(t) = (\mathbf{proc}(t) \ \ldots \ \mathbf{end}), \frac{\partial}{\partial t} x(t) = (\mathbf{proc}(t) \ \ldots \ \mathbf{end}), y(t) = (\mathbf{proc}(t) \ \ldots \ \mathbf{end}), \frac{\partial}{\partial t} y(t) = (\mathbf{proc}(t) \ \ldots \ \mathbf{end})\right]$$

Step 13: Question: can you have a function appear in your equation set which has no derivatives?

Here is a simple test :

$\dot{x} = x$

$\dot{x} = y-3$

and here is what it says:

```
restart; with(DEtools):
eq1 := diff(x(t),t) = x(t);
```

$$eq1 := \frac{\partial}{\partial t} x(t) = x(t)$$

```
eq2 := diff(x(t),t) = y-3;
```

$$eq2 := \frac{\partial}{\partial t} x(t) = y - 3$$

```
eqs := {eq1,eq2}:
funcs := {x(t),y(t)}:
inits := {x(0)=3}:
convertsys(eqs,inits,funcs,t,Y,YP);
Error, (in convertsys) unable to convert to an explicit first-order system
```

Just to make sure, here is a fancier case :

$\dot{x} = x + y$

$\ddot{x} = y-3$

```
restart; with(DEtools):
eq1 := diff(x(t),t) = x(t)+y(t);
```

$$eq1 := \frac{\partial}{\partial t} x(t) = x(t) + y(t)$$

```
eq2 := diff(x(t),t,t) = y(t)-3;
```

$$eq2 := \frac{\partial^2}{\partial t^2} x(t) = y(t) - 3$$

```
eqs := {eq1,eq2}:
funcs := {x(t),y(t)}:
inits := {x(0)=3,D(x)(0)=0}:
convertsys(eqs,inits,funcs,t,Y,YP);
:or, (in convertsys) unable to convert to an explicit first-order s
```

So we now have :

RULE #4 : All unknown functions to be solved for must have at least one derivative somewhere in the system of equations.

Step 14: Let's try to fix up the last example above

$\dot{x} = x + y$

$\ddot{x} = \dot{y}-3$ Still makes the error!

```
restart; with(DEtools):
eq1 := diff(x(t),t) = x(t)+y(t);
```

$$eq1 := \frac{\partial}{\partial t} x(t) = x(t) + y(t)$$

```
eq2 := diff(x(t),t,t) = diff(y(t),t)-3;
```

$$eq2 := \frac{\partial^2}{\partial t^2} x(t) = \left(\frac{\partial}{\partial t} y(t)\right) - 3$$

```
eqs := {eq1,eq2}:
funcs := {x(t),y(t)}:
inits := {x(0)=3,y(0)=0,D(x)(0)=0}:
convertsys(eqs,inits,funcs,t,Y,YP);
ror, (in convertsys) unable to convert to an explicit first-order system
```

So I am still not done with understanding this error! What is wrong with this last case? There is an analytic solution

```
f := dsolve(eqs,funcs);
```

$$f := \{x(t) = -3\,t + _C1,\ y(t) = -3 + 3\,t - _C1\}$$

Let's make sure this equation works:

x = -3t + a	y = -3 +3t - a
x(0) = a	y(0) = -3-a
$\dot{x} = -3$	$\dot{y} = 3$
$\ddot{x} = 0$	

Eq1: $\dot{x} = x + y$?

-3= -3t + a -3 +3t - a ?

-3= -3 ? yes

Eq2: $\ddot{x} = \dot{y}-3$?

0 = 3 - 3 ? yes

So the analytic solution is valid. But notice that you cannot set x(0) and y(0) independently!! Why is this? Those equations again are:

$$\dot{x} = x + y$$
$$\ddot{x} = \dot{y}-3$$

Rewrite as

$$\dot{y} = \ddot{x} + 3$$
$$y = \dot{x} - x$$

or

$$\dot{y} = \ddot{x} + 3$$
$$\dot{y} = \ddot{x} - \dot{x}$$

which implies that

$$\ddot{x} + 3 = \ddot{x} - \dot{x}$$
$$3 = - \dot{x}$$

So the pair of equations is really this

$$3 = - \dot{x}$$
$$y = \dot{x} - x$$

You solve the first one for x(t) and then y(t) is forced, <u>they are not independent functions</u>! But how do you know this in general from the equation system? This may involve some matrix thing that I don't know about. I can fix things by doing this instead

$$\dot{x} = x + y\dot{y}$$
$$\ddot{x} = \dot{y}-3$$

Then the error goes away. So if you just make up a random system of equations, it is not obvious that it has a general solution where you can set initial conditions in the usual manner.

I will now gather up the rules I learned during this debug:

RULE #1: Every equation must have at least one derivative!

RULE #2: Even if you system seems *obviously* first order, you can still get this error!

RULE #3: Elementary square roots are not a problem.

RULE #4 : All unknown functions to be solved for must have at least one derivative somewhere in the system of equations.