Scraps 1.14.17

It has taken a LONG TIME to get this point on 12.16.16,

Finally we get some oscillation which involves ω and involves a factor of 3. What did my PDF say about this? Here from p 124,

> Although the vertical orientation of the tether system is a stable one, there are forces which cause it to librate (oscillate) about the vertical. These weak but persistent forces include atmospheric drag due to the different air densities encountered in the northward and southward passes of non-equatorial orbits and due to solar heating and electrodynamic forces (for conducting tethers). Station-keeping and other rocket maneuvers would also contribute to driving (or damping) libration. The natural frequency for in-plane (in the orbit plane) librations is $\sqrt{3}\ \omega_o = 1.732\ \omega_o$, and $2\ \omega_o$ for out-of-plane librations (a detailed derivation is contained in Ref. 2).

So they are saying

$$\omega_{\mathtt{libration}} = \sqrt{3}\ \omega$$

and that agrees with my equation, hurray!!!

So you have to ignore the fictitious force to get this result! What is the condition for doing that? I show above that φ' = π/2 would do it, the planar case only!

What is the Ref 2 above?

2. Beletskii, V. V. and Levin, E. M., "Dynamics of Space Tether Systems," Advances in the Astronautical Sciences, Vol. 83.

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Maple Problem: Enter eq1 and eq2, eliminate z as usual, and see if Maple can do a numerical solution. Do this in "satellite in Cartesions v5.mws".

I am able to get it to fly, but the odeplot runs out of memory. Here is my code:

Enter the two equations of motion

```
eq1 := y(t)*diff(x(t),t,t) - x(t)*diff(y(t),t,t) = -w^2*y(t)*x(t)-2*w*zd*x(t);
```

$$eq1 := y(t)\left(\frac{\partial^2}{\partial t^2}x(t)\right) - x(t)\left(\frac{\partial^2}{\partial t^2}y(t)\right) = -w^2\, y(t)\, x(t) - 2\, w\, zd\, x(t)$$

```
eq2 := z(t)* diff(x(t),t,t) - x(t)*zdd =
(w^2*b^4/rpl^3)*x(t)-w^2*(b+z(t))*x(t)+2*w*diff(y(t),t)*x(t);
```

$$eq2 := z(t)\left(\frac{\partial^2}{\partial t^2}x(t)\right) - x(t)\, zdd = \frac{w^2\, b^4\, x(t)}{rpl^3} - w^2\,(b + z(t))\, x(t) + 2\, w\left(\frac{\partial}{\partial t}y(t)\right) x(t)$$

Are these correctly entered? Yes they are. Next, here is how to eliminate z:

Equations to eliminate z from the ODE system

```
> z := (t) -> sqrt(rl^2-x(t)^2-y(t)^2);
```

$$z := t \to \sqrt{rl^2 - x(t)^2 - y(t)^2}$$

```
> zd := diff(z(t),t);
```

$$zd := \frac{1}{2}\frac{-2\,x(t)\left(\frac{\partial}{\partial t}x(t)\right) - 2\,y(t)\left(\frac{\partial}{\partial t}y(t)\right)}{\sqrt{rl^2 - x(t)^2 - y(t)^2}}$$

```
> zdd := diff(z(t),t,t);
```

$$zdd := -\frac{1}{4}\frac{\left(-2\,x(t)\left(\frac{\partial}{\partial t}x(t)\right) - 2\,y(t)\left(\frac{\partial}{\partial t}y(t)\right)\right)^2}{\left(rl^2 - x(t)^2 - y(t)^2\right)^{\left(\frac{3}{2}\right)}} + \frac{1}{2}\frac{-2\left(\frac{\partial}{\partial t}x(t)\right)^2 - 2\,x(t)\left(\frac{\partial^2}{\partial t^2}x(t)\right) - 2\left(\frac{\partial}{\partial t}y(t)\right)^2 - 2\,y(t)\left(\frac{\partial^2}{\partial t^2}y(t)\right)}{\sqrt{rl^2 - x(t)^2 - y(t)^2}}$$

Next we have

Enter expression for r1p and enter constants

```
> r1p := r1^2+b^2+2*b*z(t);
```

$$r1p := rl^2 + b^2 + 2\,b\sqrt{rl^2 - x(t)^2 - y(t)^2}$$

```
> b := 1000:  r1 := 10:  w := 1:
```

This all looks good. Now we come to the dsolve call:

Setup and make the dsolve call

```
> eqs := {eq1,eq2}:
> funcs := {x(t),y(t)}:
> inits := {x(0)= 1,y(0) = 0, D(x)(0)=0,D(y)(0)=0};
```

$$inits := \{x(0) = 1, y(0) = 0, D(x)(0) = 0, D(y)(0) = 0\}$$

```
> f := dsolve(eqs union inits,funcs,type=numeric,method=classical,output=listprocedure);
```

$$f := \left[t = (\mathbf{proc}(t) \ \ldots \ \mathbf{end}), y(t) = (\mathbf{proc}(t) \ \ldots \ \mathbf{end}), \frac{\partial}{\partial t}y(t) = (\mathbf{proc}(t) \ \ldots \ \mathbf{end}), x(t) = (\mathbf{proc}(t) \ \ldots \ \mathbf{end}), \frac{\partial}{\partial t}x(t) = (\mathbf{proc}(t) \ \ldots \ \mathbf{end})\right]$$

There are only two equations, functions unknown are x(t) and y(t), and there are the inits. I start m_1 at location x = 0 and y = 0 and no velocity.

But then I ask if for f(0.1) and it locks up! So maybe I can try some other methods now. I started with classical and it hangs on f(0.1). I try rfk45 and get this

```
> f(0.1);
Error, (in dsolve/numeric/rkf45) cannot evaluate boolean
```

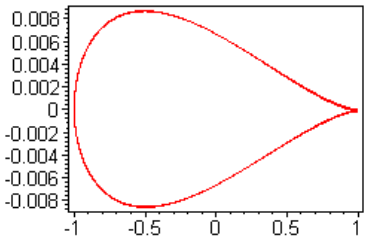
My error was that I had rp1 and r1p both present. Now it works better.

```
> f(0.1);
```

$$t(.1) = .1,\ y(t)(.1) = .002601172186851066,\ \left(\frac{\partial}{\partial t}y(t)\right)(.1) = .06422491379443354,\ x(t)(.1) = .5373794335159814,$$
$$\left(\frac{\partial}{\partial t}x(t)\right)(.1) = -8.476500496449882\Big]$$

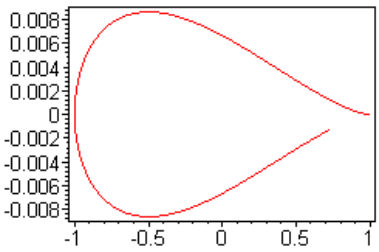
and I get finally some kind of semi-reasonable (perhaps) plot,

```
odeplot(f,[x(t),y(t)],0..2,numpoints=1000,axes = boxed);
```



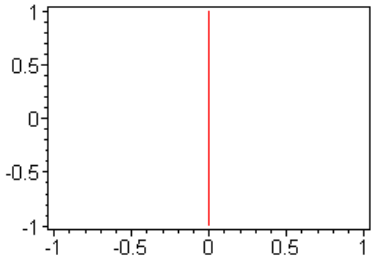
It starts at the right cusp and goes round the loop and repeat Here is just less than one loop

```
odeplot(f,[x(t),y(t)],0..0.55,numpoints=1000,axes = boxed);
```



I think as it starts to move, there is Coriolis pushing it to the right. This is I think a big breakthrough in my Cartesian satellite plotting efforts, even though I don't understand the result and I may have lots of math errors still. But it is a reasonable motion! This would be my "out of plane" motion, and I do expect a deviation.

Let's try x(0) = 0 and y(0) = 1, then I expect to get the "in plane" motion:



Here it stays in the x = 0 plane and swings back and forth. This looks VERY good, I will now take a break, it is 2:30 PM.

Now let's try for a circular path?

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$$r'_1{}^2 = r_1{}^2 + b^2 + 2bz = b^2 [\, 1 + 2(z/b) + (r_1/b)^2]$$

$$r'_1{}^{-2} = b^{-2} [\, 1 + 2(z/b) + (r_1/b)^2]^{-1}$$

$$r'_1{}^{-3} = b^{-3} [\, 1 + 2(z/b) + (r_1/b)^2]^{-3/2} \qquad x = [2(z/b) + (r_1/b)^2]$$

Now use

$$(1+x)^n = 1 + nx + n(n-1)/2\, x^2 + ...$$

$$(1+x)^{-3/2} = 1 + (-3/2)\, x + (-3/2)\,(-5/2)/2\, x^2 + ...$$

$$(1+[2(z/b) + (r_1/b)^2])^{-3/2} = 1 + (-3/2)\,[2(z/b) + (r_1/b)^2] + (-3/2)\,(-5/2)/2\,[2(z/b) + (r_1/b)^2]^2 + ...$$

$$= 1 + (-3/2)\, 2(z/b) + (-3/2)\,(r_1/b)^2 + (15/8) * 4(z/b)^2$$

$$= 1 - 3(z/b) - (3/2)\,(r_1/b)^2 + (15/2)(z/b)^2$$

so

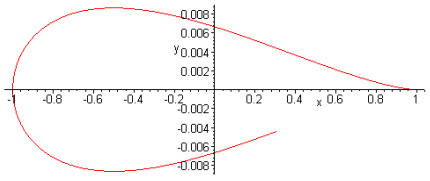
$$(b/r'_1)^3 = 1 - 3(z/b) - (3/2)\,(r_1/b)^2 + (15/2)(z/b)^2$$

and

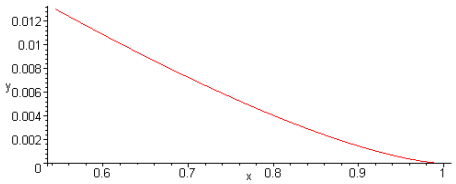
$$A \equiv (\omega^2 b^3/r'_1{}^3) \approx \omega^2[1 - 3(z/b) - (3/2)\,(r_1/b)^2 + (15/2)(z/b)^2]$$

This is accurate for A through second order!!

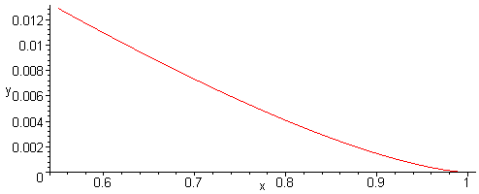
With full A I get this



With first order approx A I get this



With second order approx A I get this



There must be something wrong with my approximation for A!!!

r1p := r1^2+b^2+2*b*z(t);

Yes indeed, this is the wrong expression for r1p.

I am now keeping "the next term" for the moment. Then

$$z\ddot{x} - x\ddot{z} = (\omega^2 b^3/r'_1{}^3)bx - \omega^2(b+z)x + 2\omega x\dot{y}$$

$$= \omega^2[1 - 3(z/b) - (3/2)(r_1/b)^2]bx - \omega^2(b+z)x + 2\omega x\dot{y}$$

$$= [\omega^2 - 3\omega^2(z/b) - (3/2)\omega^2(r_1/b)^2]bx - \omega^2(b+z)x + 2\omega x\dot{y}$$

$$= \omega^2 bx - 3\omega^2(z/b)bx - (3/2)\omega^2(r_1/b)^2 bx - \omega^2(b+z)x + 2\omega x\dot{y}$$

$$= \omega^2 bx - 3\omega^2 zx - (3/2)\omega^2(r_1{}^2/b)x - \omega^2 bx - \omega^2 zx + 2\omega x\dot{y}$$

$$= -4\omega^2 zx - (3/2)\omega^2(r_1{}^2/b)x + 2\omega x\dot{y}$$

$$= -4\omega^2 zx + 2\omega x\dot{y} - (3/2)\omega^2(r_1{}^2/b)x$$

So there

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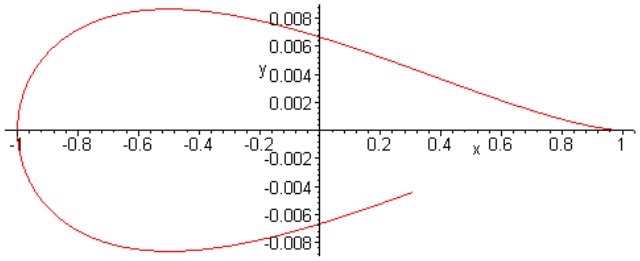
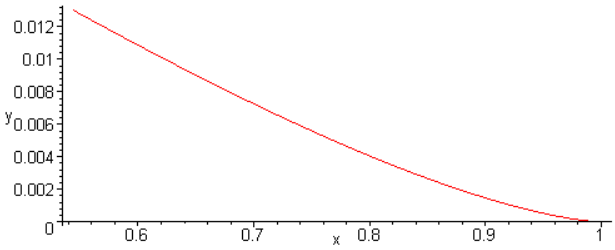
I am messing in v5 of the mws (OINLY!). With the full-bore A I get this

```
odeplot(f,[x(t),y(t)],0..0.5,numpoints=1000,axes = boxed,labels =
[x,y],thickness=1,axes=normal );
```



With the approximate A I get this

```
odeplot(f,[x(t),y(t)],0..0.5,numpoints=1000,axes = boxed,labels =
[x,y],thickness=1,axes=normal );
```



which is just the start of the previous curve. Why are these so very different? I thought my approximation for A was pretty good, but here it makes a huge difference in a trajectory!

v6:

```
eq2 := y(t)*xdd - x(t)*ydd = -w^2*x(t)*y(t)-2*w*x(t)*zd;
```

$$eq2 := y(t)\left(\frac{\partial^2}{\partial t^2}x(t)\right) - x(t)\left(\frac{\partial^2}{\partial t^2}y(t)\right) = -w^2\,x(t)\,y(t) - 2\,w\,x(t)\,zd$$

```
eq3 := z(t)* xdd - x(t)*zdd = -4*w^2*x(t)*z(t) + 2*w*x(t)*yd;
```

$$eq3 := z(t)\left(\frac{\partial^2}{\partial t^2}x(t)\right) - x(t)\,zdd = -4\,w^2\,x(t)\,z(t) + 2\,w\,x(t)\left(\frac{\partial}{\partial t}y(t)\right)$$