The Search for T PhL 1.12.17

Question: How can I get an estimate of T as I did in the angular world in (F.6.29)? There I had three equations in θ,φ and I used the first radial one to estimate T. In Cartesians I have this

$$\ddot{x} = -[(\omega^2 b^3/r'_1{}^3) + (T/m_1 r_1)]x$$

$$\ddot{y} = -[(\omega^2 b^3/r'_1{}^3) + (T/m_1 r_1) - \omega^2]y + 2\omega\dot{z}$$

$$\ddot{z} = -[(\omega^2 b^3/r'_1{}^3) - \omega^2]\ (b+z) - (T/m_1 r_1)z - 2\omega\ \dot{y}$$

$$x^2+y^2+z^2 = r_1^2 \qquad \text{(F.8.10)}$$

From (F.6.25) I know that

$$(b/r'_1)^3 \approx 1 - 3(r_1/b)\cos\theta \approx [1 - 3z/b]$$

so equations become

$$\ddot{x} = -[(\omega^2[1 - 3z/b]) + (T/m_1 r_1)]x$$

$$\ddot{y} = -[(\omega^2[1 - 3z/b]) + (T/m_1 r_1) - \omega^2]y + 2\omega\dot{z}$$

$$\ddot{z} = -[(\omega^2[1 - 3z/b]) - \omega^2]\ (b+z) - (T/m_1 r_1)z - 2\omega\ \dot{y}$$

First equation says

(I am looking for a result $T = 3m_1\ \omega^2 r_1\cos^2\theta = (3m_1\ \omega^2/r_1)r_1^2\cos^2\theta = (3m_1\ \omega^2/r_1)z^2$

$$m_1 r_1\ \ddot{x} = -m_1 r_1[(\omega^2[1 - 3z/b]) + Tx$$

$$m_1 r_1\ \ddot{x} = -m_1 r_1\omega^2 + m_1 r_1\omega^2 3(z/b) + Tx$$

I don't see my result coming out of this equation! I do know that

$$x^2+y^2+z^2 = r_1^2$$
$$x\dot{x} + y\dot{y} + z\dot{z} = 0$$
$$x\ddot{x} + \dot{x}^2 + y\ddot{y} + \dot{y}^2 + z\ddot{z} + \dot{z}^2 = 0$$

but that does not seem to help much. Suppose I drop the z/b terms in all three equations above:

$$\ddot{x} = -[\omega^2 + (T/m_1 r_1)]x$$

$$\ddot{y} = -[\omega^2 + (T/m_1r_1) - \omega^2]y + 2\omega\dot{z}$$

$$\ddot{z} = -(T/m_1r_1)z - 2\omega\,\dot{y}$$

again unclear what I can do here. Go back again to

$$\ddot{x} = -[(\omega^2b^3/r'_1{}^3) + (T/m_1r_1)]x$$

$$\ddot{y} = -[(\omega^2b^3/r'_1{}^3) + (T/m_1r_1) - \omega^2]y + 2\omega\dot{z}$$

$$\ddot{z} = -[(\omega^2b^3/r'_1{}^3) - \omega^2]\,(b+z) - (T/m_1r_1)z - 2\omega\,\dot{y}$$

$$x^2+y^2+z^2 = r_1^2 \qquad \text{(F.8.10)}$$

What happens if I eliminate the $(\omega^2b^3/r'_1{}^3)$ terms instead of eliminate T?

1 $\ddot{x} = -(A + B)x$

2 $\ddot{y} = -(A + B - \omega^2)y + 2\omega\dot{z}$

3 $\ddot{z} = -(A - \omega^2)\,(b+z) - Bz - 2\omega\,\dot{y}$

4 $x^2+y^2+z^2 = r_1^2 \qquad \text{(F.8.12)}$

When I try to eliminate A from 1 and 2, B also goes away and that gives what I already know

$$y\ddot{x} - x\ddot{y} = -\omega^2xy - 2\omega x\dot{z} \qquad \text{(F.8.13)}$$

Let's now try to eliminate A from 1 and 3

1*(b+z): $\ddot{x}(b+z) = -(A + B)x(b+z)$

3*x: $\ddot{z}x = -(A - \omega^2)\,(b+z)x - Bzx - 2\omega\,\dot{y}x$

$$= -A\,(b+z)x + \omega^2\,(b+z)x - Bzx - 2\omega\,\dot{y}x$$

Now subtract to get

$$\ddot{x}(b+z) - \ddot{z}x = -(A + B)x(b+z) + A\,(b+z)x - \omega^2\,(b+z)x + Bzx + 2\omega\,\dot{y}x$$

$$= -Ax(b+z) - Bx(b+z) + A(b+z)x - \omega^2 (b+z)x + Bzx + 2\omega \dot{y}x$$

$$= - Bx(b+z) - \omega^2 (b+z)x + Bzx + 2\omega \dot{y}x$$

$$= - Bxb - Bxz - \omega^2 (b+z)x + Bzx + 2\omega \dot{y}x$$

$$= - Bxb - \omega^2 (b+z)x + 2\omega \dot{y}x$$

and so B manages to survive. I then have

$$\ddot{x}(b+z) - \ddot{z}x = - Bxb - \omega^2 (b+z)x + 2\omega \dot{y}x$$

or

$$\ddot{x}b + \ddot{x}z - \ddot{z}x = - Bxb - \omega^2 (b+z)x + 2\omega \dot{y}x \qquad (*)$$

But I also know that

$$z\ddot{x} - x\ddot{z} = Abx - \omega^2(b+z)x + 2\omega x\dot{y}$$

so equation (*) becomes

$$Abx - \omega^2(b+z)x + 2\omega x\dot{y} + \ddot{x}b = - Bxb - \omega^2 (b+z)x + 2\omega \dot{y}x$$

or

$$Abx + \ddot{x}b = - Bxb$$

or

$$Ax + \ddot{x} = - Bx$$

or

$$\ddot{x} = - (A+B)x$$ around in a circle of course.

So unlike in spherical coordinates, there seems to be no easy want to extract an approximate expression for T because it seems you need the radial equation to do that!

<u>What happens if I just insert my known result for T into these equations?</u>

$$\ddot{x} = - [(\omega^2 b^3/r'_1{}^3) + (T/m_1 r_1)]x$$

$$\ddot{y} = - [(\omega^2 b^3/r'_1{}^3) + (T/m_1 r_1) - \omega^2]y + 2\omega\dot{z}$$

$$\ddot{z} = - [(\omega^2 b^3/r'_1{}^3) - \omega^2] (b+z) - (T/m_1 r_1)z - 2\omega \dot{y}$$

$$x^2+y^2+z^2 = r_1^2 \qquad (F.8.10)$$

$T = 3m_1 \omega^2 r_1 \cos^2\theta = (3m_1 \omega^2/r_1) r_1^2 \cos^2\theta = (3m_1 \omega^2/r_1) z^2$ dim$T = M\,T^{-2}L$ = ma correct

$(T/m_1 r_1) = 3\omega^2 (z/r_1)^2$ $T^{-2} = T^{-2}$

Go back instead to

$$\ddot{x} = -[(\omega^2[1 - 3z/b]) + (T/m_1 r_1)]x$$

$$\ddot{y} = -[(\omega^2[1 - 3z/b]) + (T/m_1 r_1) - \omega^2]y + 2\omega\dot{z}$$

$$\ddot{z} = -[(\omega^2[1 - 3z/b]) - \omega^2]\,(b+z) - (T/m_1 r_1)z - 2\omega\,\dot{y}$$

and insert $(T/m_1 r_1) = 3\omega^2 (z/r_1)^2$ to get

$$\ddot{x} = -[(\omega^2[1 - 3z/b]) + 3\omega^2 (z/r_1)^2]x$$

$$\ddot{y} = -[(\omega^2[1 - 3z/b]) + 3\omega^2 (z/r_1)^2 - \omega^2]y + 2\omega\dot{z}$$

$$\ddot{z} = -[(\omega^2[1 - 3z/b]) - \omega^2]\,(b+z) - 3\omega^2 (z/r_1)^2 z - 2\omega\,\dot{y}$$

Now drop the z/b terms where appropriate

$$\ddot{x} = -[(\omega^2[1]) + 3\omega^2 (z/r_1)^2]x$$

$$\ddot{y} = -[-3\omega^2 z/b + 3\omega^2 (z/r_1)^2]y + 2\omega\dot{z}$$

$$\ddot{z} = -[-3\omega^2 z/b]\,(b+z) - 3\omega^2 (z/r_1)^2 z - 2\omega\,\dot{y}$$

or

$$\ddot{x} = -[(\omega^2 + 3\omega^2 (z/r_1)^2]x$$

$$\ddot{y} = -[-3\omega^2 z/b + 3\omega^2 (z/r_1)^2]y + 2\omega\dot{z}$$

$$\ddot{z} = -[-3\omega^2 z/b]\,b - 3\omega^2 (z/r_1)^2 z - 2\omega\,\dot{y}$$

or

$$\ddot{x} = -\omega^2[(1 + 3(z/r_1)^2]x$$

$$\ddot{y} = -3\omega^2[-z/b + (z/r_1)^2]y + 2\omega\dot{z}$$

$$\ddot{z} = 3\omega^2 z - 3\omega^2 (z/r_1)^2 z - 2\omega \dot{y}$$

or

$$\ddot{x} = -\omega^2[(1+3(z/r_1)^2]x$$

$$\ddot{y} = -3\omega^2 (z/r_1)^2 y + 2\omega \dot{z}$$

$$\ddot{z} = 3\omega^2 z[1 - (z/r_1)^2] - 2\omega \dot{y}$$

We know that

$$x\ddot{x} + \dot{x}^2 + y\ddot{y} + \dot{y}^2 + z\ddot{z} + \dot{z}^2 = 0$$

but if velocities are small we can say

$$x\ddot{x} + y\ddot{y} + z\ddot{z} \approx 0$$

Insert now from above (with velocities ignored) to check this result

$$x\ddot{x} + y\ddot{y} + z\ddot{z}$$

$$= x\{-\omega^2[(1+3(z/r_1)^2]x\} + y\{-3\omega^2(z/r_1)^2 y\} + z\{3\omega^2 z[1 - (z/r_1)^2]\}$$

$$= -\omega^2[(1+3(z/r_1)^2]x^2 - 3\omega^2(z/r_1)^2 y^2 + 3\omega^2[1 - (z/r_1)^2]\, z^2$$

$$= -\omega^2 x^2 - 3\omega^2(z/r_1)^2 x^2 - 3\omega^2(z/r_1)^2 y^2 + 3\omega^2 z^2 - 3\omega^2 (z/r_1)^2 z^2$$

$$= -\omega^2 x^2 + 3\omega^2 z^2 - 3\omega^2(z/r_1)^2[\, x^2+y^2+z^2]$$

$$= -\omega^2 x^2 + 3\omega^2 z^2 - 3\omega^2(z/r_1)^2[r_1^2]$$

$$= -\omega^2 x^2 + 3\omega^2 z^2 - 3\omega^2 z^2$$

$$= -\omega^2 x^2$$ // same result I got last time.

so it almost works! Very unclear.

$$x^2+y^2+z^2 = r_1^2$$

$$x\dot{x} + y\dot{y} + z\dot{z} = 0$$

$$x\ddot{x} + \dot{x}^2 + y\ddot{y} + \dot{y}^2 + z\ddot{z} + \dot{z}^2 = 0$$