This is the Title PhL 1.11.15

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$\mathbf{e}_n(\mathbf{\Phi}) = R(\mathbf{\Phi})\mathbf{e}'_n$

$|\mathbf{e}_n(\mathbf{\Phi})\rangle = |R(\mathbf{\Phi})\mathbf{e}'_n\rangle$

$|\mathbf{e}_n(\mathbf{\Phi})\rangle = \mathcal{R}(\mathbf{\Phi})|\mathbf{e}'_n\rangle = |\mathbf{e}'_i\rangle\langle\mathbf{e}'_i|\mathcal{R}(\mathbf{\Phi})|\mathbf{e}'_n\rangle = |\mathbf{e}'_i\rangle [R(\mathbf{\Phi})]'_{in} = |\mathbf{e}'_i\rangle [R(\mathbf{\Phi})]_{in}$

$|\mathbf{e}_n(\mathbf{\Phi}+d\mathbf{\Phi})\rangle = \mathcal{R}(\mathbf{\Phi}+d\mathbf{\Phi})|\mathbf{e}'_n\rangle = |\mathbf{e}'_i\rangle\langle\mathbf{e}'_i|\mathcal{R}(\mathbf{\Phi}+d\mathbf{\Phi})|\mathbf{e}'_n\rangle = |\mathbf{e}'_i\rangle [R(\mathbf{\Phi}+d\mathbf{\Phi})]'_{in} = |\mathbf{e}'_i\rangle [R(\mathbf{\Phi}+d\mathbf{\Phi})]_{in}$

$|(d\mathbf{e}_n(\mathbf{\Phi}))_{S'}\rangle \equiv |\mathbf{e}_n(\mathbf{\Phi}+d\mathbf{\Phi})\rangle - |\mathbf{e}_n(\mathbf{\Phi})\rangle$

$= |\mathbf{e}'_i\rangle [R(\mathbf{\Phi}+d\mathbf{\Phi})]_{in} - |\mathbf{e}'_i\rangle [R(\mathbf{\Phi})]_{in}$

$= (R(\mathbf{\Phi}+d\mathbf{\Phi})]_{in} - [R(\mathbf{\Phi})]_{in}) |\mathbf{e}'_i\rangle$

Then

$\langle \mathbf{e}'_j |(d\mathbf{e}_n(\mathbf{\Phi}))_{S'}\rangle = (R(\mathbf{\Phi}+d\mathbf{\Phi})]_{in} - [R(\mathbf{\Phi})]_{in}) \langle \mathbf{e}'_j |\mathbf{e}'_i\rangle$

$= (R(\mathbf{\Phi}+d\mathbf{\Phi})]_{jn} - [R(\mathbf{\Phi})]_{jn})$

$= [(d\mathbf{e}_n(\mathbf{\Phi}))_{S'}]'_j$

So here suddenly I am getting this result

$[(d\mathbf{e}_n(\mathbf{\Phi}))_{S'}]'_j = R(\mathbf{\Phi}+d\mathbf{\Phi})]_{jn} - [R(\mathbf{\Phi})]_{jn}$

where the matrices on the right are the unprimed matrices! The only facts I used above were

$\langle\mathbf{e}'_i|\mathcal{R}(\mathbf{\Phi})|\mathbf{e}'_n\rangle = \langle\mathbf{e}'_i|\mathbf{e}_j\rangle\langle\mathbf{e}_j|\mathcal{R}(\mathbf{\Phi})|\mathbf{e}_k\rangle\langle\mathbf{e}_k|\mathbf{e}'_n\rangle$

$= R(\mathbf{\Phi})_{ij} R(\mathbf{\Phi})_{jk} R^{-1}(\mathbf{\Phi})_{kn} = R(\mathbf{\Phi})_{in}$

$[R(\mathbf{\Phi}+d\mathbf{\Phi})]'_{jk} = \langle\mathbf{e}'_i|\mathcal{R}(\mathbf{\Phi}+d\mathbf{\Phi})|\mathbf{e}'_n\rangle = \langle\mathbf{e}'_i|\mathbf{e}_j\rangle\langle\mathbf{e}_j|\mathcal{R}(\mathbf{\Phi}+d\mathbf{\Phi})|\mathbf{e}_k\rangle\langle\mathbf{e}_k|\mathbf{e}'_n\rangle$

$= R(\mathbf{\Phi})_{ij} R(\mathbf{\Phi}+d\mathbf{\Phi})_{jk} R^{-1}(\mathbf{\Phi})_{kn} = ????$

This is the Catch 22! Now if you think of

Back up, show time dependence t on things better.

$\mathbf{e}_n(\mathbf{\Phi}(t)) = R(\mathbf{\Phi}(t))\mathbf{e}'_n$

$|\mathbf{e}_n(\mathbf{\Phi}(t))\rangle = |R(\mathbf{\Phi}(t))\mathbf{e}'_n\rangle$

$\mathbf{1} = |\mathbf{e}'_i\rangle\langle\mathbf{e}'_i| = |\mathbf{e}_i(\mathbf{\Phi}(t))\rangle\langle\mathbf{e}_i(\mathbf{\Phi}(t))|$

so you think of this completeness AT ANY TIME t. Then you get

$|\mathbf{e}_n(\mathbf{\Phi}(t))\rangle = |R(\mathbf{\Phi}(t))\mathbf{e}'_n\rangle$

$\langle\mathbf{e}'_m|\mathbf{e}_n(\mathbf{\Phi}(t))\rangle = \langle\mathbf{e}'_m|R(\mathbf{\Phi}(t))\mathbf{e}'_n\rangle = \langle\mathbf{e}'_m|\mathcal{R}(\mathbf{\Phi}(t))|\mathbf{e}'_n\rangle = R'_{mn}(\mathbf{\Phi}(t))$

Then it follows that

$\langle\mathbf{e}'_m|\mathbf{e}_n(\mathbf{\Phi}(t)+d\mathbf{\Phi})\rangle = R'_{mn}(\mathbf{\Phi}(t) + d\mathbf{\Phi})$

and maybe this fixes the Catch 22 !!

Try this without the clutter of showing t :

$\langle\mathbf{e}'_m|\mathbf{e}_n(\mathbf{\Phi})\rangle = R'_{mn}(\mathbf{\Phi})$

$\langle\mathbf{e}'_m|\mathbf{e}_n(\mathbf{\Phi}+d\mathbf{\Phi})\rangle = R'_{mn}(\mathbf{\Phi}+d\mathbf{\Phi})$

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Let's now try to present this to the Critic Listener. Consider that viewed from Frame S' the Frame S basis vectors are *moving* and so

$\mathbf{e}_n(\mathbf{\Phi}) = R(\mathbf{\Phi})\mathbf{e}'_n$

or

$|\mathbf{e}_n(\mathbf{\Phi})\rangle = |R(\mathbf{\Phi})\mathbf{e}'_n\rangle$. (*)

Completeness can be written

$\mathbf{1} = |\mathbf{e}'_i\rangle\langle\mathbf{e}'_i|$ in the fixed Frame S' basis

$\mathbf{1} = |\mathbf{e}_n(\mathbf{\Phi})\rangle\langle\mathbf{e}_n(\mathbf{\Phi})|$ for any $\mathbf{\Phi}$ in the moving Frame S basis

For example, at a slightly later time when $\mathbf{\Phi}$ has changed to $\mathbf{\Phi}+d\mathbf{\Phi}$ we get

$\mathbf{1} = |\,\mathbf{e}_n(\mathbf{\Phi}+d\mathbf{\Phi})\rangle\langle\,\mathbf{e}_n(\mathbf{\Phi}+d\mathbf{\Phi})\,|$

Close (*) on the left with the non-moving $\langle \mathbf{e}'_m |$ to get

$$\langle \mathbf{e}'_m \mid \mathbf{e}_n(\mathbf{\Phi})\rangle = \langle \mathbf{e}'_m \mid R(\mathbf{\Phi})\mathbf{e}'_n\rangle = \langle \mathbf{e}'_m \mid \mathcal{R}(\mathbf{\Phi})\mid \mathbf{e}'_n\rangle = [R(\mathbf{\Phi})]'_{mn}$$

Then at some slightly later time where $\mathbf{\Phi}$ has changed to $\mathbf{\Phi}+d\mathbf{\Phi}$ we can write

$$\langle \mathbf{e}'_m \mid \mathbf{e}_n(\mathbf{\Phi}+d\mathbf{\Phi})\rangle = \langle \mathbf{e}'_m \mid R(\mathbf{\Phi}+d\mathbf{\Phi})\mathbf{e}'_n\rangle = \langle \mathbf{e}'_m \mid \mathcal{R}(\mathbf{\Phi}+d\mathbf{\Phi})\mid \mathbf{e}'_n\rangle = [R(\mathbf{\Phi}+d\mathbf{\Phi})]'_{mn}$$

This all seems just fine. Now consider

$$[R(\mathbf{\Phi})]'_{mn} = \langle \mathbf{e}'_m \mid \mathcal{R}(\mathbf{\Phi})\mid \mathbf{e}'_n\rangle = \langle \mathbf{e}'_m \mid \mathbf{e}_i(\mathbf{\Phi})\rangle\langle \mathbf{e}_i(\mathbf{\Phi})\mid \mathcal{R}(\mathbf{\Phi}) \mid \mathbf{e}_j(\mathbf{\Phi})\rangle\langle \mathbf{e}_j(\mathbf{\Phi})\mid \mathbf{e}'_n\rangle$$

$$= [R(\mathbf{\Phi})]_{mi}\,[R(\mathbf{\Phi})]_{ij}\,[R^{-1}(\mathbf{\Phi})]_{jn} = [R(\mathbf{\Phi})R(\mathbf{\Phi})R^{-1}(\mathbf{\Phi})]_{mn} = [R(\mathbf{\Phi})]_{mn}$$

At a later time we replace $\mathbf{\Phi}$ by $\mathbf{\Phi}+d\mathbf{\Phi}$ everywhere in the above line to get

$$[R(\mathbf{\Phi}+d\mathbf{\Phi})]'_{mn} = [R(\mathbf{\Phi}+d\mathbf{\Phi})]_{mn}$$

Therefore we have shown that

$$\langle \mathbf{e}'_m \mid \mathbf{e}_n(\mathbf{\Phi})\rangle = [R(\mathbf{\Phi})]'_{mn} = [R(\mathbf{\Phi})]_{mn}$$

$$\langle \mathbf{e}'_m \mid \mathbf{e}_n(\mathbf{\Phi}+d\mathbf{\Phi})\rangle = [R(\mathbf{\Phi}+d\mathbf{\Phi})]'_{mn} = [R(\mathbf{\Phi}+d\mathbf{\Phi})]_{mn}$$

Now consider

$$|(d\mathbf{e}_n(\mathbf{\Phi}))_{S'}\rangle \equiv |\mathbf{e}_n(\mathbf{\Phi}+d\mathbf{\Phi})\rangle - |\mathbf{e}_n(\mathbf{\Phi})\rangle$$

Close with $\langle \mathbf{e}'_m |$ to get

$$\langle \mathbf{e}'_m |(d\mathbf{e}_n(\mathbf{\Phi}))_{S'}\rangle \equiv \langle \mathbf{e}'_m |\mathbf{e}_n(\mathbf{\Phi}+d\mathbf{\Phi})\rangle - \langle \mathbf{e}'_m |\mathbf{e}_n(\mathbf{\Phi})\rangle$$

$$= [R(\mathbf{\Phi}+d\mathbf{\Phi})]_{mn} - [R(\mathbf{\Phi})]_{mn}$$

This says that

$$[(d\mathbf{e}_n(\mathbf{\Phi}))_{S'}]'_m = [R(\mathbf{\Phi}+d\mathbf{\Phi})]_{mn} - [R(\mathbf{\Phi})]_{mn} = (dR)_{mn}$$

Even though we are taking a Frame S' component on the left, we end up with the Frame S standard rotation matrices on the right!

What happens if you instead take a Frame S component?

$$\langle \mathbf{e}_m(\mathbf{\Phi}) |(d\mathbf{e}_n(\mathbf{\Phi}))_{S'}\rangle \equiv \langle \mathbf{e}_m(\mathbf{\Phi}) |\mathbf{e}_n(\mathbf{\Phi}+d\mathbf{\Phi})\rangle - \langle \mathbf{e}_m(\mathbf{\Phi}) |\mathbf{e}_n(\mathbf{\Phi})\rangle$$

$= \langle \mathbf{e}_m(\mathbf{\Phi}) | \mathbf{e'}_i \rangle \langle \mathbf{e'}_i | \mathbf{e}_n(\mathbf{\Phi}+d\mathbf{\Phi}) \rangle - \langle \mathbf{e}_m(\mathbf{\Phi}) | \mathbf{e'}_i \rangle \langle \mathbf{e'}_i | \mathbf{e}_n(\mathbf{\Phi}) \rangle$

$= [R(\mathbf{\Phi})]_{im} [R(\mathbf{\Phi}+d\mathbf{\Phi})]_{in} - [R(\mathbf{\Phi})]_{im} [R(\mathbf{\Phi})]_{in}$

$= [R(\mathbf{\Phi})]_{im} \{ R(\mathbf{\Phi}+d\mathbf{\Phi}) - R(\mathbf{\Phi}) \}_{in}$

$= [R(\mathbf{\Phi})]_{im} \{dR\}_{in}$

So this is a different result !!!!!!!!!!!!!

$= [R(\mathbf{\Phi}+d\mathbf{\Phi})]_{mn} - [R(\mathbf{\Phi})]_{mn}$