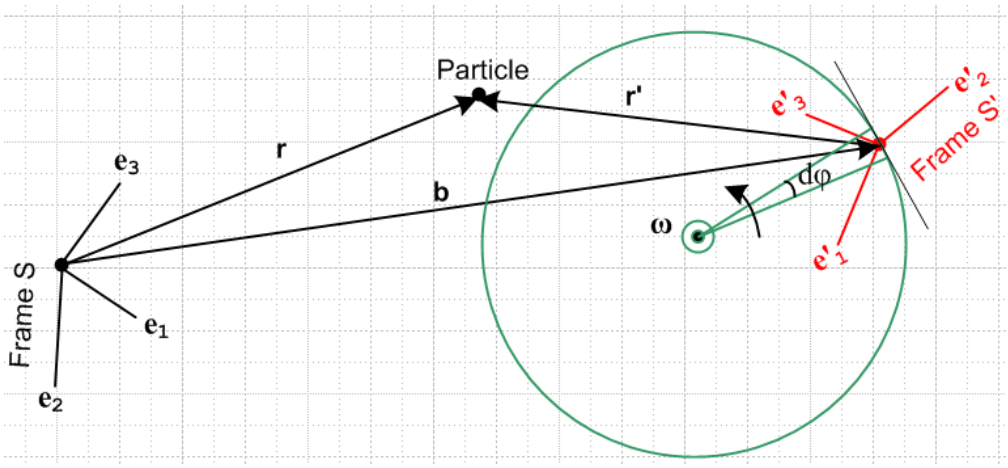
Paradox with the vector ω PhL 2.12.17

I think this paradox gets resolved within this doc, so no paradox.

On the One Hand:

Consider this picture



where Frame S is inertial. Suppose it happened that $\mathbf{e}_1$ is coming out of the plane of paper on the left. Then you would write $\boldsymbol{\omega} = \omega\mathbf{e}_1$ and then $\boldsymbol{\omega} = (\omega,0,0)$ and these are the components of $\boldsymbol{\omega}$ in Frame S.

If you treat $\boldsymbol{\omega}$ like any other vector, you could ask:
"What are the components of $\boldsymbol{\omega}$ as measured in Frame S' "?

Since the axes of Frame S' are at some weird orientation, the answer will be:
"the components of vector $\boldsymbol{\omega}$ as seen from Frame S' are complicated".

More formally, suppose $\mathbf{e}_i = R(\boldsymbol{\Phi})\mathbf{e}'_i$ and suppose $\boldsymbol{\omega} = \omega\mathbf{e}_1$. Then

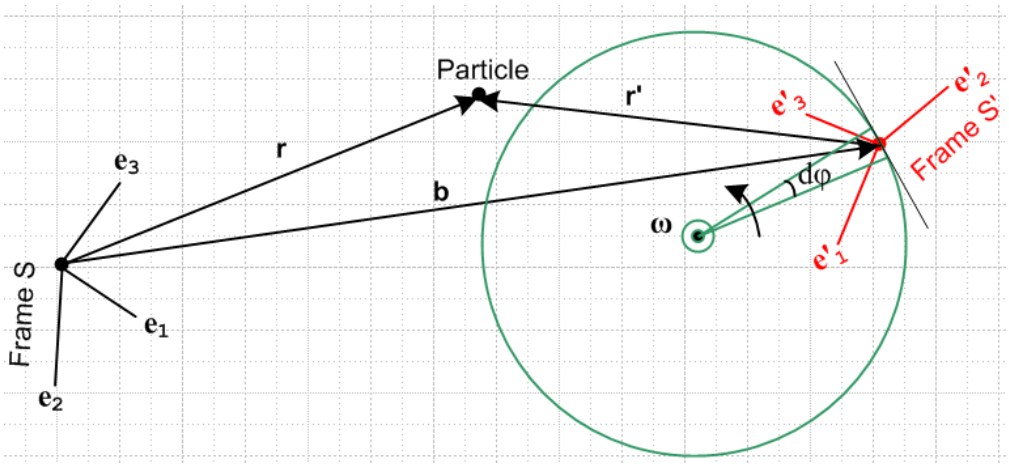
$$(\omega)'_i = \boldsymbol{\omega} \bullet \mathbf{e}'_i = \boldsymbol{\omega} \bullet [R(\boldsymbol{\Phi})^{-1}\mathbf{e}_i] = [R(\boldsymbol{\Phi})\boldsymbol{\omega}] \bullet \mathbf{e}_i = [R(\boldsymbol{\Phi})\boldsymbol{\omega}]_i = [R(\boldsymbol{\Phi})]_{ij}\,\omega_j$$

$$= [R(\boldsymbol{\Phi})]_{ij}\,\omega\,\delta_{j,1} = \omega\,[R(\boldsymbol{\Phi})]_{i1}$$

and in general this is some complicated expression for each of the $(\omega)'_i$ components.

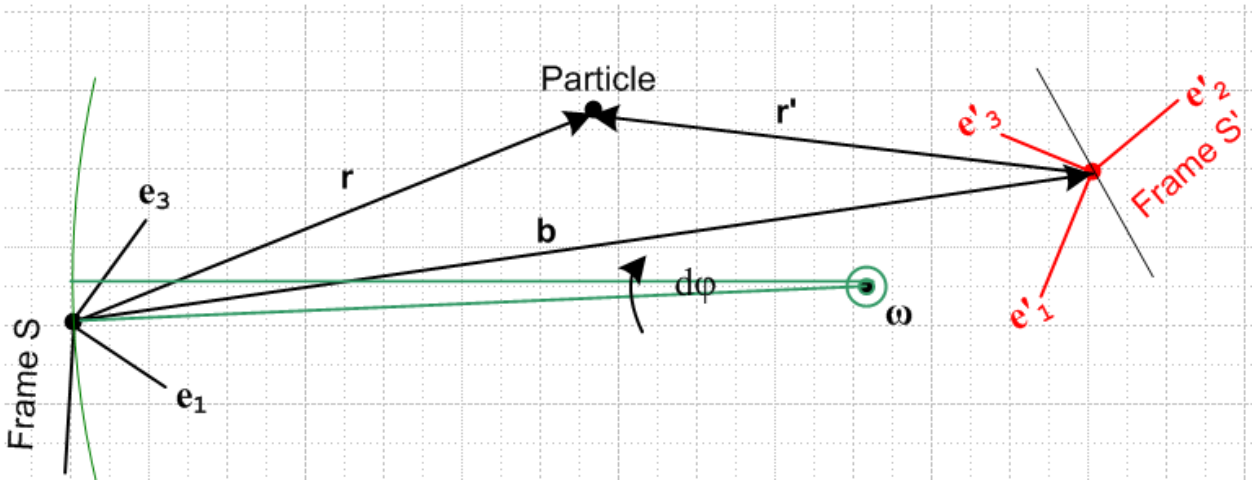
On the Other Hand:

Start again with the above picture



Here you would say: The axes of Frame S' are rotating at **ω** = (ω,0,0) as seen from Frame S.

Now hop onto a camera platform which is rotating CCW at **ω** about the same axis and this picture becomes



Now Frame S' is at rest and Frame S is rotating in effect at **-ω** . So you would say

The axes of Frame S as seen from Frame S' are rotating at **-ω** = (-ω,0,0)

Resolution:

The second item I think is valid, and you end up with the statement as claimed,

The axes of Frame S as seen from Frame S' are rotating at **-ω** = (-ω,0,0).

However, the components shown of **-ω** = (-ω,0,0) are still Frame S components, not Frame S' components. The Frame S' components are still something messy.

I jump to Goldstein rigid body here because he is going to use ω components in Frame S' !!

I work further with this stuff in some other doc.

What is the meaning of the ω_i in the GPS presentation of rotating objects?

$$\left(\frac{d\mathbf{L}}{dt}\right)_s = \mathbf{N}. \qquad \left(\frac{d\mathbf{L}}{dt}\right)_s = \left(\frac{d\mathbf{L}}{dt}\right)_b + \boldsymbol{\omega} \times \mathbf{L}.$$

or, by dropping the "body" subscript:

$$\frac{d\mathbf{L}}{dt} + \boldsymbol{\omega} \times \mathbf{L} = \mathbf{N}. \qquad (5.37)$$

So what is the meaning of $\boldsymbol{\omega}$ in this equation (5.37) ? The equation says

$(d\mathbf{L}/dt)_{S'} + \boldsymbol{\omega} \times \mathbf{L} = \mathbf{N}$

Since this is a vector equation, we have not yet talked about "components", so we keep going. Now GPS is going to take the components of the above equation!!! This is a tricky operation. If we do this within Frame S' (the rotating frame) then my Commutation Theorem (1.11.1) shows that things commute, so you can say

$(\partial_{S'}\mathbf{L})'_i = \partial_{S'}[\,(L)'_i\,] = \partial_t(L)'_i$ // using below 1.11.1

Then we really should write rigid body doc

$$\partial_t(L)'_i + [\,\boldsymbol{\omega} \times \mathbf{L}]'_i = N'_i$$

or

$$d(L)'_i/dt + \varepsilon_{ijk}(\omega)'_j(L)'_k = (N)'_i$$

and indeed it is the Frame S' components that you want!! This last equation appears as

$$\frac{dL_i}{dt} + \epsilon_{ijk}\omega_j L_k = N_i.$$

and you see how they are using a very compact notation. But I agree, their ω_i really are the Frame S' components!!!