This is the Title PhL 1.11.15

Note that page numbering is turned on in this template and view is 125%,
located in phil/roaming/microsoft/templates size about 219K.

Footnote. A vector equation valid only in Frame S' so cannot evaluate in Frame S.

Write

$$\mathbf{e}_n(t) = R(t)\mathbf{e}'_n(t)$$

at time t which may be evaluated in any Frame. Then

$$d\mathbf{e}_n(t) \equiv \mathbf{e}_n(t+dt) - \mathbf{e}_n(t)$$

$$= R(t+dt)\mathbf{e}'_n(t+dt) - R(t)\mathbf{e}'_n(t)$$

$$\approx [\, R(t)\,(\partial_t \mathbf{e}'_n(t)) + (\partial_t R(t))\mathbf{e}'_n(t)\,]\, dt \quad . \qquad \text{// neglect order } (dt)^2 \text{ etc .}$$

so

$$(d\mathbf{e}_n(t)/dt) = R(t)\,[\partial_t \mathbf{e}'_n(t)] + [\partial_t R(t)]\,\mathbf{e}'_n(t) \quad .$$

This vector equation is valid in any frame. For example.

$$[(d\mathbf{e}_n(t)/dt)]_i = R(t)_{ij}[(\partial_t \mathbf{e}'_n(t))]_j + [\partial_t R(t)]_{ij}\,(\mathbf{e}'_n(t))_j \qquad \text{// Frame S}$$

$$[(d\mathbf{e}_n(t)/dt)]'_i = R'(t)_{ij}[(\partial_t \mathbf{e}'_n(t))]'_j + [\partial_t R(t)]'_{ij}\,(\mathbf{e}'_n(t))'_j \; . \qquad \text{// Frame S'}$$

If we set $\partial_t \mathbf{e}'_n(t) = 0$ for all t, then we are *assuming* we are evaluating in Frame S' and then (G.7.22) becomes

$$(d\mathbf{e}_n(t)/dt)_{S'} = (\partial_t R(t))\mathbf{e}'_n(t) \qquad \text{// vector equation valid only in Frame S'}$$

$$[(d\mathbf{e}_n(t)/dt)_{S'}]'_i = [\partial_t R(t)]'_{ij}\,(\mathbf{e}'_n(t))'_j \; . \qquad \text{// OK, evaluated in Frame S'}$$

$$[(d\mathbf{e}_n(t)/dt)_{S'}]_i = [\partial_t R(t)]_{ij}\,(\mathbf{e}'_n(t))_j \; . \qquad \text{// NOT OK, evaluated in Frame S [why not OK?]}$$

Finally,

$$[\partial_t R(t)]' = R'(t+dt) - R'(t) = R(t+dt) - R(t) = dR = dR'$$

since for example $R'(t) = R(t)R(t)R^{-1}(t) = R(t)$ as in (1.1.35). We end up then with

$(d\mathbf{e}_n/dt)_{S'} = (dR'/dt)\ \mathbf{e}'_n(t)$ where $dR' = R(t+dt) - R(t)$ // compare (G.7.5)

In Frame S' evaluation this says

$[(d\mathbf{e}_n/dt)_{S'}]'_i = (dR'/dt)_{ij}\ (\mathbf{e}'_n(t))_j$

Later we define. $A' \equiv [(dR'/dt)\ R^{-1}(t)]$ with $A'_{ij} = (dR'/dt)_{ik}\ [R^{-1}(t)]_{kj}$